SYMBOLIC CALCULATORS IN MATHEMATICS EDUCATION FOR FUTURE ENGINEERS: TEACHING STRATEGIES AND THEIR EFFECT ON STUDENTS' PERFORMANCE

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In this paper, we present the results of a study linking teaching practices in a first year calculus course for engineering students where the use of a symbolic calculator is allowed, with student performance in problem solving. In particular, this study highlights elements of the didactical contract that influence student practices when using the calculator and the arguments they choose when writing solutions. We conclude by bringing forth ideas on how to increase the calculator's contribution to students' mathematical practices as well as ways to minimize the risks associated with its use.

Context

The TI symbolic calculator is now systematically used as a teaching and learning tool at École de technologie supérieure (ÉTS). It was initially introduced in 1999 (Picard and Pineau, 2001; Beaudin and Pineau, 2002) and was mandatory solely in the introductory calculus course; it is now used in all mathematics and science courses at our school. After a few years of systematic use of the calculator in mathematics classes, it became relevant to examine the various approaches developed by the teachers, noting their common denominator, probing reasons that would explain the similarities and the differences, and determining their visible effects on students. Our purpose was to find ways of increasing the contribution of the symbolic calculator to students' learning and mathematical practice while minimizing the risks associated with its use.

Theoretical framework

The integration of computer algebra systems (CAS) as tools for teaching and learning mathematics in engineering schools rests in part on *pragmatic* considerations, i.e. those that are result-oriented. These considerations take into account the competencies that students, destined for a profession where mathematics and technology play an important part, must develop in order to tackle the increasing complexity of problems that arise in such professions. Those competencies can be regrouped using categories that were introduced by De Terssac (1996) in the field of sociology of work: *communication* skills (translating, representing, interpreting the context, determining what is to be done and what has been done), *evaluation* skills (identifying, choosing and justifying whatever is being engaged into action), and *intervention* skills (acting upon a situation by using available knowledge and transforming encountered situations into reusable knowledge).

However, the integration of any technology in the teaching of mathematics becomes viable only if it also takes into account *epistemic* considerations, i.e. the potentialities, limits and risks associated with the integration of such tools in understanding the mathematics being taught. On the epistemic level, the initial perception of the shift from technical to conceptual work (Heid, 1988) did not take into account the specificity of new techniques made possible by such tools nor their effect on access to knowledge (Lagrange, 2000). To encourage a higher level of conceptualization, it seems necessary to coordinate the actions, representations and observations made with the tool with an understanding of the underlying mechanisms specific to that tool (Artigue, 1997; Trouche, 2000), as well as to develop a practice of writing solutions coherent with the tool's integration in problem solving (Ball, 2003). These various elements are directly affected by the nature of the *didactical contract* (Brousseau, 1986) implicitly or explicitly established between the teacher and the students.

Methodology

The study focused on 4 teachers (henceforth referred to as Alain, Bernard, Charlotte and Diane) and more than 200 of their students, all of whom volunteered for the study. In the introductory calculus course, all the teachers and students worked with the same CAS (the TI-92 Plus symbolic calculator) and the same textbook (Hughes-Hallett et al., 1999).

Teachers' modes of integration of the CAS were analysed through the tasks (graded homework and examinations) used in assessing student understanding. The grid for this analysis was structured according to the competencies involved: *evaluation*, *intervention*, and *communication* (De Terssac, 1996) crossed with a typology linked to the level of the complexity of the tasks: *association*, *comprehension*, *organisation*, and *reformulation* (Caron, 2004). The need or relevance of the CAS in order to accomplish the task was also noted.

Interviews were carried out with each individual teacher to understand better the reasons behind their favoured mode of integration of the CAS, as well as to decode the didactical contract implicit in their teaching. In order to evaluate the importance given to the various aspects of the mathematical practice as well as to validate the specific competencies targeted by each of the teachers, we also examined the instructions they added to the wording of problems (requirements, hints, etc.) as well as the scale they used when grading the final exam.

Students' competencies were analysed through their writing of the common final exam. In particular, we examined their answers to questions where the calculator was allowed. We used the same grid that was used when analysing tasks, but this time, we mapped students' errors with the competencies that appeared lacking. Student productions were further analysed when they seemed to reveal particularly interesting phenomena.

