TEENAGE GIRLS AND MATHEMATICS

Gudbjörg Pálsdóttir, Assistant Professor, Mathematics Education
Iceland University of Education

Increased attention has been focused on research on beliefs about mathematics and mathematics education and it has become one of the central elements of study in mathematics education. A qualitative research study was made on the beliefs of four Icelandic girls about mathematics, the study of mathematics, and themselves as learners of mathematics in the year 2003 and again in the year 2008. Their descriptions and thoughts are viewed in the light of theories and newfound results of overseas quantitative research on girls´ beliefs about mathematics and the study of mathematics. The main conclusions of this research are that these girls:

- *view mathematics as a process*
- *place emphasis on understanding and solving the problems at hand*
- *are self-confident, well organized and study hard*
- *seldom use elaboration strategies.*

INTRODUCTION

Mathematics is one of the main subjects in primary and lower secondary school. In the upper secondary school students can choose how many courses in mathematics they take. Extensive research has been made in the field of mathematics education in the last thirty years. The research has contributed to the understanding of how people learn and has given ideas in the process of developing new ways to organise teaching. Research in this area has drawn people's attention towards the influence of beliefs on how people learn and the outcome of the learning. I have been a lower secondary school teacher for several years and became interested in learning about beliefs and their importance. I wondered how much I knew about the beliefs of my students over the years and discovered that my knowledge is limited. My main effort had been to get to know my students as individuals and analyse their mathematical knowledge. Therefore, I was interested in deepening my knowledge on some of the already existing research on beliefs and to conduct my own research.

My main research question was:

> How do students in lower secondary schools think about mathematics and their mathematical learning?

This research question was divided into three areas, the beliefs about mathematics, the study of mathematics and the students themselves as learners of mathematics. My

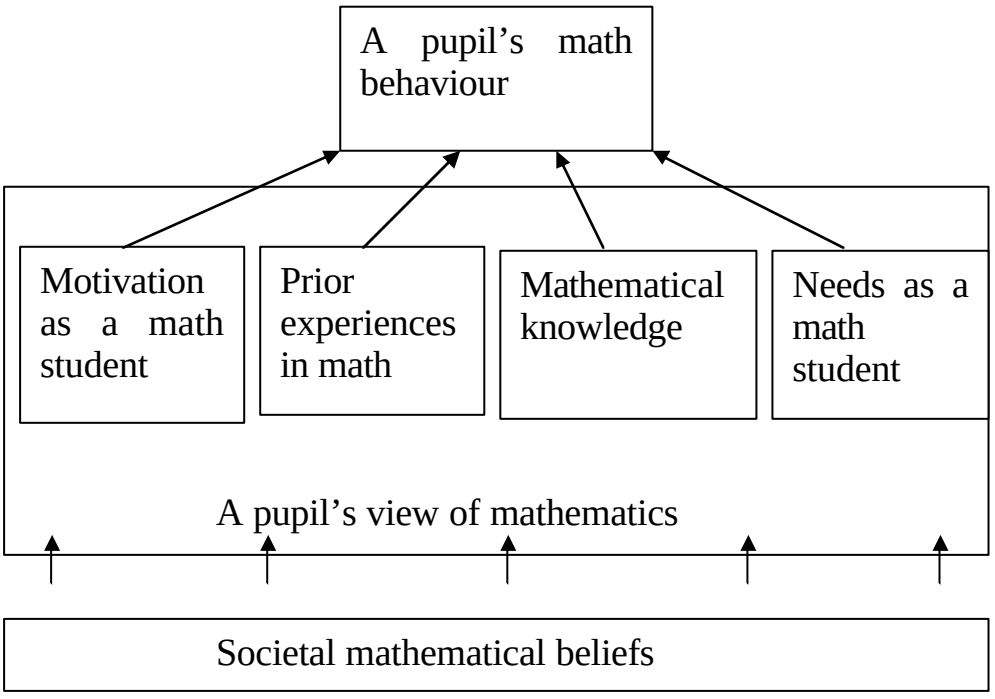
experience had taught me that teenage girls often ask many questions about learning and their interest in mathematics is decreasing. Consequently, I decided to focus on the beliefs of only a few girls. A qualitative research method was used and the study was based on individual interviews with four Icelandic girls. They were all in their final year of lower secondary school (Pálsdóttir 2004:9). This year I have been interviewing the same girls again and have asked them the same questions and some following up questions. I will in my presentation introduce my findings from the interviews for this year where I focus on in what way the girls beliefs have developed and what they analyse as an influence on their beliefs.

