

# FORMATIVE PROCESSES: SHARING LEARNING EXPERIENCES IN GEOMETRY<sup>1</sup>

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This project has as objective to point out indications of learning and professional development of teachers in pre-service, teachers in service, post-graduation students and university teachers who participated of a collaborative group, during the period of 2003-2007, in which the teaching of Geometry under different medias was conceived as the object of study and research. A formation strategy was constructed and consisted of the collaborative elaboration of classes about Geometry, development in classrooms of Basic Education in different medias and posterior discussion and analysis in the group. The group constructed a pedagogical knowledge on Geometry through a shared and systematic work. This propitiated security allowing the teacher to dare, expose and risk, as well as validated pedagogical practices that in the end assumed group characteristics: the use of the register in the Mathematics classes and to give voice and hear the student.

This paper is related to a research developed during 2003 – 2007 in a collaborative group constituted by graduation students (pre-service mathematics teachers) and post graduation students, in-service basic school teachers and university professors. The study object of the group is Geometry and its teaching, fact that gave origin to the name of the group – Collaborative Group in Geometry (Grucogeo). The group has weekly 2 hour reunions, in the university, with voluntary participation and with entries and exits of its participants in each semester, having, in average, 15 participants. In this formation space there were developed 5 researches: 1 coordinated by the authors of this paper (with external financing of MCT/CNPq); 2 scientific initiation researches and 2 Masters in Education researches.

The look to Geometry and its teaching was diversified: exploration of tasks of investigative nature with and without the use of softwares; accomplishment of a same task in different medias - in Grucogeo or with pupils from Basic Education -; exploration of the potentialities and limits of each media: folding, constructions with ruler and compass, use of manipulable materials and computational environment; discussions and analysis of the argument processes, validation and proofs in Geometry.

The proposal of the work group was centered in the elaboration/development/evaluation and analysis of activities directed to Basic Education. The group focused on some activities - brought by the teachers (in service and of the university), some of them were generated in contexts taken from actual classes of the professors, and re-elaborated in a way that they could be used with the pupils in the

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classrooms of the school of origin of the in-service teachers. Some times the pupils were brought to the institutional space of the university.

For the classroom activities, the participants were divided in sub-groups in a way that in each one of them there was at least one basic education teacher to develop the lesson with the pupils, followed by graduation students that assisted in the registry of the process and in the management of the activity. The registered material (productions of the pupils, daily field diary of the graduation students/researches and narratives of the teachers) were brought to analysis, synthesis, socialization and systematization in Grucogeo.

This work proposal is based on the use of formative strategies (Alarcão, 2003, p.58) that " have as objective to make the teachers more competent in the analysis of the questions that surround their routine, allowing them to act, not only helping in the resolution of immediate problems, but situating them in a broader horizon that perspectives theirs and the school's function in the society where we live".

Amongst the strategies suggested by this author, in Grucogeo were used: "the questioning of other educative actors", "the confrontation of opinions and approaches"; "the discussion groups or study circles" and "the pedagogical questions". There was in the group a culture of questioning and problematization of the questions relative to Geometry and its teaching.

We also adopted the strategy of production of narratives (oral and written) about the daily practices of the teachers (Alarcão, 2003; Cunha, 1997). Cunha (1997, p. 2) when referring to the narratives as formative processes, affirms that "at the same time that the subject organizes the ideas for the narrative – written or oral - it reconstructs its experience in a reflexive way and, therefore, it ends up making an auto-analysis that creates new bases of understanding of its own practice. The narrative provokes changes in the way people understand themselves and the others ".

