

# SOME ASPECTS OF THE SOCIOCULTURAL PRACTICE OF PROVING IN A UNIVERSITY COURSE WITH SUPPORT OF CABRI

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## Abstract

The purpose of this paper is to illustrate three aspects of the sociocultural practice of proving. They acquire relevance when a group of students and its teacher are considered as the unit of analysis, particularly while they engage in the collective building of a portion of an axiomatic system for Euclidean plane geometry. These three aspects are: (i) students' legitimate peripheral participation; (ii) teacher's role, and (iii) role of the dynamic geometry software. We use data collected in a course of Euclidean plane geometry in the second semester of secondary school mathematics teachers training at the Universidad Pedagógica Nacional (Bogotá, Colombia). The analysis leads us to state that Lave and Wenger's (1991) sociocultural theory offers a fruitful theoretical framework to understand important aspects of the process of learning to prove.

## Introduction

The concern about teaching and learning to prove, in all educational levels, is an important issue in mathematics education research today. In the last two decades, many papers and research reports published on this topic reveal its importance. Several theories and epistemological, cognitive and sociocultural trends have been used to create both theoretical frameworks and methodological tools to approach this question. Studies on beliefs and difficulties (Bell, 1976; de Villiers, 1992; Dreyfus, 1999), argumentative processes (Duval, 1991; Garuti, Boero and Lemut, 1998), types and levels of proofs (Balacheff, 1988; Harel and Sowder, 1998; Fiallo and Gutiérrez, 2007) and the role of DGS in teaching and learning to prove (Jones, 2000; Healy and Hoyles, 2001) are combined with sociocultural approaches which claim the need of taking advantage of social interactions in teaching and learning to prove (Alibert and Thomas, 1991; Mariotti, 2000; Marrades and Gutiérrez, 2000; Sackur, Drouhard and Laurel, 2000; Blanton and Stylianou, 2003).

In recent years, Lave and Wenger's social practice theory (Lave and Wenger, 1991; Wenger, 1998) has emerged as a new and useful conceptual framework for describing and understanding several cultural issues involved in the complex task of getting students to make sense of proving. The combination of Lave and Wenger's sociocultural perspective with mathematics education theories about teaching and learning to prove offers a fruitful framework to give light on important aspects of these processes.

The purpose of this paper is to illustrate three aspects of the sociocultural practice of proving: Proving as a joint enterprise, including a shared repertoire of routines, and a mutual engagement of students and teacher. Those aspects acquire relevance when we observe a group of students and their teacher while they are engaged into the collective building of a portion of an axiomatic system for Euclidean plane geometry. First, we briefly give account of the theoretical framework of our study. Secondly, we describe the relevant components of the methodology. Thirdly, we present excerpts from an episode to be analyzed. And, finally, we analyze the episode under the light of the three above mentioned aspects of their practice of proving.

## Theoretical framework

Several researchers have pointed out (Hanna, 2000; Mariotti, 2006) that proving is a central activity of mathematicians. By proving theorems they organize the contents of a particular mathematical domain into a formal deductive discourse aimed to validate them within a theoretical system accepted by the mathematical community. But the principles and proof rules used to produce such a discourse are established by a specific human group, so we can claim that the activity of proving has a sociocultural character, conditioned by the context where it takes place and by the specific mathematical domain where its authors are acting (Alibert and Thomas, 1991; Radford, 1994; Hoyles, 1997; Godino and Recio, 2001; Mariotti, 2006). Additionally, we claim that the activity of proving makes sense if it is linked to other mathematical activities such as experimentation, argumentation and axiomatization.