Findings - Common denominator

Undoubtedly related to the use of a common textbook, there was a relative homogeneity in the distribution of the complexity of the tasks submitted by the teachers. In general, symbolic computation was less solicited than graphical or numerical capabilities. This can be explained by the fact that learning traditional calculus methods still constitutes one of the objectives of this introductory course.

Most tasks involved a certain form of association (e.g. the recognition of an obvious mathematical property or the application of a known method). Between 65% and 85% of the tasks involved at least some degree of comprehension (e.g. the identification of an underlying mathematical object or the interpretation of a result), and approximately half of the tasks called for some form of structured reasoning (e.g. the justification of a result or the use of a property to direct the resolution). Reformulation was rarely required, and mostly occurred in optimization and modelling problems (Figure 1).

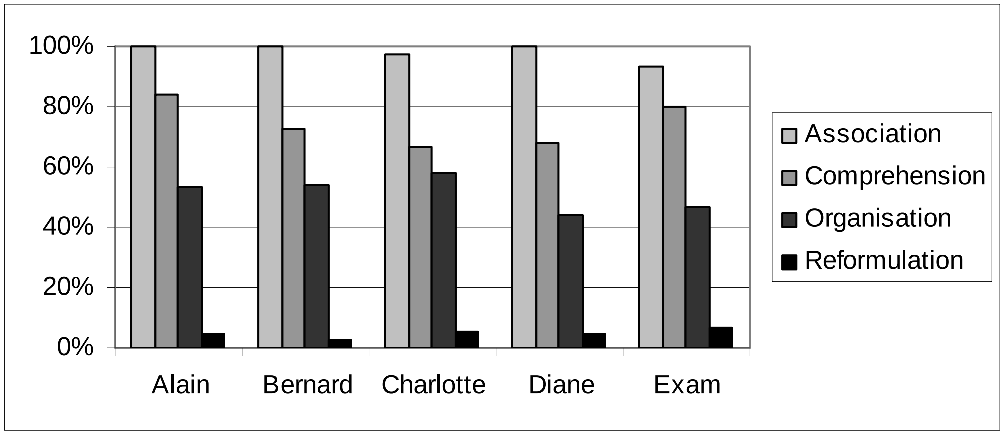

Figure 1 - Tasks across complexity levels

There was also little difference between teachers when it came to the need or relevance of the CAS to accomplish tasks (Figure 2). The teachers used the calculator as a tool for illustration. Thus, their focus was more on the calculator's *epistemic* value than on its *pragmatic* function. The complexity of most problems submitted to students did not require the use of the computational capabilities of the tool to produce a solution.

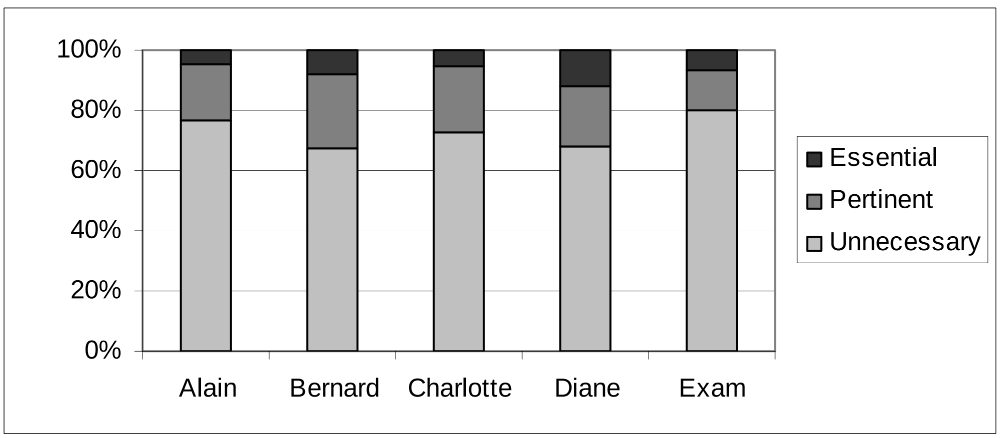

Figure 2 - Need or relevance of CAS to accomplish tasks

Overall, no significant differences in performance in the final exam emerged from the data. This can be attributed to the nature of the problems that reflected both the common textbook and the relative homogeneity of tasks that were submitted by teachers. However, detailed observation of the student productions to one of the questions of the final exam did reveal interesting differences in how they approached the problem, solved it, and communicated their actions and thought processes. Elements of the didactical contract, specific to each teacher, appear to explain some of the differences in the competencies displayed.