The following definition of the concept beliefs is used in the study.

> Students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine in close interaction with each other and with students' prior knowledge their mathematical learning and problem solving in class.
>
> Op 't Eynde, P., Corte E. de & Verschaffel L. 2002:27

THEORETICAL BACKGROUND

Since the seventies, great interest has been shown in the study of students' beliefs and ideas. It can be expected that this research-interest is based on the attitude that beliefs influence how people understand themselves and their surroundings, and how they deal with their lives.



(Pehkonen & Safuanov 1996:34)

The main view is that on the basis of ideas on a specific matter every individual develops beliefs about it. His beliefs evolve from simple perceptual beliefs, experience, ideas and expectations. Beliefs are built up from many factors and their interactions are complicated. Students develop their beliefs in interaction with their surroundings and they also influence their surroundings. Mathematical ideas and beliefs act as a filter that influences all their thoughts and actions concerning mathematics. Prior experience of mathematics and the learning of mathematics influence both beliefs towards learning and the use of mathematics. Societal mathematical beliefs also influence students' beliefs.

More factors could be mentioned that influence the mathematical behaviour of students. A network of influences from the people in one's surroundings influences the individuals' beliefs and how or if they try to learn mathematics. Beliefs towards subjects and learning are, in addition to cognitive factors, the basis of learning. Beliefs have to do with factors such as motivation, self-confidence and how positive students are. These factors do not only support the learning, they are a part of it (Pehkonen & Safuanov 1996:31-35; McLeod & McLeod 2002:115).

Gender is one of the factors that have been found to be of great influence, but not to the same extent on the performance as on the beliefs and thereby on the motivation and the purpose of learning mathematics. Around the turn of the century some research (Brandell 2002:10; Pehkonen 1994:43-44) showed that the beliefs of students in lower secondary schools were changing towards mathematics, the study of mathematics and the experience of being a learner of mathematics. The students expressed such beliefs that mathematics was more for girls than boys, the girls worked better in maths-class and were more successful. More awareness had risen that some social factors, inside and outside the classroom, had some influence. Some research had shown that the majority of students no longer saw mathematics as a male-dominated subject. There was, however, a clear difference in what the sexes considered important in the learning of the subject. In the reports from PISA 2000, similar conclusions were drawn (Organisation for economic co-operation and development 2003:7–25, 82–90, 127–142). In PISA 2000, the boys scored higher but the difference was not significant. The ways the sexes studied were different. The boys were more confident and showed more interest in mathematics. They believed they could cope with learning difficulties, used elaboration strategies and enjoyed competition. The girls were more concerned with what to learn and used more time on their mathematical learning. They paid more attention to organising their study, were able to concentrate more easily and used more control strategies. The gender difference was there but it was in beliefs and ways of learning. The conclusion was that in order to work for equality it was necessary to work with beliefs and learning methods.

A big research project, the GeMa-project, on teenagers' beliefs in mathematics has been conducted in Sweden. In the year 2003, some results were published (Brandell et.al 2003). The focus is on gender and the comparison of the beliefs of girls and

boys. From this, many things of interest appeared, underlining that the beliefs of teenagers are very diverse. More than half of them thought that mathematics is neither a male nor female domain. To give some ideas of how Swedish teenagers thought I have chosen three examples:

- The girls are thought to work hard in lessons, get encouragement from the teachers and the expectation is that they will do well.
- The boys are supposed to be disturbing in class, like to use computers, like challenging problems, expect mathematics to be easy and that they will need mathematics in their future jobs.
- Girls think that it is important to understand mathematics and get worried if they are not succeeding.