Mariotti (1997, 2005) points out that developing a deductive organization of Euclidean geometry without any experimental and intuitive support, based on experiences related to space and shape, may produce an activity without meaning. We believe that producing a proof is meaningful when it responds to meaningful questions in the dynamics of mathematical activities. We also believe that it is necessary to consider the relationship among argumentation and proof, in spite of the differences in their nature, objectives, discourse styles and truth values, since argumentation and proof are more or less related depending on the social group engaged in the activity of proving. In an educational context, we agree with Mariotti (2005, 2006) that the argumentative activity may constitute the key opening the access to the proving activity, since students are familiar with argumentation. Nevertheless, it is necessary to prepare specific didactic interventions to help students pass from argumentation to proof. Furthermore, Mariotti (2000) asserts that proofs cannot be considered outside a mathematical theory regulating the production of deductive chains built to prove a statement. Accordingly, we state that the exercise of constructing a deductive organization of theorems through the establishment of relationships of logical dependence between propositions may be didactically relevant.

According to our above mentioned perspective about proof, we conceive its learning as a sociocultural process where apprentices assume and learn to use the epistemic values and communicative conventions used by mathematicians when they develop activities related to proofs and proving (Goos, 2004). We see a mathematics classroom as a community of practice (Wenger, 1998) where students have the opportunity to learn to prove when they share a repertoire of practices characteristic of the proving activity. Whereas students participate in these practices they achieve competence on them, they develop an idea of what does prove mean and they learn how they can be legitimate participants of the community.

Then, learning is a process of negotiating meanings inside a community of practice Wenger (1998). The negotiation emerges from the interaction of two processes: *participation* and *reification*. Participation is the process in which we establish relationships with other people, we define our way to belong to the community, we take part in social enterprises, and we develop our identity through the participation. Reification is the process of giving form to our experience by producing material objects that capture this practice. When we do this, we create focus points around which we can carry out the negotiation of meanings. Reification is essential in all practices and it includes different processes that produce symbols, descriptions, representations, terms, concepts that capture the practice of the community.

According to Wenger (1998), there are three dimensions that distinguish a community of practice: (i) the evidence of a *joint enterprise* built collectively as the members assume responsibilities, discuss about what they value, what things to show, what is the product of the community; (ii) the identification of a *shared repertoire* of actions that are transformed into

routines, procedures, own practices and particular styles of acting; and (iii) the existence of a *mutual engagement* with actions whose meaning is negotiated by means of the relationships established in the practice.

## **Methodology**

The teaching experiment reported here was carried out with 21 pupils (17-20 year old), 6 women and 15 men, studying a course on Euclidean plane geometry scheduled in the second semester of the secondary mathematics teachers training career at the Universidad Pedagógica Nacional (Bogotá, Colombia). The objective of the course was to teach the students interpret and write proofs by using postulates, definitions and previous theorems, to organize all them in a deductive network. The mathematical content of the course was a set of relationships among points, straight lines, planes, angles, triangles and quadrilaterals. The course lasted for 16 weeks, with 5 hours per week.

Contrary to the traditional style of teaching mathematics at the university, where the teacher exposes the contents using a text as a guide, in this teaching experiment the teacher tried to develop the topics collectively through small group activities, guided conversations and whole class discussions. Students solved mathematical tasks that gave them the opportunity to be involved in social practices related to proof, organized according to the theoretical framework presented in the previous section. In a climate of social interaction, teacher and students communicated their ideas in the form of mathematical statements or arguments, criticized the others' ones, argued about their certainty, and produced proofs based on elements of an axiomatic system that grew step by step as each new statement was proved.

Dynamic geometry software (DGS) was used as a mediating tool during the processes of teaching and learning to prove. By solving problems that provide the students with opportunities to produce conjectures and useful ideas for proving them, the community established a link between the experimental activity of constructing geometrical figures with the DGS and the deductive activity of proving those conjectures. As an objective of this course was to build a portion of an axiomatic system for Euclidean geometry, the didactical contract in the class specified that 'all Cabri constructions have to be made by using only Cabri tools that had their match (axiom, definition, theorem, ...) in the part of the axiomatic system already built' and that 'each step of a Cabri construction has to be justified with statements included in the part of the axiomatic system already built'.

