Teaching Advanced Calculus with APOS and Coursecompass.

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Abstract

This paper is about an experience teaching two different courses in Advanced Calculus, at the same time, with the ACE cycle proposed by E. Dubinsky using Pearson´s Course Compass as a support for the homework and tests.

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1 Introduction

The problem of teaching mathematics has been widely discussed in the literature for the past thirty years. One of the theories that has been developed is a constructivist one based on Piaget's ideas called APOS (Actions, Processes, Objects and Schemas). This theory is still evolving and is accompanied by a method ACE (activities, class discussion, exercises) to apply the theory in an undergraduate classroom. The method is based on the idea that reflective abstraction is a tool to design the instruction to improve significantly the way in which students acquire mathematical concepts.

The instructor has to design activities to cause reflective abstraction on the students working in a team and from there guide them to comprehend the concepts through class discussion. After that the students have to solve exercises and problems in order to attain the mathematical knowledge. For the purpose of designing relevant activities the instructor has to do a genetic decomposition of the concept to try to cover, in the best way, all of the elements present that gave rise to it. This might involve some history of the problems posed and solved by the concept and also an analysis of all the previous knowledge a student must have to be able to understand and construct the concept. This genetic decomposition is not universal and unique. This decomposition has to take into account the particular conditions over which one has to apply the method and has to be refined. These conditions refer to the country, school and interests of the students where one is trying to apply the method.

The problem is to do research in the concepts involved in Advanced Calculus courses for Economics students, using the APOS theory [1], at ITAM (Instituto Tecnológico de México). This theory uses the ACE (activities, class discussion, exercises) cycle that involves the use of computers at the activities part of the cycle and a dynamic feedback for the students both in the class discussion as well as for the exercises.

2 The settings

At ITAM we do not have teaching assistants and there are not enough computer labs to be used during the semester in a regular weekly basis. Also the classrooms are designed for a traditional conference type class. On top of that, in almost all their classes the students are working on an individual basis and it is not easy to get them involved in team work. These settings give rise to the situation of designing the activities to be done by the students in a classroom as a team. The activities have to be marked by hand by the instructor and returned as soon as possible to the students for feedback. When they use the computers for the activity they get immediate feedback when their programs run properly and produce the desired results. Moreover the exercises also have to be marked and returned to the students so they can get immediate feedback and learn from their mistakes. The exercises are done individually.

For the instructor that wants to do some research using the APOS method these settings mean an enormous amount of work. The instructor has to do the genetic decomposition of all the concepts and from there design the activities, mark them every week, prepare the exercises for the subject and mark them also every week. At ITAM the teaching load, per instructor, is three courses, which means at least 13 weekly hours in a classroom, with at least 30 students in each course.

Course Compass is the platform developed by Pearson publishing company that runs over Blackboard and provides the students and instructors with a tool to manage the contents, exercises, exams of different books and to work in a collaborative way through the internet. In particular they have the Calculus book written by Thomas, Weir, Hass an Giordano completely developed. The instructor can choose from within all the exercises in the book for the exercise part of the ACE cycle. The exercises are solved individually by the students, from their homes through the internet, using a friendly editor designed in Java. Each exercise has three levels of assessment for the student. The first level gives only a hint, the second level gives the student a similar example and the last sends the student to the book where the theory is explained. If none of these work the student can establish direct communication with the instructor to ask how to solve the exercise. The only problem for some of the exercises is the inflexibility accepting answers, for example if the student types 0.5 and the right answer for the editor is $\frac{1}{2}$, then Course Compass will mark it wrong.

