A TECHNOLOGY INTERVENTION USING MULTIPLE REPRESENTATIONS ON MATHEMATICS

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This paper is research oriented

Theme: Integration of technology into school and other learning environments

Abstract

The use of multiple mathematical representations promotes students' understanding of mathematical concepts. Research indicates that positive gains in understanding of mathematical topics appear in cases when multiple modes of mathematical representations are used effectively. Technology integration facilitates multiple modes of representations which improves transitions from concrete manipulation to abstract thinking, and provides a foundation for continued learning. This study investigates the effectiveness of a web-based learning tool with three groups or 20 middle grades students randomly assigned to one of three conditions. In the first condition students use the learning tool for 10 minutes, the second condition 20 minutes, and in the third condition 30 minutes three times per week. The results indicate that condition three showed the greatest performance gains while showing a reduction in the use of the kinesthetic models. Additionally, the longer students used the tool the less they depended on auditory and audio-visual representational models.

A Technology Intervention on Multiple Representations

This study investigates students' rational number performance taught with multiple modes of representation through a technological intervention. From previous work in this area, there are two crucial issues: (a) the use of multiple modes of representations in mathematics, and (b) meaningful integration of computer activities to achieve multiple modes of mathematical representations.

Definitional and Conceptual Issues

The National Council of Teachers of Mathematics' (2000) *Principles and Standards for School Mathematics* emphasizes the necessity of representations on students' understanding of mathematical concepts and relationships. As representations have been an essential part of mathematics teaching, appropriate use of them is a critical factor for effective teaching together with accurate content knowledge (Hiebert & Carpenter, 1992).

Representation

A representation is defined as any configuration of characters, images or concrete objects that can symbolize or *represent* something else (DeWindt-King & Goldin 2003; Gagatsis & Elia, 2004; Goldin & Kaput, 1996). Individual representations create a larger representational system consisting of (a) characters or signs including letters, numbers, words, real-life objects, and (b) rules and practices for combining and operating on the signs (Goldin, 2003). Representational systems are both internal and external in nature. Internal representation systems are those that exist within the mind of the individual; external representation systems can easily be shared with and seen by others. Internal representations consist of constructs to assist in describing the processes of human learning and problem solving in mathematics (Goldin, 1998), and external representations consist of such things as diagrams, formal language, and symbolic notations (Goldin & Steingold, 2001). Using multiple representations to demonstrate the same concepts helps not only to develop a better conceptual understanding but also strengthens one's ability to solve problems (Amato, 2004; Gagatsis & Elia, 2004).

Technology and Manipulatives

In order to facilitate the transition from concrete manipulation to abstract thinking, teachers should integrate computer activities into mathematics meaningfully (Ball, 1988). Manipulatives are "physical objects specifically designed to foster learning" (Zuckerman et al., p.859) and digital (virtual) manipulatives are "computationally enhanced versions of physical objects, created in an effort to expand the range of concepts that children can explore through direct manipulation" (Zuckerman et al., p.860). The replication of physical manipulatives in the form of computer applications provides additional features and advantages over traditional manipulatives (Reimer & Moyer, 2005; Suh et al., 2005; Zuckerman et al., 2005).

Virtual manipulatives are advantageous in their capability to connect dynamic visual images with abstract symbols where physical manipulatives have limitations. Unlike physical manipulatives, virtual manipulatives use graphics, numbers and words on the computer screen to connect the iconic with the symbolic mode (Suh et al., 2005) and virtual manipulatives can record user interaction with the virtual manipulative and record such as movements and screen capture across time so the teacher or researcher can understand the false starts as well as the final submitted solution. Dynamic visual representations can be manipulated in the same ways that a concrete manipulative but (Moyer, Bolyard, & Spikell, 2002), include features that make the manipulative more useful for self-directed learning. A computer mouse is most commonly the interface for interacting with dynamic visual representation. Using a mouse the student can slide, flip, and turn in similar ways as concrete manipulatives.