These studies use quantitative methods and they gave me a good overview over this research field. Pekhonen´s theories also gave me some inspiration as to what questions to ask.

METHODOLOGY

Very little research has been done on the mathematical beliefs of Icelandic teenagers. Iceland has participated in some multinational research, recently PISA 2000, PISA 2003 and PISA 2006. This research is entirely quantitative. I thought it would be more interesting to conduct a qualitative research where I could study thoroughly the beliefs of a few individuals and give some ideas of how they express their beliefs. Many studies look at gender as a factor of great influence (Brandell et.al. 2202, Gothlin 1999). I decided to narrow my research and only study girls. That gave me opportunity to view my findings in regard to the big multinational studies and some studies of girls' beliefs about mathematics. The four girls I interviewed in the year 2003 were all 15 years old attending a lower secondary school in the capital city of Iceland, Reykjavik. They were volunteers from a class of 12 girls (and 10 boys). In the year 2008 I have interviewed the same girls again.

There are many different research approaches in the field of qualitative research. In interviews, participants have good possibilities to use their own words and the interviewer gets real examples of how the participants express their experience and what concepts they use. I found this an interesting approach for me as a researcher entering a new field. I wanted to find out what ideas the girls had, interpret them, react and ask further. I prepared some open questions and divided the subject into three main areas, the subject, the study and being a learner. The research question was as well divided in three main questions: What is mathematics? What is important in the study of mathematics? How does it feel to be a mathematical learner?

The data was mostly gathered via computers. In the year 2003 I met the girls in the beginning to explain my study/research and then I sent them three letters, one at a time, with questions. The first one was about their beliefs about mathematics, the

next one was about the learning of mathematics and the last one was about their feelings about themselves as mathematical students/students. When they had sent some answers I sent them my interpretation of their letters about their beliefs and asked them to make comments. Finally I met with the girls to discuss their beliefs and get some further information. In the year 2008 I have used almost the same procedure. I did contact each girl and asked her to participate again. They agreed and then I sent them a letter. Now I am in the process of analysing the data. I have got all the letters from the girls but have not yet met them to discuss my interpretation of their beliefs. I will at ICME 11 present my conclusions of my following up study.

DESCRIPTION AND ANALYSIS OF THE 2003 STUDY

The first interview was about the girls' beliefs about mathematics. I asked how they would describe mathematics and mathematical knowledge and how they felt about it. I used in my analysis three main perspectives of the nature of mathematics, traditional, formalist and constructivist perspective (Pehkonen & Törner 2004:21-37). In the traditional perspective mathematics is seen as a set of skills or a toolbox. In the formalist mathematics is logic and rigour and the focus is on the system. In the constructivist perspective the process in building up understanding is most important.

Helga, one of the girls, answered as follows the question: What is mathematics?

> Mathematics makes it possible for you to calculate sizes and helps you in your daily life to know what you need to know and it also tells you why things are the way they are.

In the interview Helga mainly showed the traditional perspective. She

- talked about the importance of arithmetic
- said that geometry was about using the right formulas
- thought everybody used mathematics
- saw mathematics as a big subject and thought she had yet much to learn
- said that mathematics was about explaining relations.

The other girls expressed similar beliefs. However, they had different views as to how they valued the usefulness of mathematics and influence in the society.