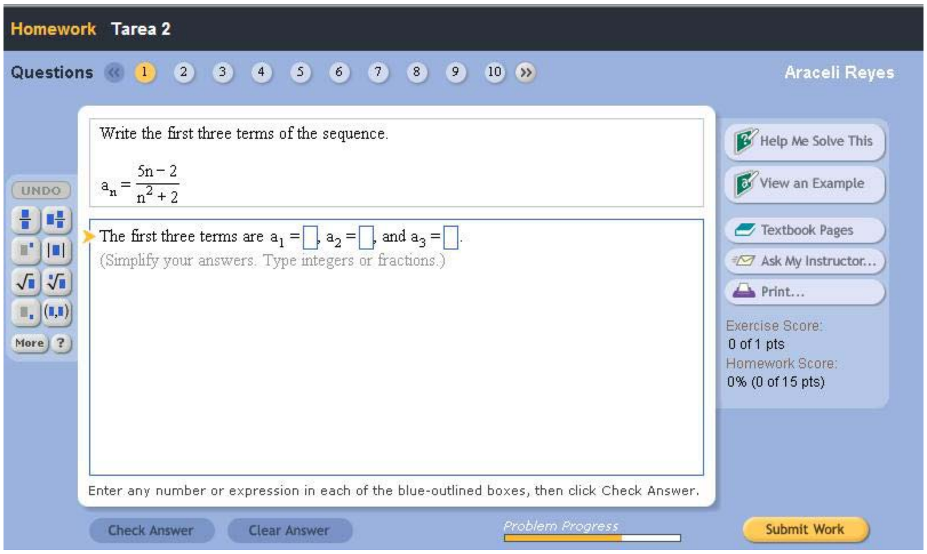

Figure 1: Exercise in Course Compass

It also has videos with explanations from an expert instructor to help the students understand the subject. The instructor can also upload his own questions, problems and exercises to Course Compass. Each exercise is marked immediately and the student gets a mark at the end of the session.

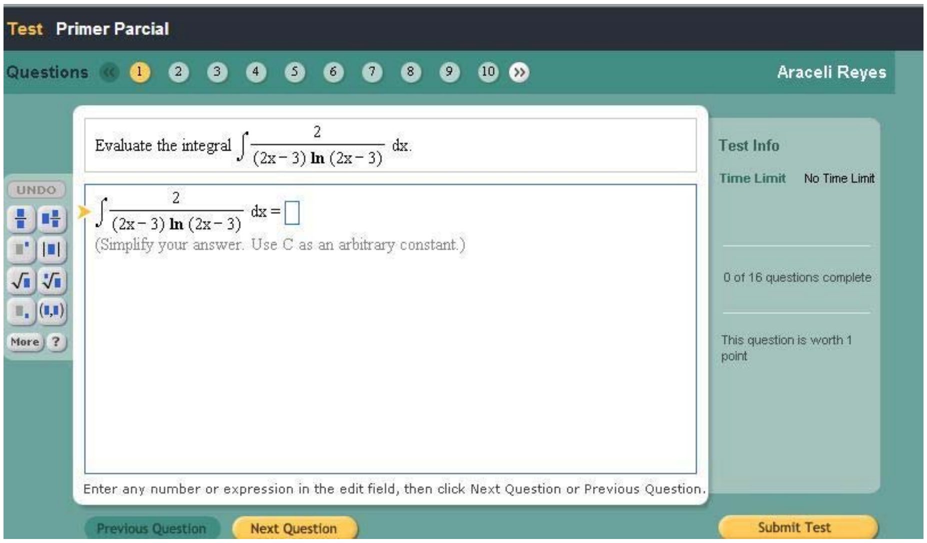

Figure 2: Exam question in Course Compass

The Calculus syllabuses for Economics students are reordered with respect to a classical calculus course because in their economy classes the students use many of the concepts very soon in the semester. So the Calculus II course starts with trascendental functions, the derivative and the integral as well as the inverses and then it moves on to vector calculus and optimization. The emphasis is on optimization with restrictions covering equalities and inequalities, the implicit functions theorem and envelope theorems. We used Taylor´s series to handle extreme values and concavity and convexity of several variables functions. The Calculus III course starts with integration techniques and continues with, sequences and series, multiple integrals, homothetic and homogeneous functions. We also included vectorial calculus and curve length to introduce the line integral and exact differentials to get the students ready for their course in Mathematics for Economy that basically is on Dynamic Systems. The general reordering was a departmental decision together with the Economy Department. This means that the epistemological decomposition for the concepts of the courses is completely different from what could be found in the literature.

3 The operational problem

The main and most important question was if Course Compass could help an instructor to use the ACE cycle to start doing some research in the comprehension of the concepts involved in the Advanced Calculus courses with the restrictions described above.

The first problem was if the students would accept using the internet to solve tests and exercises and get assessment in the way described. In the past when trying to have any class work done with the computer has proved to be very difficult. The students would complain about the difficulty of writing answers to mathematical problems with the computer and claimed that they could not get along with them (the computers).

A secondary question, no less important, was if this approach would convince the students to work as a team when doing the activities.

4 The theoretical problem

There is not much material developed for these concepts and in this order.[6] There is some evidence that the students do not understand concepts such as a multivariate function [3] which is needed to understand Advanced Calculus. The instructor had to do an epistemological decomposition to satisfy the needs of the Economy Department and develop all the activities accordingly.

4.1 Goals

The goals of this project were to develop the activities for the Calculus II and III courses based on a genetic decomposition and apply them to teach with the support of Course Compass.