The present report is based on the transcription of audio and video recordings of all class sessions, supplemented with field notes that a member of the research team took while she was in class as a non participant observer. We intend to design a methodological tool that let us use Lave and Wenger's (1991) sociocultural theory in the analysis of the transcriptions. To do it, we have developed a coding scheme to interpret and analyze, from the sociocultural theory, those episodes during the classes having relevance by their connection to the teaching and learning of mathematical proof. In this paper we shall refer to three aspects of the class culture decisive to evaluate if the class can be considered a community of practice whose enterprise is learning to prove: (i) legitimate peripheral students' participation, (ii) teacher's role, and (iii) role of DGS. The analysis presented here is referred to some events happened during the classes corresponding to an episode that we denominated 'Three non collinear points determine two segments that bisect each other'.

The above mentioned coding scheme has been designed after the identification of the following categories of analysis, based on the theoretical framework:

***Joint Enterprise:*** Collective organization of contents in a hypothetical deductive discourse in order to validate them within a theoretical system (Hanna, 2000; Mariotti, 2006). As the

enterprise is clarified, students identify organizational and discursive practices associated with the task of proving and, as the same time, they acquire a meaning for it.

**Shared repertoire:** Set of routines of experimentation, argumentation or axiomatization (Mariotti, 1997; 2000; 2005; 2006) which are useful to explore and give meaning to statements of Euclidean plane geometry. The community use them to argue about the certainty of those statements and discuss how they could be justified inside the axiomatic system, with the aim of producing proofs agreeing with a set of established social and sociomathematical norms (Yackel y Cobb, 1996).

**Mutual Engagement:** Signs of giving by the teacher the responsibility of validation back to the students, through a didactical contract that the teacher settles down (Marrades y Gutiérrez, 2000), aiming that students assume the task of validation inside their community of practice.

In a first step to organize the data, we divided the transcriptions of the class sessions into episodes corresponding to the topics dealt with, like study of a statement, matters related to a shared routine (e.g., using figures, or organizing some statements in a relational network), checking the answers to a task, or development of a Cabri workshop. From those episodes, we identified 114 primary documents that were analyzed to create the coding scheme. For each code, we identified groups of excerpts that we considered useful to do a complete analysis of the cultural practice of proving in this class. As we advanced in the analysis of the episodes, the codes were object of a cyclic debugging process in which we revised their meaning, deleted useless codes, and identified new codes. Each change in the coding scheme made us review again all the already analyzed episodes. The final result of this process was a set of 66 codes; each of them associated to one or more categories of analysis. Table 1 includes a description of the codes used to analyze the episode in next sections.

Code Name	Code Description
<b>Joint Enterprise</b>	
FormalGrade	Excerpts showing evidences of the grade of formalism demanded to write proofs.
RelationProcedureTheory	Excerpts where members of the community establish a relationship between a procedure and the underlying theory.
<b>Shared Repertoire</b>	
DeductiveArgumentToProof	Arguments made aiming to build a deductive chain for validating a statement.
WithWhichWeCount	Summary of the information available to the community that could be used in a proof.
UnlockProveProcess	Suggestions, usually made by the teacher, to unlock a proving process in course when students don't know how to carry it on.
StrategyCabriConstruction	Interventions letting us identify the strategy used by a group of students to make a construction with Cabri.
FormulateTask	Interventions, usually made by the teacher, to begin a mathematical activity, either implicitly or explicitly.
Institutionalize	Interventions, usually made by the teacher, where she puts the community's production in correspondence with the cultural mathematical practice.
Paraphrase	Interventions, usually made by the teacher, where she paraphrases a student's utterance aiming to improve the communication.
ProofProduction	Interventions showing a proof, made either collectively or individually, to be evaluated by the class.

ProposeWayProve	Suggestions of ways to do a proof, even when they are not correct or pertinent.
SynthesizeProof	Summary of a proof made after a student's request or to compile the parts of a process than has been interrupted several times.
<b>Mutual Engagement</b>	
Assign/AssumeResponsibility	Assignment of an explicit responsibility, to be voluntarily carried out outside of class, that is relevant for a good working of the community of practice.
ImportantContribution	Student's contributions that show his/her relevant role in the development of the course and that neither the teacher nor other student had considered.

