ICME-11 TSG 34 Research and Development in Task Design and Analysis
Proposal for Presentation

Variational Tasks in Dynamic Geometry Environment

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INTRODUCTION

Dynamic geometry environment is a powerful milieu in which the teaching and learning of plane geometry could be re-conceptualized due to the drag-mode (see for example the discussion in Lopez-Real & Leung, 2006). Indisputably, dragging in dynamic geometry plays a crucial role in the formation of geometrical concepts and conjectures (see for example, Arzarello et al, 2002; Leung and Chan, 2005). In particular, the epistemological values of robust and soft constructions with respect to dragging have been the foci of many studies in dynamic geometry (see for example, Laborde, 2005). Though fruitful insights have been obtained, there is yet a deep understanding on how students perceive the relationship between the spatio-graphical representations (what they actually see on the computer screen) and the theoretical knowledge (the traditional school geometry they learnt). The variation visualized from the drag-mode in dynamic geometry may give students a new reasoning pattern that diverges from the traditional deductive thinking. In particular, the (pseudo) accurate representation of geometrical objects and measurements under dragging offers a confluence of simultaneities that could bring about discernments that might be different from a static paper-pencil environment. Sinclair (2004) concluded in her study on Grade 12 students working on pre-constructed dynamic geometry sketches that “students who focused on visual information and drew conclusions based on the appearance of the onscreen image did not have the tools to develop and communicate a proof based on visual concepts” and this was due to students’ unawareness of the accurate representation in dynamic geometry sketches. It would be interesting to probe into the impact of accurate representation and variation under dragging on students’ spatio-graphical reasoning that leads to explanations of phenomena in dynamic geometry environments. In this respect, we initiated an experimental instrument to assess students’ geometrical understanding in the context of ICT activity in a mathematics competition for regional primary schools in Hong Kong (Lee, Wong & Tang, 2004; Wong, Lee & Tang, 2005). The instrument was in the form of dynamic geometry manipulative tasks in which students can vary a point (a dimension of variation) in a geometrical configuration via dragging. Students were asked to drag the point to a position that would satisfy certain required condition. A coded and varying (as the point was dragged) numerical value was associated to the point for the purpose of recording students’ answers. This opened up a new arena for quantitative analysis in dynamic geometry research that might yield interesting collective information on students’ different ways of interpreting spatio-graphical data in dynamic geometry environment. Subsequently, we have experimented with groups of secondary 1 and 2 (Grade 7 and Grade 8) students in Hong Kong using this instrument. After patterns are observed in the quantitative analysis, selected students are invited for one-to-one clinical interviews during which they will work on a specific manipulative task while the interviewer will probe them with questions on geometrical understanding of the task. The instrument was refined and upgraded recently as a web-based platform to facilitate management and delivery of tasks, collection and organization of students’ responses. In particular, students’ work can be recorded and collated as ‘example-spaces’ (Watson and Mason, 2005) for research and pedagogical purposes. This supporting platform is crucial for ongoing development of tasks as building blocks of classroom teaching and learning activities.

TASK DESIGN

In this presentation, we will illustrate the essential features of the instrument using a series of variational tasks designed for the web-based platform for diagnosis and learning of basic school geometry concepts. Each task consists of a dynamic geometry figure with an instruction for students to perform a particular manipulation on the figure. We use Geogebra to create these dynamic figures which are then exported as Java applets accessible by students through ordinary web browsers.

The basic component of a task is a dynamic geometric figure. The primary mode of manipulation is dragging of movable points in the figure. Through this direct manipulation, a student is also simultaneously varying the configurations, measurements and their relations inside the figure. When designing a task, attention is given to the following major features.

(A) A specific area of the geometry curriculum is chosen as the focus of each task. This may help students quickly relate a task with their familiar content of school geometry. However, this does not mean students only need a narrow range of tools and knowledge for completing a task. In fact, there are often opportunities for students to consider a number of concepts not immediately linked together in usual textbook exercises.