The first step was making a list of the concepts involved in the courses. The second step was developing a first approach to epistemological decomposition of the concepts in order to develop the activities for the team work. [1]. In the figure we present the decomposition for the first part of the course:

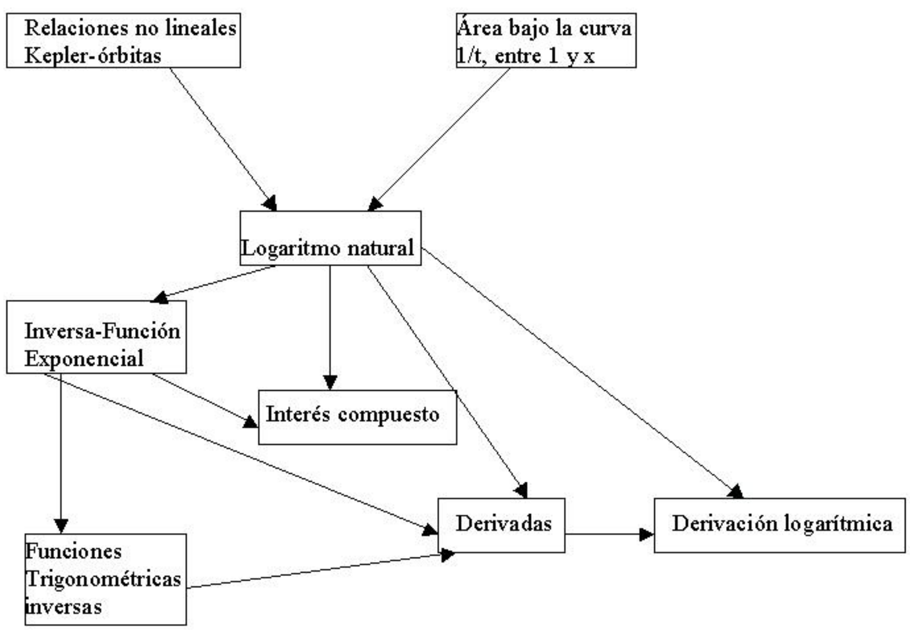

Figure 3: Epistemological Decomposition for the first Subject for Calculus II

There were three activities associated to this subject. They were written in Spanish and you can find them in Appendix 1..

The third step was designing the homework associated to each activity. An the last step was designing the tests.

There were four tests applied during the semester, one every four weeks. The second test was applied using paper and pencil for practical reasons but the other three, including the final were done in Course Compass. In Appendix 2 there is a photograph of the students taking the test.

To begin the course the instructor designed an exercise to let the students get familiarized with the Java editor so they could solve the exercises. This was an extremely easy exercise with no relevance in the final note of the students.

The instructor also gave the students a brief introduction of how to install the platform in their own machines an gave a demonstration in the classroom of how the editor worked. Each student had to register in the platform an had a password to login.

5 Conclusions

The use of Course Compass allowed the instructor to perform the decomposition for the two Calculus courses and design 18 activities for the Calculus II course and 16 activities for the Calculus III course.

The 65 students that took the course gave an opinion of their use of Course Compass. They were very pleased with the usefulness of it., the instructor designed a questionnaire inside Course Compass and the students answered it. In the Cálculo III course only five of the 31 students failed the course. In past courses the rate of failure was 17%, 18% and in this course was 9%. Only one student in this course did not use Course Compass properly during the exams. Neverhteless her opinion was favourable to using Course Compass

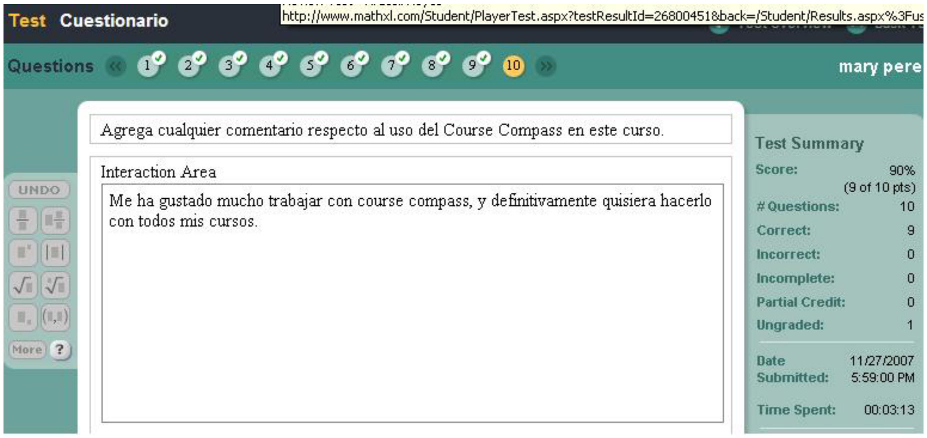

Figure 4: Question from the questionaire

For the other group the outcome was better, only 3 of the 34 students failed the course and all of them used the Course Compass properly, both during the exams and the exercises. For this course in the past the rate of failure was 40% and 16% and in this course 9%.

