Task Design based on *Purpose* and *Utility*

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Framework which guides task design

This framework arises from the conjecture that engaging purposefully in the use of mathematical ideas in a well-designed task leads to learning which is different from that which might arise when practicing an associated technique or exploring why that technique works in more traditional classroom tasks (Ainley, Pratt and Hansen, 2006). It offers a new perspective on the issue of creating opportunities for pupils to learn about the ways in which mathematical ideas are useful, using the linked constructs of *purpose* and *utility*.

Purpose, within this framework, refers to the perceptions of the pupil rather than to any uses of mathematics outside the classroom context. There is considerable evidence of the problematic nature of pedagogic materials which contextualise mathematics in supposedly real-world settings, but fail to provide *purpose* to which the learner can relate, either in terms of the overall task, or the ways in which mathematical ideas are used within it (see for example Ainley, 2000; Cooper and Dunne, 2000). The purpose of a task, as perceived by the learner, may be quite distinct from any objectives identified by the teacher, and does not depend on any apparent connection to a 'real world' context. The purpose of a task is not the 'target knowledge' within a didactical situation in Brousseau's (1997) sense. Indeed it may be completely unconnected with the target knowledge. However, the purpose creates the necessity for the learner to use the target knowledge in order to complete the task, whether this involves using existing knowledge in a particular way, or constructing new meanings through working on the task. Movement towards satisfactory completion of the task provides feedback about the learner's progress, rather than this being judged solely by the teacher (Ainley et al, 2006, Ainley and Pratt, 2005).

Within such purposeful tasks there is the possibility of creating opportunities to understand the *utility* of mathematical ideas. The *utility* of a mathematical idea is defined as how, when and why that idea is useful. Traditional approaches to teaching mathematics in school address instrumental understanding of procedures, and relational understanding of mathematical concepts (Skemp, 1976), but generally fail to address the *utility* of these ideas. The pedagogic tradition, embodied in textbooks around the world, is to begin with procedures and relationships, and to address utilities as the final stage in the pedagogic sequence (if at all). We conjecture that *utility* is not merely an application of a concept but a separable dimension of mathematical understanding, alongside the instrumental and relational components.

Some explicit design principles

This section contains some heuristics for creating purposeful tasks that have the potential to associate that purpose with defined utilities.

a) The task has an explicit end product that the pupils care about. This might be a physical object, a virtual object created on the computer, or (as in the case of the task outlined below) a method to achieve a particular goal.

b) The task involves making something for another audience to use. Creating a something for younger pupils to use can be particularly powerful, but even an imagined audience imposes constraints which help to focus attention on aspects of the task.

c) The task is well focused, but still contains opportunities for pupils to make meaningful decisions about the ways in which they work on it.

d) Tasks which involve optimisation problems are often sufficiently intriguing to generate purposeful activity.

An example of a task based on the framework

The context for this task is a computer-based game for a school fair in which players try to make a target number (61 in the solution in Figure 1) by placing the numbers 1 to 5 in the cells in the left-hand column of this spreadsheet:

	A	B	C	D	E
1	1				
2	3	4			
3	5	8	12		
4	4	9	17	29	
5	2	6	15	32	61

Figure 1: a completed game

The target number is set by the stall holder. Each time a player wins by achieving the target, the stall holder must choose a new set of five starting numbers and set a new target. The overall purpose of the task is for pupils to be able to advise the stall holder on what the target number should be for any particular set of five starting numbers.

The task is set in three stages. The first step asks pupils to identify the ‘rule’ that is used to calculate each number in the grid, and then set up their own spreadsheet to play the game. This involves using spreadsheet formulae. Once they have done this, their challenge is to play the game and make the highest total they can with the numbers 1 – 5.

The next stage is to find a strategy for arranging any set of starting numbers to get the highest total. At this point pupils have the freedom to experiment with different sets of numbers, and to increase or decrease the size of the set.

Once pupils have a robust strategy, they are challenged first to explain why it works, and then to find a method that they can give to the stallholder to calculate the highest total for any set of starting numbers. This must be a method which the stallholder can use quickly, each time a new target is set. Pupils have used spreadsheet notation to express the rules for calculating the grid, but the nature of the spreadsheet formulae disguises the underlying structure of the problem. The need to identify this underlying structure is driven by the purpose of providing advice for the stall holder. Pupils are therefore encouraged to move away from the computer and use a more standard algebraic notation to represent the numbers, combining and simplifying expressions to make this structure more transparent.

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