Findings - Interesting phenomena

Answers to question 2 were particularly interesting to analyse, as this was the only question where the calculator was absolutely necessary.

Question 2. Find the positive value k such that the area of the region between the graphs $y = k\cos(x)$ and $y = kx^2$ is 2. *Clearly specify the definite integral you use.*

Answering the question required computing the limits of integration (i.e. the *x* values, *a* and *b*, of the points of intersection of the two curves, noting their invariance with respect to *k*) and solving equation $\int_a^b \left(k\cos(x) - kx^2\right)dx = 2$ for *k*.

Intervention skills - Variable reserve in using the tool

In this part of the exam, students were allowed, in principle, to use their calculator without restriction. For this particular question, the calculator was essential to produce the limits of integration. Although not necessary, it was possible to use the calculator to evaluate the integral. Interestingly, several students seemed to self-impose limits on their choice of tasks for which they used it. This was particularly apparent among the students of Alain who admits to taking "*marks off when there is an abusive use of the TI*".

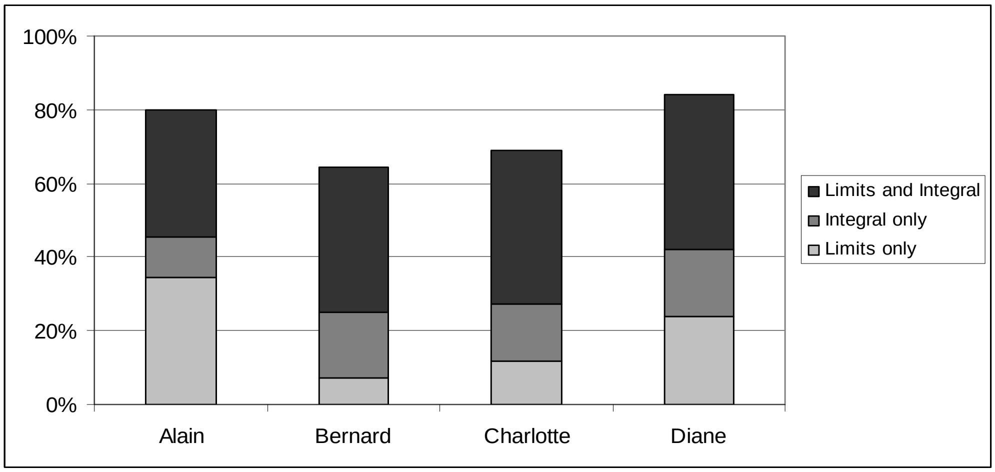

Figure 3 - Student calculator use for Problem 2

Consequently, less than half of Alain's students used the CAS to evaluate the integral, as opposed to nearly 60% of the other teachers' students (Figure 3). This may have been an attempt on their part to demonstrate their capacity to determine the appropriate use of the tool. However, students' caution in using the calculator only when it was necessary seemed to fade as soon as the object was not directly linked to calculus content. Indeed, after having showed that they could integrate by hand, some of Alain's students did not hesitate to use the "solve" function of the CAS to solve a simple one-variable linear equation (Figure 4).

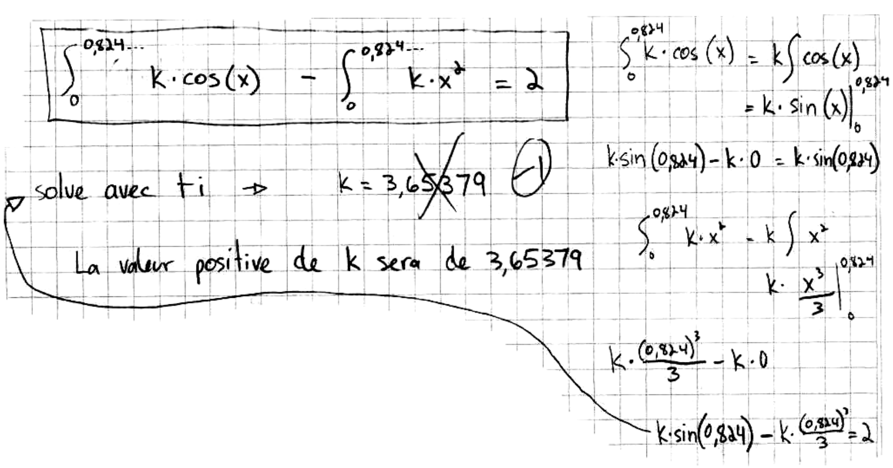

Figure 4 - Paradoxical use of the calculator

Evaluation skills – Legitimacy of registers

Although all teachers recognize the epistemic value of the graphical and numerical registers in teaching and learning mathematical concepts, they do not always consider these registers legitimate for solving problems that could be solved analytically within the symbolic register.