Mathematical knowledge must be the aim of the study of mathematics. Some mathematics educators have found it useful to distinguish between two types of mathematical knowledge, conceptual knowledge and procedural knowledge. In his book *Elementary and Middle School Mathematics,* Van de Walle (2004:27) writes about these two main perspectives about the nature of mathematical knowledge, Conceptual Knowledge and Procedural Knowledge. Conceptual knowledge consists of logical relationships constructed internally and existing in the mind as a part of a network of ideas. Learning is based on the individual as he is building up his own knowledge. He needs to be reflective and active. He must ask his own questions and

think about how he can use his knowledge to understand new things. The learning of mathematics is about making sense of how the mathematical ideas connect. The understanding will be better as the individual makes more connections. The math-teacher gives out problems that help the student to construct knowledge on a specific area. Discussions are important both between students but also between the student and the teacher. Elaboration strategies are in focus and students have to draw conclusions from their work.

Procedural knowledge is the knowledge of the rules and the procedures that one uses in carrying out routine mathematical tasks and also the symbolism that is used to represent mathematics. Students have to build up their knowledge step by step. Students have to be active in memorising the procedures. They use various ways to improve their memory without focusing on their mathematical understanding, as when they use rhymes to remember how to calculate fractions. The aim with procedural knowledge is to build up instrumental understanding that students can use quickly when solving routine problems. The teachers' role is to build up a logistic sequence of small pieces of mathematical information. Most important is that the teacher is able to explain and make the learning easier for the students.

In the interviews on mathematical learning the girls found it hard to talk in general terms about mathematical learning. They could nevertheless describe clearly their ideas of a good mathematics teacher and how he should play his part in the learning process.

Lara, one of the girls, said that in teaching the teacher should go from the simple to the more complicated. The teacher's role is to organise and build up a learning process. The learner's role is to follow the process and practice on many problems. She considered the role of the teacher to be very important. She thought the relationship between the learner and the teacher was meaningful. She found that the learning is most successful if the learner studied under the guidance of the teacher. Lara described a good teacher of mathematics like this:

> Mathematics teachers should take one step at a time when explaining. He should be patient and have various ways to explain the same concept. She says it is an advantage if the learner knows the teacher and the teacher knows the learner on personal terms.

The girls all showed a strong tendency to look at mathematical knowledge as procedural knowledge. Their view was that the learning of mathematics was important for everyone and to succeed in the learning the student should work well every day and organise his work precisely. They thought that good performance involves the mastering of arithmetic skills and strategies. However, they did express, especially Soffia, that it is important that the learning advances the thinking skills and logical thinking. They also proclaimed the importance of independent students' work when building up an understanding of the procedures.

When discussing with the girls how it felt to be a mathematical learner I used as a background some research on how people sense their own ability (Guðbjörnsdóttir

1994:171, Magnúsdóttir 2003:34. Fennema 2000). This research shows that one's estimation of one's own abilities influences how one organises and performs an act. It also influence on what kind of problems one deals with, how much effort one puts into it and for how long one tries. Further elaboration on the results shows that gender influences the students' choice of challenging mathematics courses. These ideas are in coherence with the result of the PISA study (Organisation for Economic Co-operation and Development 2003).

Meece (1996) discusses the studies of Fennema and Peterson in the nineties. They developed a model for learning behaviour to try to explain the gender difference in how the sexes succeeded in solving mathematical problems. They proclaim that in order to be able to solve complicated problems you need to act in a specific way. The students have to be able to get deeply involved in the problem, be able to work alone and show persistence and concentrate. The model consists of four elements:

- Evaluation of the student's own mathematical abilities.
- Sense of how useful mathematics is.
- Learning abilities.
- Sense that mathematics fits your gender-role.

(Meece 1996:117)

These are the main components of the Fennema and Peterson model. It has been used in many research-studies. The fourth element in the model seems to have little influence, though the results from the other parts of the model seem to be gender biased. Fennema and Peterson claim that the difference in learning style is due to the socialisation in the classroom. Reports show that it is very common that teachers support boys in problem-solving strategies. It is also found that girls rather than boys avoid risks and problem-solving and seek guidance. This has influence on how teachers encourage their students to take part in mathematical discussions and thinking. According to Fennema and Peterson, boys are more enterprising, more active in their study and more likely to start discussions with their teacher about what they are studying than girls.