We also started from the presupposed that the heterogeneity of the group – in-service teachers and graduation students - allied to the adopted formative strategies contribute with the constitution of a repertoire of knowledges and with the professional development of the teachers. Although future teachers have left school recently, in the processes of initial training they need to transform themselves from students to teachers, what implies a change in the way of looking the school and classroom culture. In this direction, the in-service teachers assume the role of trainers and co-constructors of the professional identity of the future teachers, in a process of mutual involvement, approaching Rogoff's concept of "guided participation" (1998, p. 129-130). The author considers that this participation can be "tacit or

explicit, face-the-face or distant, involved in joint efforts with people (...). It includes deliberate attempts to teach, and incidental commentaries or actions that are heard by chance or seen as well as the involvement with materials and particular experiences that are available, indicating the direction in which people are encouraged or discouraged to go".

The teaching activity is culturally organized; the classroom is a space with its rhythms, rites and symbols that the in-service teacher knows and dominates and, therefore, can contribute with the future teachers. This way, in the heterogeneity of the group, the different actors assume distinct roles in a process of mutual aid. The in-service teachers bring their practices so that they can be reflected and questioned, reaching new meanings and assuming, together with the university professors, the role of educators/trainers of these future teachers who, in the start, participate in Grucogeo in search of specific Geometry knowledge. The sharing of these graduation students with the teachers allows the appropriation of professional teaching knowledges. On the other hand, these graduation students, immersed in the academic culture, in the confrontation of opinions and approaches with the teachers, contribute with different ways of geometric thought (mainly in the processes of proofs and validations of which the teachers are distant), as well as in the use of the computational media. The university professors, also educators/trainers, at the same time that they appropriate the actual practices of basic school, contribute with the group, bringing the look of a researcher (theoretical and methodological basis), with problematization and pedagogical questions.

This existing movement in Grucogeo allows us to recognize the collaborative dimension of the group which potentializes the learning of the teachers and their professional development. Having this in mind, in this text we analyze the indications of learning in/of and about teaching (Cochran-Smith; Lytle, 1999) and of the professional and teaching development of the participants. We present one of the experiences lived in the group, in which we underline the moments where the participants acted as apprentices, as teachers and as researchers. Regarding the research material, the group has: audio and video recordings of the group meetings; video recordings of the pupils in resolution of activities sessions in the university; written narratives of the teachers; written registers of the meetings.

### **The activity of triangular inequality: shared learnings**

The activity of triangular inequality appeared when the university professors suggested to the teachers to think, from the contents that were being worked in their classrooms, some situation that could be worked in different medias.

Teacher Paulo revealed the desire to work with the property of triangular inequality. At this moment the group, in the condition of apprentice, searched to know this property, mainly the graduation students who did not know it (it's important to underline that these students are products of a Brazilian school system that for some years has abandoned geometry teaching). However, in the initial discussion the subject and the pedagogical knowledge were connected, since that for the graduation students it was a moment of learning the geometric concept and for the in-service teachers there was a concern in how to pedagogically treat such concept.

Paulo suggested that the first media to be used could be a game with three dices and previously cut sticks, with determined measures, going from 1 unit to 6 units of length - for each measure, there would be three equal sticks. The pupil would play the three dices and would grab the sticks corresponding to the value designated in the dices. For example, if the dices had 3, 4 and 6 points, the pupil would grab these three lengths of sticks. The objective is to verify if the three sticks allow or not to form the representation of a triangle. From the conclusions, the pupils would register their own conclusions - in a table format, or in other formats - searching a generalization of the property.

The group thought that the activity was pertinent and gave suggestions related to the organization of the material. It was also consensual in the group that this would be carried through with different medias (the game, the use of the Cabri Géomètre software and construction with ruler and compass). As previously underlined, the heterogeneity of the group allowed the magnification of ideas. Post-graduation student Jorge, using his experience as a mathematics and computer sciences teacher, contributed with the group presenting suggestions of how this activity could be developed in software.

After planning the dynamics of the activity in the 5th and 6th grades (11 and 12 years old) of the public school, in which teacher Paulo worked, it was agreed that he would be followed by the Scientific Initiation scholarship graduation student Thiago, also a participant of the group. This pair would act in the classroom as teachers.