According to student interviews and attitude survey results, Reimer and Moyer (2005) found that virtual manipulatives

a. helped students learn more about fractions by providing immediate and specific feedback,
b. were easier and faster to use than kinesthetic models,
c. enhanced students' enjoyment while learning mathematics.

Summary

The use of multiple mathematical representations has been shown to increase students' understanding of mathematical concepts. Nonetheless, while research indicates positive gains in student learning of mathematical topics, these gains appear in case when the multiple modes of mathematical representations are used effectively. Furthermore, transition from concrete manipulation to abstract thinking is promoted by technology integration into mathematics activities.

Research Questions

There is strong support in the mathematics education community that students can grasp the meaning of mathematical concepts by experiencing multiple mathematical representations (e.g., Amato, 2004; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Goldin & Steingold, 2001; Perry & Atkins, 2002; Suh, Moyer, & Heo, 2005).

Three major questions arose that guided the current study:

1. What representational models (aural, audio-visual, and virtual kinesthetic) influence mathematics achievement?
   1.1. What representational models facilitate use of the technological tool (i.e., aural, audio-visual, or dynamic virtual kinesthetic)?
2. What is the relationship between treatment duration (i.e., 10, 20, and 30 minutes) and the representational model use?
   2.1. What is the relationship between familiarity with the instrument and representational models students use to complete the problems?
3. Does mathematics achievement change across groups based on the amount of use of multiple representations?
   3.1. Is there a difference in performance between students who choose to use the virtual fraction strips as compared to students who use virtual geoboard?

Method

Participants

Sixty students were selected for this study. The students were randomly assigned to one of three treatment groups (n=20): (a) 30-minute group, (b) 20-minute group, and (c) 10-minute group. All groups used the on-line instrument without teacher or researcher assistance. The only difference among groups was the amount of time they used the on-line instrument per week. The on-line instrument was designed to allow the researchers to limit access to the tool to a specific amount of time per day and number of days per week.

Procedure

Each group was given different URLs to login to the web-based learning tool to ensure each group spent only their allotted time with the tool. They used online instrument for three days per week for three weeks. To improve the applicability and usability of the instrument and to determine the usefulness of multi-modal on-line help all text is clickable, the text is read to students, and they can have the information read as many times as they need. Brief Flash Videos (FLV®) are included to orient students to the on-line tools for working with the virtual fraction strips and the virtual geo-board, and students can watch the videos as many times as they need. None of the videos teach either how to use the virtual geo-board or how to use virtual fraction strips.

Instrumentation

The instrument is an interactive internet based computer software program designed to present randomly generated addition, subtraction, multiplication and comparison, problems where each fraction is less than 1 and the sums, and products are all equal to or less than one (see Figure 1 and Figure 2). The instrument collected additional information including all clicks, starts or mis-starts with the virtual manipulatives, active time/passive time, and total time using audio, audio-visual, manipulatives, the final solution in both algorithm and manipulative representations, and the feedback to the students on the suitability of their solution.