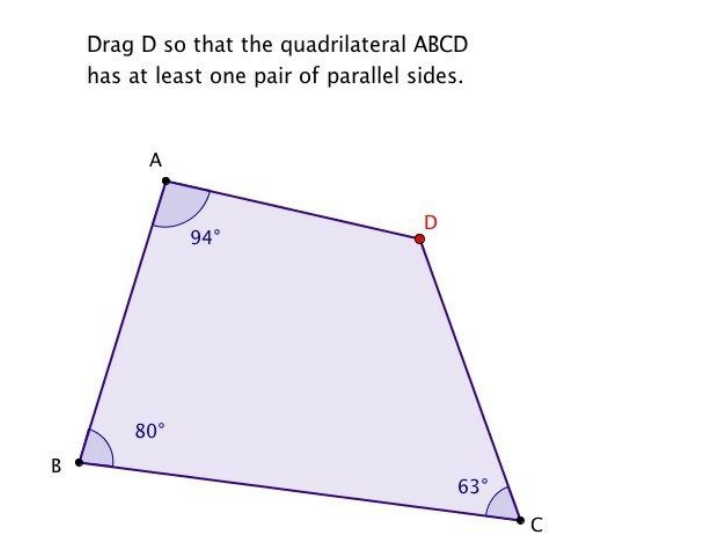
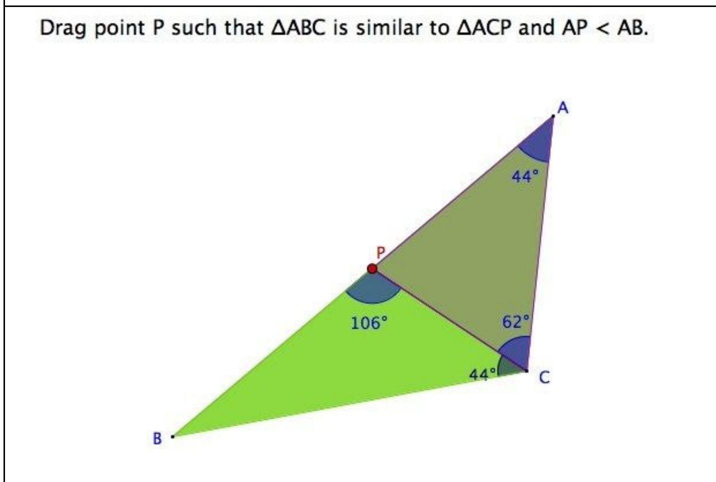
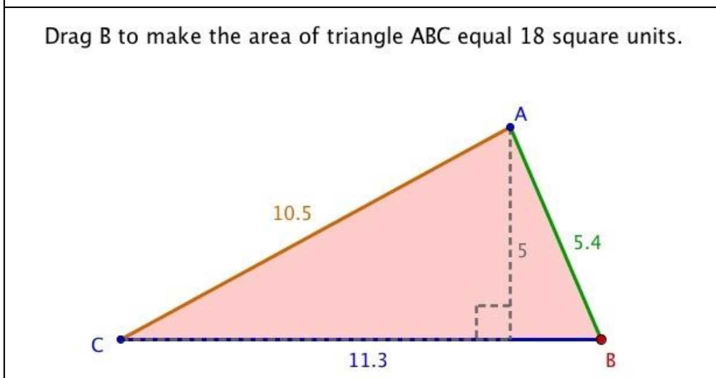
(B) Possible manipulation of a figure is determined according to which parts of it and how they are allowed to be dragged. Other parts are fixed or constrained due to the way the figure is constructed. A task usually starts with a relatively free figure where students are required to drag it until certain conditions appear.