Table 1. Codes in each category used to analyze this episode.

### The episode 'Three non-aligned points determine two segments that bisect each other'

In this section we summarize this episode that shall be analyzed in next section. The episode took place during three class sessions in the second month of the course. Up to that moment, six postulates and ten definitions had been stated, and eight theorems had been proved, establishing relationships among points and straight lines, the existence of the midpoint of a segment<sup>1</sup>, and the property that the distance from the midpoint to the ends of the segment is half the length of the segment. The first session of the episode was in the computer room, and students were asked to solve, in pairs, part *a.* of this problem:

*Problem:* Given three non-aligned points  $A, B, C$ , construct [with Cabri] a point  $D$  such that segments  $AB$  and  $CD$  bisect each other.

*a. Describe the procedure of construction and justify each step in it.*

*b. Write the theorem that can be stated after the solution to this problem, and prove it.*

Table 2 shows a summary of the process of construction students did<sup>2</sup>. The right column informs on the statements in the axiomatic system giving theoretical mathematical support to the tool used.

Steps in the construction	Cabri tools used	Theoretical support
1. Draw non-aligned points $A, B, C$ .	1. Point.	1. Postulate P1: Planes are sets of points.
2. Draw straight line $\overline{AB}$ .	2. Line.	2. Postulate P3: Given two different points, there is exactly one straight line containing them.
3. Draw segment $\overline{AB}$ .	3. Segment.	3. Definition D6: Segment $\overline{AB}$ is the set of points $A, B$ , and all points [in line $\overline{AB}$ ] between <sup>3</sup> $A$ and $B$ .
4. Draw point $P$ as the midpoint of segment $\overline{AB}$ .	4. Midpoint.	4. Theorem T7: Every segment has a midpoint.

<sup>1</sup> Definition 10: The midpoint of a segment  $\overline{AC}$  is a point  $B$  between  $A$  and  $C$  such that segments  $\overline{AB}$  and  $\overline{BC}$  are congruent [i.e., they have the same length].

<sup>2</sup> A file with this construction can be downloaded from <http://www.uv.es/Angel.Gutierrez/archivos1/problem.fig>

<sup>3</sup> Definition D4, of 'betweenness': Point  $B$  is between points  $A$  and  $C$  when  $A, B$  and  $C$  are in a straight line and their distances verify  $AB + BC = AC$ . Notation:  $A-B-C$ .

5. Draw straight line $\overline{CP}$ .	5. Line.	5. Postulate P3.
6. Calculate the distance $CP$ .	6. Distance or length.	6. Postulate P5: The distance $AB$ between two points $A$ and $B$ in a straight line is the absolute value of the difference among their coordinates.
7. Draw ray $\overline{CP}$ , or draw point $J$ such that $P$ is between $C$ and $J$ , and draw ray $\overline{PJ}$ .	7. Ray.	7. Definition D8: A ray $\overline{AB}$ is the set containing segment $AB$ and every point $C$ such that $B$ is between $A$ and $C$ .  Theorem T4: Given points $A$ and $B$ in a straight line, there is a point $C$ such that $B$ is between $A$ and $C$ .
8. Transfer twice the distance $CP$ in ray $\overline{CP}$ , from $C$ , or once in ray $\overline{PJ}$ , from $P$ . Label as $D$ the endpoint of last transfer.	8. Measurement transfer.	8. Theorem T6: Given ray $\overline{AB}$ and a positive number $x$ , there is exactly one point $P$ in $\overline{AB}$ such that [distance] $AP$ is $x$ .

Table 2. Steps in the construction with Cabri of point  $D$ .