Alain and Bernard are particularly vigilant in having students use the symbolic register by asking for solutions in their exact form. Note the student's surprise (Figure 5) when he is unable to find an exact solution to the equation $\cos x - x^2 = 0$. Such surprise reveals that even when a student appears to know *when* to use the CAS, he does not always know *why* he is doing so. With a tool that automatically chooses the method of resolution associated with a command, and thereby relieves the student from making that choice, the reasons for moving from an algebraic method to a numerical algorithm may remain unclear to the student.

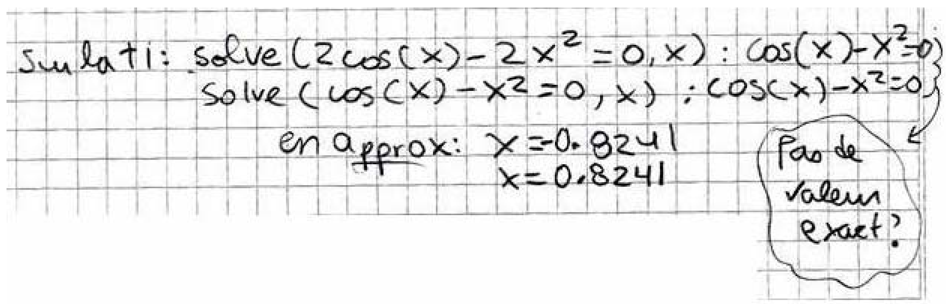

Figure 5 - Evidence of surprise at not finding an exact solution

For Alain, who warns students against "abusive" use of the CAS, numerical resolution is only admissible when the analytical methods (or calculations in exact form) are insufficient to produce solutions. Moreover, these numerical solutions remain preferable to simple observations on a graph or in a table of values. For students who are introduced to all these registers in a relatively short time, this hierarchy is not always obvious.

As the graphical register opens the door to less mathematical rigour (when observation of graphs replaces analytical reasoning) and with the traditional methods of derivation and integration being two of the foci of this calculus course, the symbolic register still dominates, even when calculations in this register are entrusted to the CAS.

Communication skills – Juggling with languages

Alain and Bernard, who value the pragmatic function of the tool for problems where no solution can be found analytically, tolerate, in such cases, incomplete documentation of the solution and elements of language that are specific to the calculator.

In comparison, Charlotte and Diane insist on finding certain elements (e.g. equations, justifications) while refusing the tools' instructions and syntax in written solutions. This imposes another hierarchy possibly as obscure to students as the hierarchy of registers: students are simultaneously learning the vocabulary and syntax of two new languages, that of the tool and that of calculus, but only the latter must be visible in their productions.

Explicitly reflected in their assessment scale, Charlotte's and Diane's communication requirements do not necessarily translate into better communication competencies, but they do seem to reduce the number of *organisation* errors by encouraging students to structure their thoughts using more generally accepted mathematical language. However, such guidelines generate records that do not reflect all the possibilities and constraints that are associated with CAS use, and consequently hide the technical aspects of the solving process. Not only does it make it difficult to assess student mastery of the mathematical content, it also deprives the teacher of key information on the instrumentation process.

Figure 6 shows how one of Charlotte's students abruptly interrupted her work. As was ultimately determined, the student simply had not used the multiplication key between *k* and cos(*x*) and this technical requirement was at odds with the syntax accepted by the graphing calculator that the student had used in college and with which she was familiar.