Also the instructor interviewed about a third of the students and here are some of their answers to "How did Course Compass contribute to your learning? "

Student 1

Course Compass has helped me a lot to understand better the exercises. It is a great support to the course.

Student 2

It allows you to practice a lot more if you used only the book because you get immediate feedback.

The rest of the answers (21) were mostly the same.

6 Bibliography

1. Dubinsky, E. M.; Asiala, A.; Brown, D.; DeVries, D.; Mathews, K. Thomas. A Framework for Research and Curriculum Development in Undergraduate Mathematics Education, 1996. Research in Collegiate Mathematics Education II, Issues in Mathematics Education (CBMS), American Mathematical Society.
2. Dubinsky, E. M. Pedagogical Change in Undergraduate Mathematics Education. Purdue University, W. Lafayette, Indiana.
3. Trigueros, M.; Martínez-Planell,R. Students ideas on Functions of two variables: Geometrical aspects.
4. Thomas, Jr. George B. et al, Calculus. 11th edition. Pearson Education Inc. Addison Wesley. 2005.
5. Thomas, Jr . George B. et al, Cálculo undécima edición. Pearson Education Addison Wesley. 2006.
6. 2006 Conference on Research in Undergraduate Mathematics Education. Contributed Papers and Preliminary Reports. Abstracts
7. http://www.coursecompass.com

7 Appendix 1

Actividad 1

1. *Aproxime el área por abajo y por arriba de la hipérbola* $\frac{1}{t}$ *en el intervalo* $[1,6]$ *con* 10 y 20 *sub intervalos. Estime el valor del límite cuando el número de sub intervalos aumenta.*
2. *Aproxime el área por abajo y por arriba de la hipérbola* $\frac{1}{t}$ *en el intervalo* $[1,2.71828]$ *con* 10 y 20 *sub intervalos. Estime el valor del límite cuando el número de sub intervalos crece.*

3. *Calcule los logaritmos naturales de* 6 *y* e=2.71828. *Compare con las estimaciones que hizo en los incisos a y b, respectivamente.*
4. Encuentre una función continua f con dominio en R tal que $f'(x)=f(x)$ y $f(0)=1$. La solución a este problema es la función e^x.
5. Verifique que las funciones e^x y $\ln(x)$ son inversas una de la otra.

Actividad 2

A continuación se presentan los datos de las órbitas de los planetas del sistema Solar y sus periodos

Planeta Semieje Mayor a Excentricidad e Periodo T Unidad

Planeta	Semieje Mayor a	Excentricidad e	Periodo T	Unidad
Mercurio	57.95	0.2056	87.967	días
Venus	108.11	0.0068	224.701	días
Tierra	149.57	0.0167	365.256	días
Marte	227.84	0.0934	1.8808	años
Júpiter	778.14	0.0484	11.8613	años
Saturno	1427	0.0543	29.4568	años
Urano	2870.3	0.046	84.0081	años
Neptuno	4499.9	0.0082	164.784	años
Plutón	5909	0.2481	248.35	años

1. Conviertan los periodos a la misma unidad. Sugerencia: utilicen días terrestres multiplicando los periodos en años por 365.256.
2. Intenten poner en una gráfica *XY* estos datos.
3. Saquen logaritmo natural de las columnas de semieje mayor y periodo en días.
4. Grafiquen estos números como puntos de R^2.
5. Supongan que existe una relación lineal entre ambas columnas de la forma

$$R = m\, t + b$$

donde t es el logaritmo natural del periodo y r es el logaritmo natural del semieje mayor. Estimen los valores de m y b.

Actividad 3

1. Un capital C_0 se invierte a un interés compuesto del i% mensual duarnte T años.
   (a) ¿Cuál va a ser el capital al cabo de uno, dos y tres meses?
   (b) ¿Cual va a ser el capital al cabo de T años?

Calculen el siguiente límite y justifiquen su respuesta

$$\lim_n \left(1+\frac{1}{n}\right)^n$$

Expliquen que tiene que ver el límite del inciso (2) con el interés compuesto.

(a) Si se depositan \$30000 a un interés compuesto continuamente de 4.2% durante 8 años ¿cuál va a ser el capital al cabo de esos 8 años?

8 Appendix 2