Table 2 raises some restrictions students had when using the tools in Cabri. To draw segment  $AB$ , students could not do it just by using the tool Segment, because, in the part of the axiomatic system already built, a segment had been defined as a subset of a straight line, so students had to draw first line  $AB$  (step 2) and then segment  $AB$  (step 3). In the same way, to draw point  $D$  on line  $CP$ , students could not use the tools Circle nor Compass, because they did not have theoretical support. Instead they had to use the tool Measurement Transfer. Therefore, students had to draw the ray in line  $CP$  with endpoint  $P$  and not containing point  $C$ . To do it, they had to draw a point  $J$  such that  $P$  is between  $C$  and  $J$ . Some groups of students drew ray  $CP$  and transferred twice the distance  $CP$  from  $C$ ; other groups drew ray  $PJ$  and transferred distance  $CP$  from  $P$ .

In the second session of this episode, the teacher asked the students to answer part *b.* of the problem, i.e. to state and prove a theorem related to the construction. The teacher promoted a whole class discussion in which 12 students participated effectively. The teacher asked the students to propose the statements that would made the deductive sequence and to justify them. When necessary, the teacher made comments to students' outcomes, she corrected their ways to express statements, and she wrote on the blackboard the statements and reasons of the proof (in a two columns style) when they had been correctly stated and accepted by the group (see Table 3). The group advanced in writing the proof by associating steps in the proof to steps in the construction made with Cabri (compare Tables 2 and 3). When two students proposed two ways to proceed in step 7 (draw rays  $CP$  or  $PJ$ ), the group decided to accept the proposal to draw ray  $PJ$ . After the students proved the existence of point  $D$ , the teacher had to make them note that, though the construction in Cabri was finished, the proof was not complete because they had not proved that point  $P$  is the midpoint of segment  $CD$ . Some students proposed correct deductive arguments to prove the equidistance from  $P$  to  $C$  and  $D$ , but the arguments they proposed to prove that  $P$  is between  $C$  and  $D$  were not correct. The teacher asked the students, as homework, to think again about this step of the proof. The blackboard showed the part of the proof made during the class (statements 1 to 9 in Table 3).

Statements of the proof	Reasons for the statements
1. Let $A, B, C$ be three non-aligned points.	1. Given.
2. Straight line $\overline{AB}$ can be obtained.	2. Postulate P3.
3. Segment $\overline{AB}$ can be obtained.	3. Definition D6, and statement 2.
4. Let $P$ be the midpoint of segment $\overline{AB}$ .	4. Theorem T7, and statement 3.
5. Straight line $\overline{CP}$ can be obtained.	5. Postulate P3, and statements 1 and 4.
6. Let $J$ be a point in line $\overline{CP}$ such that $C-P-J$ .	6. Theorem T4, and statement 5.
7. Ray $\overline{PJ}$ can be obtained.	7. Definition D8, and statements 5 and 6.
8. Let distance $CP = r$ , with $r > 0$ .	8. Postulate P5, and statements 1 and 4.
9. There is a point $D$ in $\overline{PJ}$ such that [distance] $PD = r$ .	9. Theorem T6, and statements 7 and 8.
10. [Distances] $PD = PC$ .	10. Replace $r$ in statement 9 by $CP$ (statement 8).
11. [Possible cases of betweenness:] $C-D-P$ , $D-C-P$ , or $C-P-D$ . From case $C-D-P$ follows a contradiction [ $C = D$ ]. From case $D-C-P$ follows a contradiction [ $C = D$ ]. [Therefore] $C-P-D$ [is true].	11. Definition D4, and statements 1 and 10. Definition D4, and statements 1 and 10. [Previous parts of] statement 11.
12. $P$ is the midpoint of segment $\overline{CD}$ .	12. [Definition 10, and statements 10 and 11.]

Table 3. Proof of the theorem.

In the third session of this episode, the whole class discussion continued. Now 13 students participated effectively. Three students suggested new ideas to complete the proof:

Juan proposed to draw ray  $CJ$  instead of ray  $PJ$ , but students didn't know how to prove that  $P$  is between  $C$  and  $D$ .