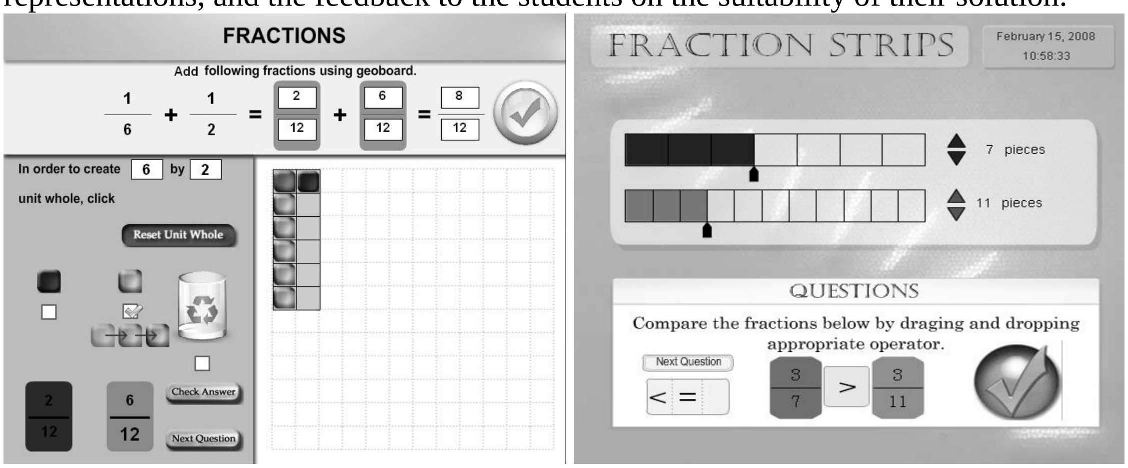

Figure 1. Screenshots of Virtual Geo-board and Fraction Strips.

Virtual geo-board instrument. The instrument employs several research protocols to improve the quality of the data collected and to ensure that data gathered is not the result of computer or internet based resources. The instrument uses screen capture every 5 seconds to record the cursor position and the work area content. The purpose of this screen capture is to provide precise information about student progress both as they make successful progress in the solution process as well as capturing any false starts so we have a complete accounting of each attempt. Additional protocols are: (a) the total time expended on each item is recorded (b) the internet protocol (IP) address is captured so we can be somewhat certain the data collected is from the target group, (c) the graphical problem/solution status is captured every 2 seconds and recorded to allow the recreation of those models.

Virtual fraction strip instrument. The instrument contains many of the same features as the virtual geo-board with the visual variation of the screen layout.

Results

Descriptive statistics for variables are shown in Table 1. Students' answers to the questions recorded as either correct "1" or wrong "0". Cumulative success ratio (CSR) is the ratio of students' answers to the total number of questions they answered. We used CSR because, every student solved a different number of questions within their allotted time. The ratio of the time students spent on different representation tools (i.e., audio, video, and manipulatives) was obtained by dividing the time spent *using audio* (TUA), *video* (TUV), and *manipulatives* (TUM) by the total group time assigned to the web-based learning tool. Audio-visual representation is a video supplemented with audio to provide instructions and to orient students to the online tool. *Learning preferences* (LP) are students' preferences among representational models; aural "1", audio-video "2", and virtual kinesthetic "3". *Manipulatives used* (MU) is the manipulative that students used to solve the questions; either fraction strips "1" or geo-board "2."

Table 1

Descriptive Statistics

	CSR	Group	TUA	TUV	TUM	LP	MU
M	.52728	2.0000	.1156	.0975	.4533	2.1167	1.4333
SD	.290668	.82339	.17118	.06926	.24292	.82527	.49972

Note. CSR=cumulative success ratio, TUA=time using audio, TUV = time using video, TUM=time using manipulatives, LP=learning preference, and MU=manipulatives used.

A multiple linear regression analysis was conducted between the dependent variable, CSR, and the independent variables, group, TUA, TUV, TUM, LP, and MU to answer questions 1 and 1.1. Table 2 shows the summary of linear regression analysis for independent variables predicting CSR scores of students. With alpha set at .05, group, TUA, LP, and MU are statistically significant predictors of CSR. The relatively large beta weights indicate group, TUA, and LP were important. When considering structure coefficients, group, LP, and MU were important predictors; they accounted for the greatest amount of variance in the dependent variable.

Table 2

Summary of Regression Analysis for Variables Predicting Cumulative Success Ratio

Predictors	β	*p*	r_s
Group	.860	< .01	-.578
Time Using Audio	.593	< .01	-.182
Time Using Video	.096	.465	-.164
Time Using Manipulatives	-.120	.359	-.114
Learning Preferences	.392	< .01	.653
Manipulative Used	-.246	< .01	-.385

Note. R^2 = .667 and adjusted R^2 = .629 (p = < .01). β = beta weight and r_s = structure coefficient.

Analysis using bivariate correlations among variables were used to answer questions 2, and 2.1. With alpha set at .05, there was a statistically significant negative relationship between group and TUA, TUV, and TUM (see Table 3). This result suggests that students who spent more time on the web-based instrument tended to rely less on supporting representations.