The interviews on how it felt to be a mathematical learner were lively. The girls' expressions were strong and personal. Their vocabulary was greater and more varied when discussing this question. They became eager in their expressiona and their thoughts went deeper. Helga said:

> I find it is rather easy to understand mathematics though I sometimes need some explanations. I think it is very important that the teacher is able to explain the same thing different ways. I feel irritated if I don't understand.

Helga felt that mathematics can both give positive and negative feelings. She felt happy when she succeeded in solving difficult problems but unhappy when she failed. From her point of view, mathematical learning is about solving the problems the teacher gives.

Lara finds it important to have peace, quiet and space when she is studying because then she finds it easier to think. She claimed she was good at understanding mathematics.

> Usually I find it rather easy to understand mathematics. It is very important to understand because otherwise you will get problems later. It is also a fact that if you understand it is not a problem to remember the strategy.

Lara emphasized the importance of understanding. She said that understanding was a precondition for her to find it interesting and challenging to do mathematics. When she was asked about the text in the mathematics books she said:

> I often read the explanations in the mathematics books but I find the strategies and methods they describe complicated. I like it much better when people show me how I am supposed to solve the problems.

Sigrun said that she found it very important to understand the things she was working with in her mathematics study. She did not feel comfortable if she finished a problem without understanding what she was doing. The mathematics was sometimes troubling her, not least the algebra. When the learning was going badly she felt she was "a loser" but when she understood she felt like "a genius". She connected these emotions mostly to dealing with new material. She said:

> ...always when you understand you get a good feeling, you become proud of yourself and you are confident that the rest will be easy.

Sigrun emphasized the importance of organizing her study and found it easier to concentrate in peace and quiet.

> I find it most rewarding to work on projects that are challenging but not too difficult so that you can solve them after trying for a while. Then I have to use my brain and I try hard to solve problems like that.

Soffía did not find it hard to learn mathematics.

> I think I have mathematics in my genes. I understand everything, at least when I have given it a thought. It doesn't take a long time for me to learn something.

Soffía thought that in the learning of mathematics analysing what you are looking for is most important. She did not find it useful to read the text in the mathematics books or listen to the explanations from the teacher. She believed in dealing with the problems. Soffía thus described successful learning:

> I find it most rewarding to work on problems where I really have to think. I prefer problems that take me up to 30–60 minutes to solve ... and I gain most if I find the solution without any help.

They main conclusions on the girls' beliefs about themselves as mathematical learners were that these four girls felt very confident about their abilities to learn mathematics and to use them. Sigrun and Soffia expressed that it was most rewarding to struggle with the problem solving them by themselves, while Helga and Lara thought it was best to get at once an explanation if they found it difficult to

understand. All the girls found it easy to organise their study and felt good when they were working on mathematical problems. All of them experienced joy when they understood things they had been working on. They all thought they were able to work independently in their study within the teacher's framework.

My main findings are that these girls express similar beliefs as girls in the overseas research-studies I used as a reference. Their beliefs about mathematics, the study of mathematics and themselves as learners of mathematics are in tandem with the conclusions of the PISA-studies and the Swedish GeMa-project.

DISCUSSION AND CONCLUSIONS

Qualitative research study is a way to give expressions of some individuals a space and a depth so that their views can be analysed from many angles. A qualitative research study based on interviews also gives the researcher opportunity to ask for the meaning of the words the individual expresses. In my study, I focused on getting a description of the girls' beliefs and not so much on how they had developed those beliefs or why. There is always a dilemma as to how to interpret research conclusions and how you can use them to understand the rest of the world. In qualitative research, the main conclusion is saying something about the individuals involved but, at the same time, being careful about using the results to draw conclusions about other people. The results can, however, often be used to understand how people think because a good insight and understanding of one individual can make it easier to understand how others think. I feel at least that my experience has made it easier for me to discuss beliefs on mathematics and mathematical learning with other teenagers and also other mathematical students. I also feel that my study has validated the idea that a research study in the Scandinavian countries and some other European countries can give some ideas of the situation in Iceland. I know that it is not possible to compare results from a qualitative research to results from quantitative research. Nevertheless, I have found it helpful to use results from a quantitative research in my analysis and to gain some understanding and knowledge of the field.