Henry proposed not to draw any ray, but to use postulate P6<sup>4</sup> in line  $CP$  to assign coordinate 0 to  $C$ , and coordinate  $r$  to  $P$ , then to use postulate P2<sup>5</sup> to find a point  $D$  in the line  $CP$  with coordinate  $2r$ . As  $0 < r < 2r$ , after theorem T2<sup>6</sup>, we can conclude that  $P$  is between  $C$  and  $D$ .

Ana proposed to draw first ray  $PC$  and then the ray opposite to it, based on a new point  $D$ . In this way it could easily be proved that  $P$  is between  $C$  and  $D$ , but this suggestion was not adequate because there was not any element of the axiomatic system guaranteeing the existence of the ray opposite to a given ray.

The group analyzed and discussed each proposal, but some time later the teacher asked them to complete the proof with Juan's proposal. As students were not able to prove that  $P$  is between  $C$  and  $D$ , the teacher suggested them to consider all possible betweenness relationships among points  $C, P$  and  $D$ . This teacher's guidance helped students to focus on each relationship and they proved that two of them were not possible, so the last implication in the theorem was obtained. The summary written on the blackboard were statements 10 to 12 in Table 3. Finally the teacher institutionalized this statement as a new theorem because

<sup>4</sup> Postulate P6: Given two points  $P$  and  $Q$  in a straight line, it is possible to define a coordinate system in the line such that the coordinate of  $P$  is 0 and the coordinate of  $Q$  is positive.

<sup>5</sup> Postulate P2: There is a one-to-one correspondence between the points in a straight line and the real numbers (i.e., each element of either set is matched with exactly one element of the other set).

<sup>6</sup> Theorem T2: Given three points  $A, B, C$  in a straight line, having respectively coordinates  $x, y, z$ , if  $x < y < z$  then  $B$  is between  $A$  and  $C$ .

she foresaw that it could be useful later (theorem T5: Given three points  $A$ ,  $B$ ,  $C$  in a straight line, one and only one of them is between the two other points).

### Analysis of the episode

From the point of view of Lave and Wenger's (1991) sociocultural theory, learning can be conceptualized as a process of change in the kind of participation of students in the community of practice of the classroom, from a peripheral participation when they are novice, to a full participation when they become expert. Such process, the legitimate peripheral participation, is one of the characteristics that allow identify a class as a community of practice. Students' participation does not consist of a compromise just at certain moments or in some activities, but their continuous collaboration in the activity of configuring the community's practice itself, and their participation in building an identity with the other members of the community (Wenger, 1998). Let's analyze in this episode some aspects of students' practice that we consider indicators of students' legitimate participation, their participation being peripheral because the community of practice was constituted a short time ago, and students didn't have any previous experience in deductive proofs.

Due to the work students did when solving the problem with the DGS, some students had ideas to share and they could participate actively in the production and justification of statements to be integrated in the proof, by relating the procedure used to get point  $D$  to statements in the axiomatic system they were building. Students' participation was monitored by the teacher, who guided them to progress in getting the proof, and showed them the formal style the proofs should have. The excerpt below shows the interactions in the whole group discussion, with the right column showing the codes associated to the interventions:

136	Teacher (T):	[...] Well, the idea in this exercise was that you do the construction in Cabri, to see ... well, how do we prove the theorem? ... three non-aligned points determine two segments that bisect each other. How do we do it? That is the theorem.	<i>(FormulateTask)</i>
137	Student 1 <sup>7</sup> :	Let $A$ , $B$ , $C$ be [paused by the teacher]	<i>(ProofProduction)</i>
138	T:	So we begin statement-reason. You more or less began ... because you tried to justify each action made in the graphic calculator <sup>8</sup> with something from our theory. Then, how do we begin? Which is the first step?	<i>(RelationProcedureTheory)</i>
139	Student 2:	$A$ , $B$ , $C$ given.	<i>(ProofProduction)</i>
140	T:	Let $A$ , $B$ , $C$ be three non-aligned points; Given [she wrote statement 1 (Table 3) on the blackboard]. This is what I have, nothing else; anything else [we need], we have to generate it. Right? Which is next step?	<i>(FormalGrade)</i> <i>(WithWhichWeCount)</i>
141	Ignacio:	Draw straight line $AB$ .	<i>(StrategyCabriConstruction)</i>
142	T:	I don't need this line in the calculator, because it allows me to draw the segment, but in our geometry ... if I only have two points and I want to have the segment, segment was defined as a subset of a line, so I need the line. Therefore, the second step would be: The line $AB$ can be drawn. Why?	<i>(RelationProcedureTheory)</i>
143	Student 3:	Postulate of the line [P3].	<i>(ProofProduction)</i>
144	T:	Postulate of the line, by using ...? By using what? The last step? Something giving me the points, of course, I need the points	<i>(Paraphrase)</i> <i>(FormalGrade)</i> <i>(ProofProduction)</i>