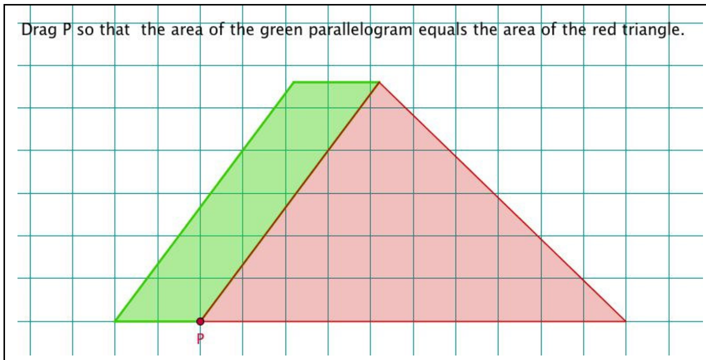
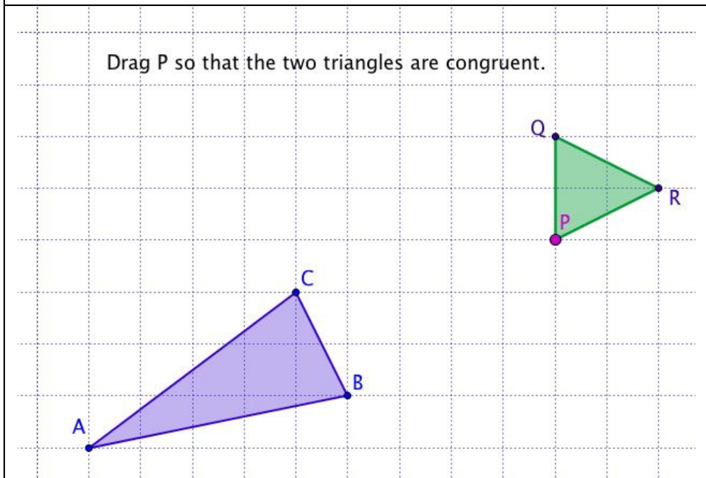
(C) Manipulation can be guided (or misled) by the simple measurements provided. These can be lengths, areas, angle sizes of parts of a figure, which are displayed and continuously updated under dragging. Provision of a square grid or other patterns in the background may also allow some indirect measurements or manipulation in a different way.

(D) Results of students' dragging are easily recordable. This is possible because variation on the whole figure is the result of changes in one or few critical movable parts. Consequently, comparisons among students' responses are possible (particularly with our specifically designed platform).

These features, although basic in dynamic geometry environment, correspond to important principles in the theory of variation (Leung, 2003; Marton & Tsui, 2004) that underlie the design and analysis of these tasks. Each dynamic figure is basically the result of a geometric construction, either 'soft' or 'robust' (Laborde, 2005), constrained by the ways parts are put together by the authors. Such constraints, and associated freedom, give rise to the 'dimensions of possible variation' and 'ranges of permissible change' which are usually at the heart of the design of task or learning activities initiated by the task (Mason & Johnston-Wilder, 2006).Without knowing the details of a construction, students explore the variability of the figure by dragging. Based on their understanding of how the figure may vary deriving from their dragging experiences, they are required to drag to make certain conditions appear or disappear in the figure (e.g. to make two sides of a quadrilateral parallel). In contrast with Van Hieles' characterization of different levels of geometric thinking of students (van Hiele and van Hiele-Geldof, 1986; Burger and Shaughnessy, 1986; Fuys et al, 1988), we concur with the view on a student's structure of attention proposed by Mason and Johnston-Wilder (2004, 2005) where what aspects a student is attending to when dealing with a geometrical situation is not considered as indication of that student's ability but only what he/she is attending to at any moment.

EXAMPLES OF TASKS

	Task 1: Learners are required to drag the point D to make at least one pair of parallel sides in the quadrilateral. D is the only movable point in the figure and can be dragged in any direction. Measurements of three of the angles of the quadrilateral are given.
	Task 2: The only movable point is P, confined to the segment AB. Learners are required to make two specified triangles similar and some angle measurements are given. This task was developed by a group of teachers. It resembles some typical exercises in textbook for proving similar triangles when some angles are given. These teachers would like to know whether the students could really perceive a pair of similar triangles when they change the shape.
	Task 3: Point B can be moved horizontally. Lengths of BC and BA will vary with B. The altitude from A is fixed and shown.

	Task 4: When P is gradually dragged from left to right horizontally, the green part remains a parallelogram and its area increases while that of the triangle decreases. Compared with task 3, this one also deals with area, but learners do not directly work with numerical measurements. Understanding how area varies is crucial for solving this problem.
	Task 5: P can be dragged in any direction. The other points are fixed. This task focuses on the conditions for making triangles congruent, but relies very much on comparison of lengths. Compared with task 4, this one requires more sophisticated use of the square grid.