After working with the game media together with Thiago, professor Paulo brought to Grucogeo the registries of the pupils so that they could be socialized. In the socialization of these lessons - oral narrative - he pointed some questions that allowed reflections in the group and possible ways for the continuity of the activity: the non familiarity of the pupils with group work and the resolution of an open nature activity; the difficulty of understanding the proposal of the problem demanding constant interventions of the teachers; the difficulty of

written registry of the pupils; and, mainly the conceptual gap of the pupils in relation to the triangle concept.

Paulo still told that, because of the conceptual gap of the pupils, he requested them to answer "What it is a triangle?" The answers of the pupils to this question, given the incompleteness of the definition, generated a discussion in Grucogeo, mainly originated by the question of Paulo: how, in the teacher condition, can I intervene to supply this conceptual gap, in other words, how to deal with the written registries of the pupils? The discussions were centered in questions as: the inadequate use of mathematical language; the prototypic geometric object and the necessary and sufficient conditions for the existence of a triangle. These questions emerge of a practice of teaching geometry, that when present in the classroom, is based in nomenclatures destitute of meanings and the use of stereotyped geometric figures. This is because the pupils defined the triangle as being a figure with 2 equal sides and a different "base" (isosceles triangle); or as being a figure with 3 equal sides (equilateral triangle) and always in the same position, calling the parallel side to the line of the notebook the "base". The pupils could not advance in the discussions about the property of triangular inequality, because they did not admit the existence of the scalene triangle.

This discussion allowed reflections in the group of two different natures: the epistemological questions related to the concept of triangle and to geometric thought and the pedagogical questions related to the challenges and ways that the teacher needs to face because of the difficulties and conceptual gaps presented by the pupils. Such gaps were evidenced by the use of an open activity complemented by the registry.

The group, in the occasion, presented suggestions of continuity of the activity, with two actions: (1) the return to the pupils of some of the answers registered about triangular inequality and (2) the continuity of the activity with the use of the Cabrí software.

In this second stage teacher Paulo returned to the classroom and presented to the pupils 3 registers about triangular inequality and proposed that they analyzed them and choose one of them as being the generalization of the property; for each one of the definitions he suggested the pupils to present examples and counter-examples. This stage was complemented with the use of the Cabrí software, from the suggestions given by Jorge: he supplied three segments and the triangle constructed from them; with the use of the mouse the pupils would move the vertices of the triangle, getting new measures for the segments and would complete a table with the obtained data, searching the regularity and, consequently, the generalization.

In the socialization of the results of the second stage of the activity in Grucogeo, Paulo underlined that the dynamics of group work and the arguments of the pupils when confronted with the presented definitions allowed them to approach the property of triangular inequality and extended the definition of triangle. According to teacher Paulo, even with the use of the Cabrí software, the argumentative answers of the pupils were still incomplete. In the concrete material (in the game), they were close to the inequality relation; but these were not brought to the computational environment. This means that the pupils were not capable of relating the two medias. On the other hand, the use of the Cabrí software allowed the use of numbers that were not whole (it was a limitation of the game media), opening spaces for the manipulation of measures for the set of rational numbers, as well as contributing for the generalization of the property. The group analyzed that the medias, in a way, complemented themselves, because the limitations of one media were surpassed by the use of the other media, strengthening the importance of pedagogically working with different medias in the school environment. So, it was suggested to Paulo that, by the fact that the computational media also has limitations (as the size of the screen of the computer), another media could be explored: the ruler and the compass. However, as mathematics teachers, there would still be the necessity of searching a formal validation - the demonstration of the validity of triangular inequality - such stage was not taken to the classroom because of the scholarization level of the pupils. However, in Grucogeo the participants, again as apprentices, studied this demonstration.