<sup>7</sup> This label is used to refer to participations of unidentified students.

<sup>8</sup> The students used some times computers and other times, like this one, graphic calculators TI-92.

- to have the straight line. Ok [she wrote statement 2 (Table 3) on the blackboard]. And then?
- 145 Ignacio: We drew segment  $AB$ . *(StrategyCabriConstruction)*
- 146 T: You drew the segment right there, so segment  $AB$  can be drawn. And, what is the justification? *(Paraphrase)*  
*(FormalGrade)*
- 147 Student 4: The definition of segment. *(ProofProduction)*
- 148 T: The definition of segment, by using [step] 2 [she wrote statement 3 (Table 3) on the blackboard]. Of course, we need the segment to make next step. Which one is it? *(Paraphrase)*  
*(ProofProduction)*  
*(WithWhichWeCount)*
- 149 Juan: There is point  $P$ , which is the midpoint [of  $AB$ ]. *(StrategyCabriConstruction)*

The students also collaborated by proposing deductive arguments that would be parts of the deductive chain of the proof. This kind of participation was not frequent in this episode, but we can show a case:

- 208 T: We still have to prove that  $P$  is the midpoint of  $CD$ . Ok?
- 209 Juan: By the definition of midpoint we can affirm that the length of  $CP$  plus the length of  $PD$  is equal to the length of  $CD$ . *(DeductiveArgumentToProof)*
- 210 T: Is that the definition of midpoint? *(RelationProcedureTheory)*  
[...]
- 216 Juan: That the length of  $CP$  is equal to length  $PD$ , and the length of  $CP$  is half the length of  $CD$ .

Students also contributed ideas when the task was to select statements from the part of the axiomatic system already built that could be used to prove a conjecture. This kind of tasks favours abductive reasoning, since students have to decide which one among several available statements can be used as hypothesis to prove the truth of the given conjecture. The excerpt below is an example:

- 240 T: Let's think a bit. I want to show a relationship of betweenness. Which elements of my axiomatic system let me conclude: "ah ... so, do we have a betweenness?" Which ones? I want to show this [she writes on the blackboard  $CP + PD = CD$ ]. Which elements of my axiomatic system tell me "if such thing, then betweenness"? *(WithWhichWeCount)*  
[...]
- 243 Efraín: The first theorem of betweenness [theorem T2]. *(WithWhichWeCount)*
- 244 T: The first theorem of betweenness says ... theorem: If coordinate of  $C$  smaller than coordinate of  $P$  smaller than coordinate of  $D$ , then ... betweenness. So now it seems I should have to work with coordinates, because this is the only I have that can help me, because if I have alignment, ... but, how do I prove this [the betweenness]? What do I need to prove this? *(WithWhichWeCount)*

Eventually, students' participation became even more legitimate, thanks to the compromise assumed by some students (code *Assign/AssumeResponsibility*) to solve the difficulty issued when they followed Juan's suggestion of proof (see previous excerpts). Those students suggested new ways to do the proof (code *ProposeWayProve*), that were analyzed by the

group and, in some cases, produced new statements that were included into the axiomatic system. In this way, the content of the course was not the one previously planned by the teacher, but it was shaped by the group. This teaching methodology is unusual in the university courses of mathematics, where teachers propose the sequence of postulates, definitions and theorems organized to make available the necessary results before a new theorem is proved. Although our teaching methodology may be less rigorous than the traditional one, it has the advantage that students live an experience of creating mathematics nearer to the professional mathematicians.