We focus on simple manipulation in each task, which then allows a new methodology for analyzing students' work. One crucial step is to capture and organize students' responses to these tasks to facilitate diagnosis and further use in classroom learning situations. This is implemented through a website specially designed for managing and delivering the tasks. Students can submit their 'answers' to the website when they finish manipulating a figure. In some open-ended tasks, such as Task 1, multiple submissions are allowed. When receiving a student's submission, the database will store the entire figure as well as some specified numerical indicators. For example, in Task 1, the coordinates of D will be specifically recorded although the entire submitted figure is also captured. In Task 2, the author specifies that the value of angle ACP will be recorded with the figure.

Apart from answer checking, the numerical indicators provide a new quantitative means to analyse students' dragging behavioir on dynamic figures, and in particular for these tasks, their perceived possible variations subject to the given constraints. For example, regarding Task 1, Figure 1 shows the positions of point D chosen by the students. A frequency count (not directly shown in the scatter plot) indicates that the majority gives a parallelogram. An interesting pattern, indicated by the red dots, reveals a special group of choices making a pair of equal opposite angles but no parallel sides (e.g. Figure 2). Figure 3 shows an example of another type of interesting results where the angles at B and C are made equal. Dots along the line through CD in Figure 3 indicate other choices of this kind. Similarly, there are also cases where another pair of adjacent angles is made equal (Figure 4).

Task 2 is designed by a group of teachers, reflecting their interest in knowing how well students can conceive a pair of similar triangles in a common textbook diagram. The dot plot in Figure 5 effectively summarizes students' choices based on the values of the recorded angle ACP in the figure. In the middle of Figure 5, there is a tabulation of students' results on the website that is available to the teachers. The right-most column shows the values recorded for each submission. Clicking any value there will open a pop-up dynamic figure submitted by that student. In the next stage of this website development, there will be thumbnails automatically generated from a class' submissions, which can be clicked and enlarged, more or less like a photo album. Without indicating correctness and students' identity, teachers can discuss with the class some representative results.

Figure 6 and 7 show more tabulation of results in Task 3 and 4. For Task 3, the length of the base BC of the triangle is recorded. Apart from the correct answer, students' choices concentrate quite obviously on two lengths of 3.6 and 6.0. For the Task 4, length of the base of the parallelogram is recorded. Students' choices concentrate in the interval from 4 to 5. While no numerical measurement is given, it is interesting to know what aspects of the figure that students focus on when comparing the areas.

Making use of 'learner generated examples' and the shared 'example space' (Watson & Mason, 2005), or 'outcome space' (Marton & Booth, 1997), will be the focus of integrating these variational tasks into pedagogic situations. We expect that inside the classroom, when various typical figures generated by students can be easily retrieved and examined from the system (without indicating which ones are correct), students can justify and compare their choices, sensitize to different ways of understanding the geometric figure.

DISSCUSSION

Geometry learning through practical tasks has been the focus of pedagogical innovations since the beginning of last century (Fujita & Jones, 2003). Better integration of the intuitive aspect of geometry learning with the deductive theoretical components is still a great challenge for geometry teaching. Practice in all sorts of drawing and construction does not merely develop skills in making correct and accurate diagrams; cognitively it should help learners focus on or be aware of essential relations and properties present in the figures. Often accurate measurements have to be considered in order to support careful examination of geometric relations (e.g. Godfrey & Siddons, 1919; Eggar, 1903).