Some questions were object of reflection in the group, from this activity: 1) the production of written registries is fundamental when it is intended to discuss the procedures of mathematical validation. The capacity to elaborate well based arguments is essential for the processes of mathematical proofs; 2) the investigative position of the teacher in the direction of searching ways of questioning with the pupils without supplying ready models. This demands time and its insertion in the school curriculum implies reviewing the way of how the educational programs are organized, with little space for the teacher who would like to carry through activities like this with his classes; 3) although the pupils did not manage to generalize and find the law of valid formation for any triangle, the task was very rich in the direction of allowing the appearance of other questions: the concept of triangle; the question of the shape - what is shape in Geometry?; the question of writing - the pupil does not have motivation to write because he does not have a reader for his text; perhaps this can be a new culture of mathematics classes. As teacher Paulo underlined, he, in the majority of time, does not know what to do with the writing of the pupils; 4) the pedagogical intervention: in what

moments to intervene and in what moments to leave the pupils more independent to search their own ways.

### **Indications of teacher learning about/in/of practice**

The analysis of the experiences, when told to the group allowed moments of reflection and discussion about the teaching of Geometry, and about the mathematics classroom culture. The importance of giving voice to the pupil - these are moments where we can identify the concepts that were already or not appropriated -, the importance of the pupil registry and what to make with it - this registry can be returned to the pupil for analysis of the used arguments and can be a reference to the teacher for evaluation of how the pupil is appropriating the concepts that are being worked - and alternative ways to propitiate the conceptual elaboration in classroom - we believe that the movement of involving with the task, searching hypothesis, to argue them looking for validation, to socialize with the colleagues and to reach consensual syntheses in the classroom, constituted the authentic mathematical activity. In the written narrative about his experience, professor Paulo underlines:

*The written registry was fundamental so that I could identify the existing gaps in the conceptual elaboration of triangle by the pupils and could make adequate interventions. This process was enriched with the moments of negotiation of meanings, where the pupils argued in favor or against a determined hypothesis. In this dynamic the mathematical truths are validated by the pupils and not imposed by the teacher [...] the participation in GRUCOGEO and the possibility to share my doubts, uncertainties and unreliabilities related to a new approach, as well as the systematization of the experience in the present text were fundamental to my professional constitution (narrative produced for the presentation in the 16th Congress of Reading, June of 2007)*

Another question that was present in the studies of Grucogeo is related to the importance of using different medias in the teaching of Geometry. Evidences of the appropriation of these discussions and knowledges produced in the group and adoption of the same ones in the practices of the future teachers are found in the narrative produced by graduation students Valéria and Renata, about the activity of training in which they explored the cevians of a triangle in different medias.

*When one works with different didactical resources in the classroom, as was the objective of our project of intervention, the pupil constructs and produces knowledge from the different representations approached by the different medias. It can be said that these passages of one representation to another allow the pupil to develop abilities and start to appropriate a concept [...] We recognize that our "bravery" in working with computational environments in the regency classes was decurrent of the shared experience in GRUCOGEO.*

In the perspective of Bolzan (2002, p.148), the group allowed the weaving of a net of relations where "a voice joined the others and, in the course of the interactions, they had meanings and ideas about pedagogical knowledge were being shared". The group constructed a pedagogical knowledge on Geometry through a shared and systematic work. The statement of teacher Mirian evidences such construction:

*The Geometry workshops [Grucogeo] allow me to explore the biggest amount of properties, regularities that exist in Geometry. How many things that I see in definitive way and that other people see it in another way; this makes me extend my knowledge and I can deal with these questions in the classroom where there are 40 heads reasoning differently from the others. (Online interview Mirian, December 2005)*

In this construction movement everyone appropriated the meanings produced by the group, transformed developed professionally. Rogoff points out (1998, p.136): "a person who participates in events modifies itself in way that it becomes different in subsequent events". Thus, the development is seen as something dynamic and the appropriation "a transformation process" (p.134). The graduation students are appropriating the strategies adopted in the group and that start to be used their regency training classes (to know about and in practice); the teachers are come systemizing their experiences and discussing them with their peers in other institutional spaces, turning themselves into researchers of their own practice (knowledge in and of practice) and the university professors are constructing new theoretical and methodological basis of research (knowledge in and of practice).

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