Table 3

Correlations (n=60)

	CSR	Group	TUA	TUV	TUM	LP	MU

CSR	1.000						
Group	.522**	1.000					
TUA	-.164	-.744**	1.000				
TUV	-.149	-.638**	.673**	1.000			
TUM	-.103	-.380**	.675**	.641**	1.000		
LP	.590**	.249	-.163	-.096	-.232	1.000	
MU	-.348**	-.082	.148	-.078	.114	-.248	1.000

Note. CSR=cumulative success ratio, TUA=time using audio, TUV = time using video, TUM=time using manipulatives, LP=learning preference, and MU=manipulatives were used.
**$p < .01$

Helmert contrast of CSR and CSR without using virtual manipulatives across groups was performed to answer questions 3, and 3.1. There was a statistically significant difference between group 1 (10 min. group) and groups 2 and 3 (20 min. and 30 min. groups, respectively) together as well as between group 2 and group 3 on both CSR and CSR without using virtual manipulatives (see Table 4). Figure 3 shows the means plots of respectively CSR and CSR without using virtual manipulatives by groups. The mean plots show that the students who had longer duration with the instrument had better performances. Specifically the performance difference between the students who spent 30 minutes and the students who spent less time with the instrument suggests an optimal time of 20-30 minutes with the instrument. Another observation is that CSR modestly decreased from group 1 to group 2 without considering manipulative use, whereas, CSR increased from group 1 to group 2 when considering manipulative use. In the first case, it is possible that students in group 2 attempted to transition away from the manipulation too soon resulting in a slightly reduced CSR.

Table 4
Helmert Contrast across Groups

	Contrast Estimate	
	Group1 vs. Later	Group2 vs. Group3
CSR	-.182 (p = .007)	-.373 (p < .01)
CSR without Using Virtual Manipulatives	-.900(p < .01)	-1.380(p < .01)

Note. CSR=cumulative success ratio, Group1=10 min., Group2=20 min., and Group3=30 min. use of web-based learning tool per day.

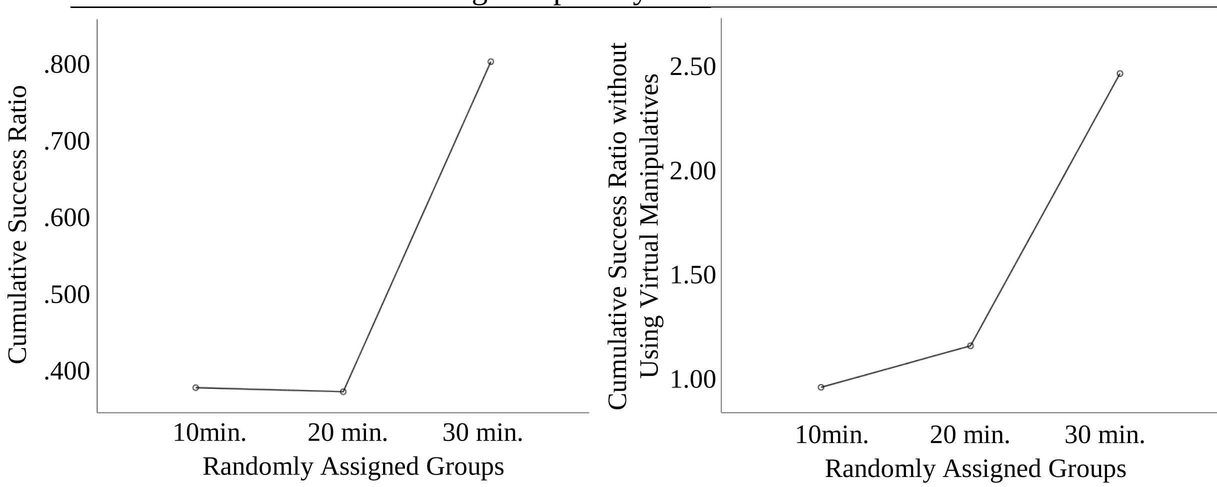

Figure 3. Mean Plots of Groups on Cumulative Success Ratio and Cumulative Success Ratio without Using Virtual Manipulatives.

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