Measurement

In our dynamic geometry tasks, learners are basically engaged in 'constructing' specific geometric objects. This kind of construction in a broader sense, although not requiring usage of sophisticated tools to create figures from scratch, does involve learners' choices in making instances of geometric relations through dragging. Use of measurement results, in this process, is found to be more crucial to our task design than originally expected. First, choice of measurements to be displayed constitutes formation of different dimensions of possible variation. Second, students' interpretation of the given measurements during their work in the tasks may be very different from teachers' normal assumptions. The traditional approach to curriculum design usually emphasizes the learning of measurement as a practical side of geometry and encourages disassociation from the study of theoretical properties. Analysis of the use of measurements in the design of our tasks may help us better integrate the experimental and theoretical aspects in the learning of geometry. In Olivero and Robutti (2007), proposed strategies in using the measurement tool in dynamic geometry was discussed in the context of forming conjecture and proof. We do not only refer to the use of direct measurement results shown on the screen. Learners also have to deal with implicit measurement concepts for accurately manipulating a figure. One common scenario is handling figures on a square grid. The ability to judge reasonably change and relation in length, area and direction of geometric objects on the grid while dragging a figure is closely linked to understanding of various basic geometric notions (Brock & Price, 1980; Vighi, 2005). Moreover, dragging a figure to change its measurements turns the measurements into objects of operation. Consequently, on could focus on understanding how measurements 'behave' rather than how they can be found (Wittmann, 1996, pp.158-9).

Visualization

An ever ongoing quest in the integration of technology in mathematics education is how visual information could link to, or even merge with, formal representations of mathematical concepts. Noss, Healy and Hoyles (1997) explored the relationship between learners' actions, visualizations and the means by which these are articulated in a computer dynamic environment. They remarked that "a central challenge for the design of mathematical learning environments is to make visible that which is normally visible only to the mathematical cognoscenti." (ibid. p.231) Technology indeed has the power not only to make visible, but even to amplify our dynamic imagination that often contributes significantly to the development of mathematical knowledge. In Presmeg's survey paper on research on visualization in learning and teaching mathematics, she remarked that "the software facilitates visualization processes … which may clarify and further the solution to a mathematical problem by providing insight, thus suggesting productive paths for reason and logic." (Presmeg, 2006, p220) Fischbein(1993) proposed the idea of figural concept which is a fusion between figure and concept. Visual image "possesses a property which usual concepts do not possess, namely, it includes the mental representation of space property." (Fischbein, 1993, p141) We look forward to using our task design together with the web-based platform to explore students' visual reasoning in a dynamic geometry environment to see if patterns or structure of awareness emerge.

We recognize that this type of manipulatable dynamic figures for geometry learning abound in many educational websites providing learning materials in the form of Java applets or Flash movies. Therefore, this project is not only about putting dynamic geometry tasks on the web. Our focus is, on one hand, developing a platform that can facilitate quantitative and qualitative analyses on students' responses; in hope of bringing about deeper insight on how students understand geometrical concepts. On the other hand, we focus on the development of dynamic geometry tasks design by school teachers using common dynamic geometry software or by other resource developers using general authoring tools. At the moment we are not exploring students' problem solving process that requires familiarity with the tools provided by dynamic geometry environment other than dragging.

For discussion in this TSG, participants can access some tasks at the following link and get some more details about the project website there:
http://web.hku.hk/~amslee/geocad/