The excerpt below shows two students proposing different ways to try to get the proof that, furthermore, induced the discovery and proof of new theorems:

- 269 Juan: I have another way. Something homologous. *(ImportantContribution. Thanks to this suggestion, a new theorem of betweenness, T5, was stated, proved and included into the axiomatic system)*  
 Drawing again [line]  $CP$  and ray  $CP$ , not [ray]  $PJ$ , but [ray]  $CP$ .  
 [...]
- 283 Ana: And with opposite rays ... could us work this way too? As we have ray  $PC$ , then we define ... *(ImportantContribution. This proposal led to prove the existence of the ray opposite to a given one, and to include this new theorem into the axiomatic system)*

As students practiced more and more in writing proofs, the grade of formalism demanded by the teacher was negotiated (code FormalGrade), and students acquired a better feeling of the meaning of proving into their community of practice. The excerpts above show the compromise acquired by the students to participate in the class activity; this is a necessary condition for the existence of a community of practice.

### Role of the teacher

In an educational context like a course at the university, one should not expect that all members of the community of practice (the group of class) shall act in the same way, since the teacher is the expert in charge of introducing the students in the new mathematical practices. But, according to Wenger's (1998) theory, a classroom where the teacher is the only bearer of knowledge cannot be considered as a community of practice. Then, to constitute a community of practice in a university class, the teacher has to transfer responsibility of mathematical practices to the students and has to assume the role of illustrating how to legitimately participate in the community.

The analysis of the episode described on the light of the codification we have designed gives us some signs of the instructional scaffolding (Bruner, 1984) designed by the teacher to delegate mathematical practices to the students and help them learn to do such practices. A significant aspect of this scaffolding was the formulation of tasks to generate a legitimate mathematical activity focused to the objective of learning to prove theorems although, at the same time, the teacher, as the only expert in the community, had to retain the responsibility of the functions of scaffolding that cannot be transferred to the students, like giving suggestions to unlock a stagnated proof (code UnlockProveProcess), synthesize the work made (code SynthesizeProof) or institutionalize it (code Institutionalize). An example is to formulate the task (code FormulateTask) to construct point  $D$  with the DGS, then to prove its existence based on links among the steps of its construction and the axiomatic system, to state the

theorem raised from the construction, and to look for ways to prove it. At the same time, the teacher retain the responsibility to control the communication in the group, by paraphrasing students' sentences (code Paraphrase) and showing them how to express correctly a statement, how to use mathematical symbols and how to organize a proof (code FormalGrade).

### **Role of the DGS**

The episode described is a good example of the characteristics of the interaction of students with the DGS. The ways they used Cabri have a very particular style, specific of this community of practice and mediated by its norms of use of the software. As we have shown, it is possible to identify in the conversations among students frequent references to theoretical justifications of their actions, dragging made and tools used (code StrategyCabriConstruction) as a way to verify the correctness of their constructions, which is a part of the didactical contract regulating the community of practice.

The norms about the way of using DGS established in the community cause a limitation in the possibilities of use of Cabri, but this is necessary to make the DGS serve the objective of the community and to be a shared resource. Although we have not shown here any example, in the episode described students used dragging to verify that the construction was well made and its steps could be linked to statements of the axiomatic system. Those links helped to establish a strong relationship between experimentation, argumentation, and proof.

### **Conclusion**

The analysis made in this paper is an example of the usefulness of Lave and Wenger's (1998) theory when we adopt a sociocultural position to give light to important aspects of learning mathematical proof different from epistemological and cognitive ones. A global analysis of the teaching experiment, still to be made, should show the progress of students in learning to prove, parallel to an evolution in their practice of proving, a higher engagement within the community of practice, a wider shared repertoire of routines, actions and tools, and a deeper understanding of mathematical proof.

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