Conducting a research study is very rewarding for the researcher. I learned a lot while I was looking into these girls' minds and their ways of thinking. I have been interested to make a follow-up study where I would study how the girls´ beliefs have developed. In January 2008 I started to gather some new data from those four girls. I will at ICME 11 be discussing how those girls' beliefs have developed and what they think is has been influencing their thinking of mathematics and mathematics teaching.

References

Brandell, G., Nyström P. & Staberg E. (2002). Matematik i grundskolan – könsneutralt ämne eller inte? Lund, Lund Universitet, Matematikcentrum.

Fennema, E. (2000). *Gender and mathematics: What is known and what do I wish was known?* From the Fifth Annual Form of the National Institute for Science Education. Detroit, Michigan. From the Web 4.1.2004. http://www.wcer.wisc.edu/nise/News_Activities/Forums/Fennemapaper.htm

Gothlin, E. 1999. *Kön eller Genus?* Göteborg, Nationella sekretariatet för genusforskning.

Guðbjörnsdóttir, G. (1994). Sjálfsmyndir og kynferði. *Fléttur: Rit rannsóknarstofu í kvennafræðum,* (pp. 135–202). In R. Richter og Þ. Sigurðardóttir (Eds). Reykjavík, Rannsóknarstofa í kvennafræðum, Háskólaútgáfan (University of Iceland Press).

Magnúsdóttir, B. R. (2003). *Orðræður um kyngervi, völd og virðingu í unglingabekk.* [Ópr. M.A. ritgerð] Háskóli Íslands (University of Iceland), Félagsvísindadeild (Faculty of Social Sciences), Reykjavík.

McLeod, D.B. & McLeod S.H. (2002). Synthesis – Beliefs and Mathematics Education: Implications for Learning, Teaching, and Research. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), Beliefs: A Hidden Variable in Mathematics Education, (pp.115–127). Dordrecht, Kluwer Academic Publishers.

Meece, J.L. (1996). Gender Differences in Mathematics Achievement. In M. Carr. (Ed.) Motivation in Mathematics (pp. 113–131). New York, Hampton Press.

Op 't Eynde, P., de Corte, E. & Verschaffel, L. (2002). Framing Students' Mathematics-related Beliefs. A Quest for Conceptual Clarity and a Comprehensive Categorization. In G.C. Leder, E. Pehkonen & G. Törner (Eds.), Beliefs: A Hidden Variable in Mathematics Education, (pp. 13–39). Dordrecht, Kluwer Academic Publishers.

Organisation for Economic Co-operation and Development. (2003). Education at a Glance – OECD indicators 2003. Paris, OECD.

Pálsdóttir, G. (2004). Viðhorf til stærðfræðináms. [Ópr. M.Ed. ritgerð] Kennaraháskóli Íslands (Iceland University of Education), Reykjavík

Pehkonen, E. (1994). Seventh-graders' Experiences and Wishes about Mathematics Teaching in Finland. Nordisk matematikk didaktikk 2, 1: 31 – 47.

Pehkonen, E. & Safuanov I. (1996). Pupils' View of Mathematics Teaching in Finland and Tatarstan. Nordisk matematikk didaktikk 4, 4: 31– 60.

Pehkonen, E. & Törner G. (2004). Methodological Considerations on Investigating Teachers' Beliefs of Mathematics and its Teaching. Nordisk matematikk didaktikk 9, 1: 21–49.

Van de Walle, J. A. (2004). Elementary and Middle School Mathematics. New York, Longman.