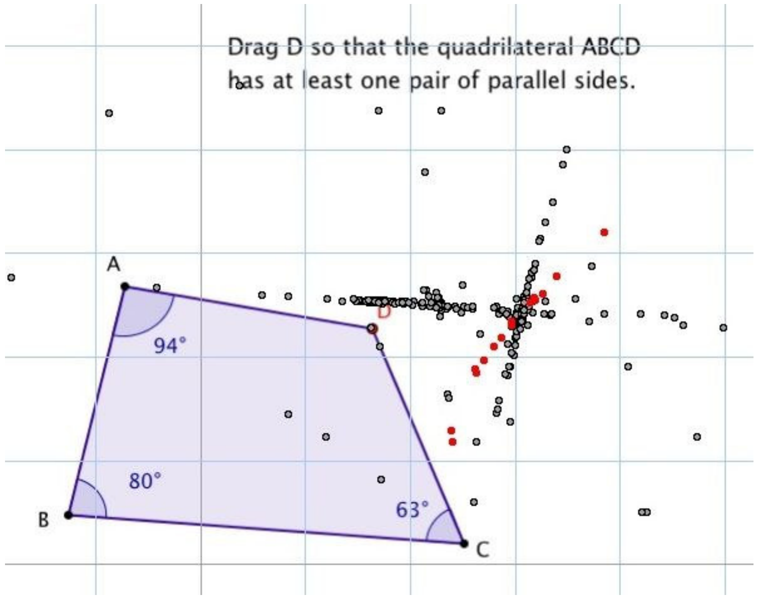


Figure 1

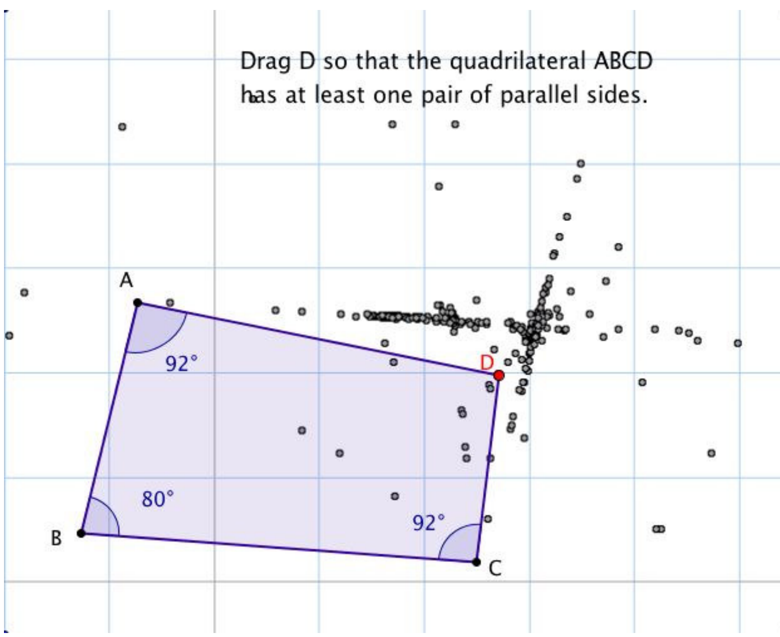


Figure 2