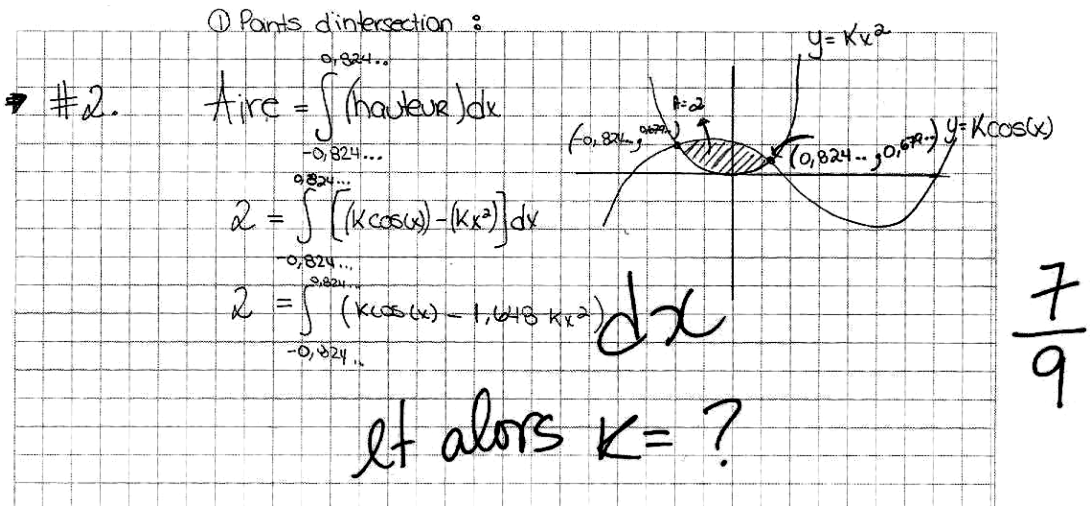

Figure 6 - Incomplete records

With students having experience with a larger array of technical tools, it becomes essential to draw attention to such specificities in the communication protocol. Moreover, rethinking assessment to include expression of the technical component should enable better assessment and provide opportunities for instructional adjustments.

Conclusion

With the use of symbolic calculators, it becomes possible to significantly increase mathematical intervention, but this requires the development of new communication competencies and the extension of evaluation competencies to cover the actions carried out by the calculator as well as the results it produces.

During the learning process, clear instructions that specify requirements (e.g. acceptable registers, expected forms of communication, obligation to verify results, etc.) would help the student better understand what is expected and why. In particular, there should be a means for the students to record instructions they give to the tool and to comment on the results produced. In addition, if students are to benefit from the new possibilities of exploration with CAS, it would be wise to grant heuristic exploration an official status, and support the emergence of rigour through appropriate questioning.

References

Artigue, M. (1997). Le logiciel Derive comme révélateur de phénomènes didactiques liés à l'utilisation d'environnements informatiques pour l'apprentissage, *Educational Studies in Mathematics*, 33(2), 1-30.

Ball, L. (2003). Emerging Practices for Recording Written Solutions in a CAS Classroom, *Actes du Colloque européen sur l'intégration des technologies dans l'enseignement des mathématiques* (ITEM-2003), Reims.

Beaudin, M. and K. Pineau. (2002). Examples of how Symbolic, Hand-Held Calculators Have Changed the Way we Teach Engineering Mathematics, *Proceedings of the annual conference of the American Society for Engineering Education,* Montreal, Canada.

Brousseau, G. (1986). Fondements et méthodes de la didactique, *Recherche en didactique des mathématiques*, 7(2), 33-115.

Caron, F. (2004). Niveaux d'explicitation en mathématiques chez des étudiants universitaires, *Revue des sciences de l'éducation*, Vol. XXX, no 2, 279-301.

De Terssac, G. (1996). Savoirs, compétences et travail. In J.M. Barbier (Ed.), *Savoirs théoriques et savoirs d'action*, Paris : Presses Universitaires de France, 223-247.

Heid, M.K. (1988). Resequencing skills and concepts in applied calculus, *Journal for Research in Mathematics Education*, 19(1), 3-25.

Hughes-Hallett, D, Gleason A.M., et al. (1999). *Fonctions d'une variable*, Montréal : Chenelière/McGraw-Hill.

Lagrange, J.B. (2000). L'intégration d'instruments informatiques dans l'enseignement : Une approche par les techniques, *Educational Studies in Mathematic*s, 43(1), 1-30.

Picard, G., and K. Pineau. (2001). Symbolic Hand-held Calculators in Teaching Engineering Mathematics, *Proceedings of the 12th Canadian Conference on Engineering Education*, Victoria, British Columbia.

Trouche, L. (2000). La parabole du gaucher et de la casserole à bec verseur : étude des processus d'apprentissage dans un environnement de calculatrices complexes, *Educational Studies in Mathematics,* 41, 239-264.