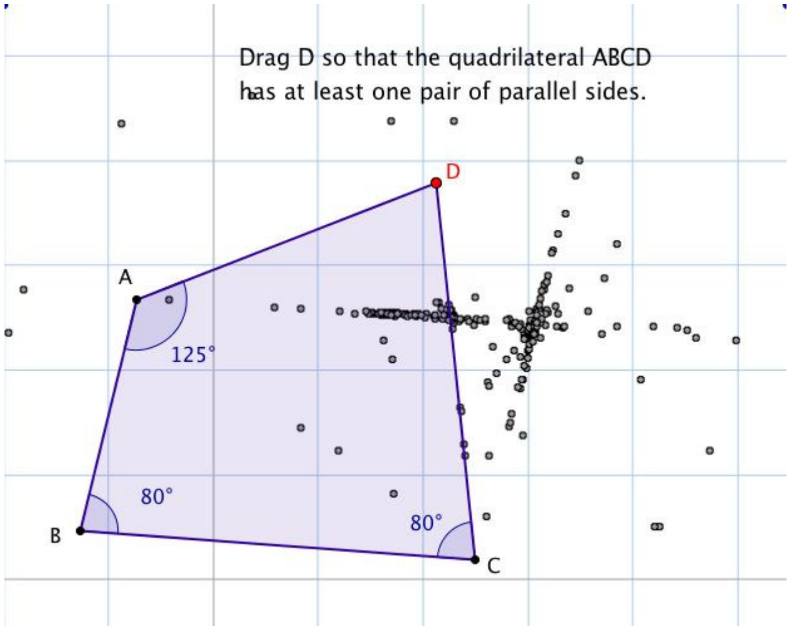


Figure 3

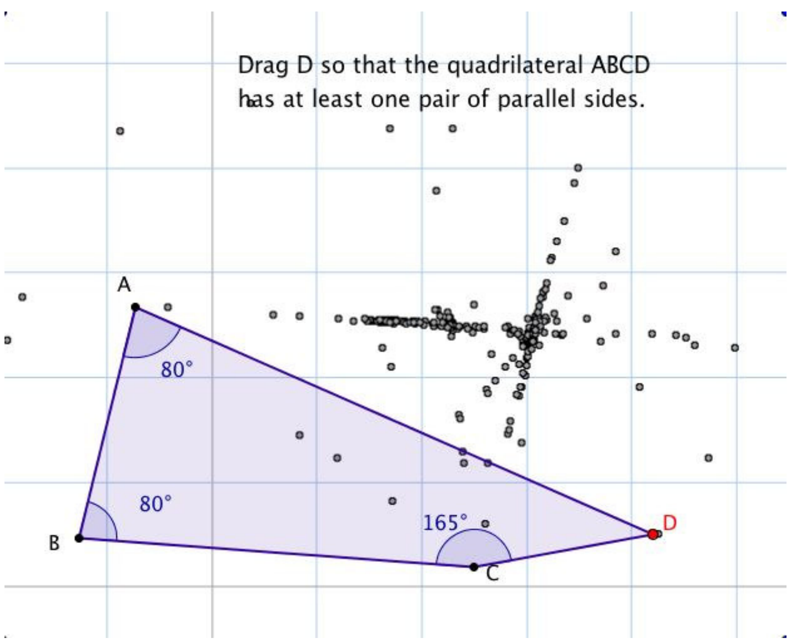
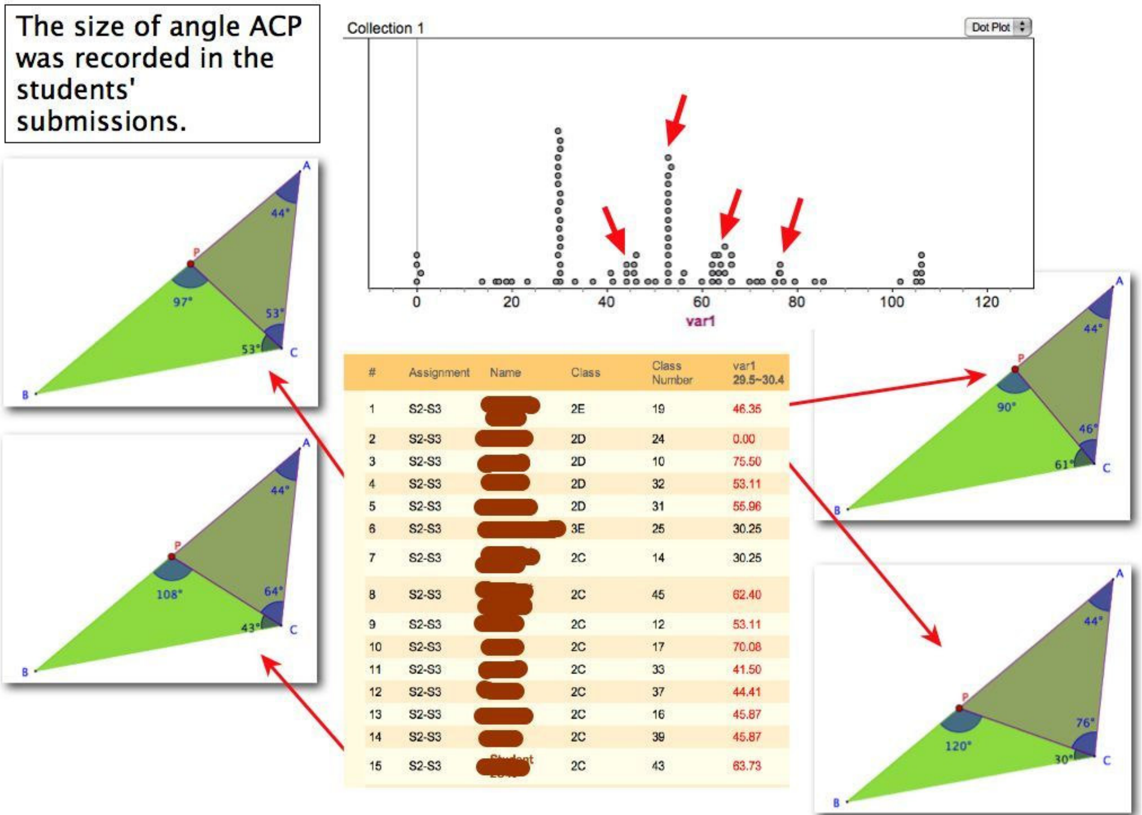


Figure 4



#	Assignment	Name	Class	Class Number	var1 29.5~30.4
1	S2-S3	[redacted]	2E	19	46.35
2	S2-S3	[redacted]	2D	24	0.00
3	S2-S3	[redacted]	2D	10	75.50
4	S2-S3	[redacted]	2D	32	53.11
5	S2-S3	[redacted]	2D	31	55.96
6	S2-S3	[redacted]	3E	25	30.25
7	S2-S3	[redacted]	2C	14	30.25
8	S2-S3	[redacted]	2C	45	62.40
9	S2-S3	[redacted]	2C	12	53.11
10	S2-S3	[redacted]	2C	17	70.08
11	S2-S3	[redacted]	2C	33	41.50
12	S2-S3	[redacted]	2C	37	44.41
13	S2-S3	[redacted]	2C	16	45.87
14	S2-S3	[redacted]	2C	39	45.87
15	S2-S3	[redacted]	2C	43	63.73

Figure 5

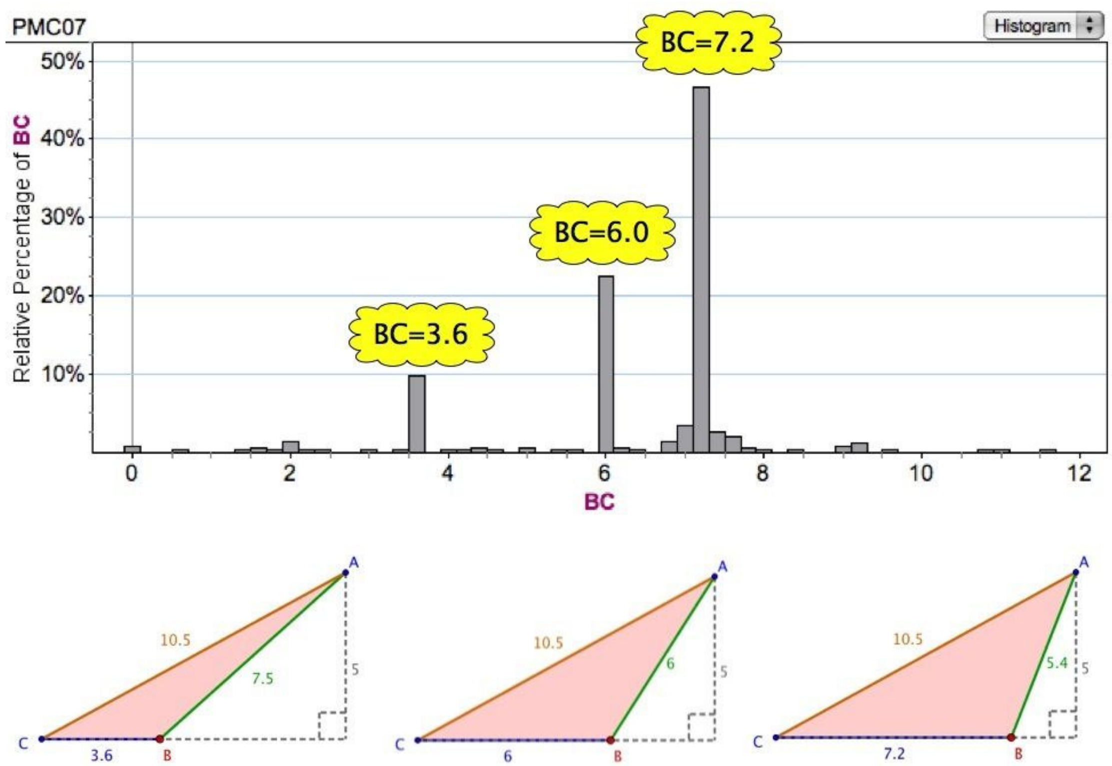


Figure 6

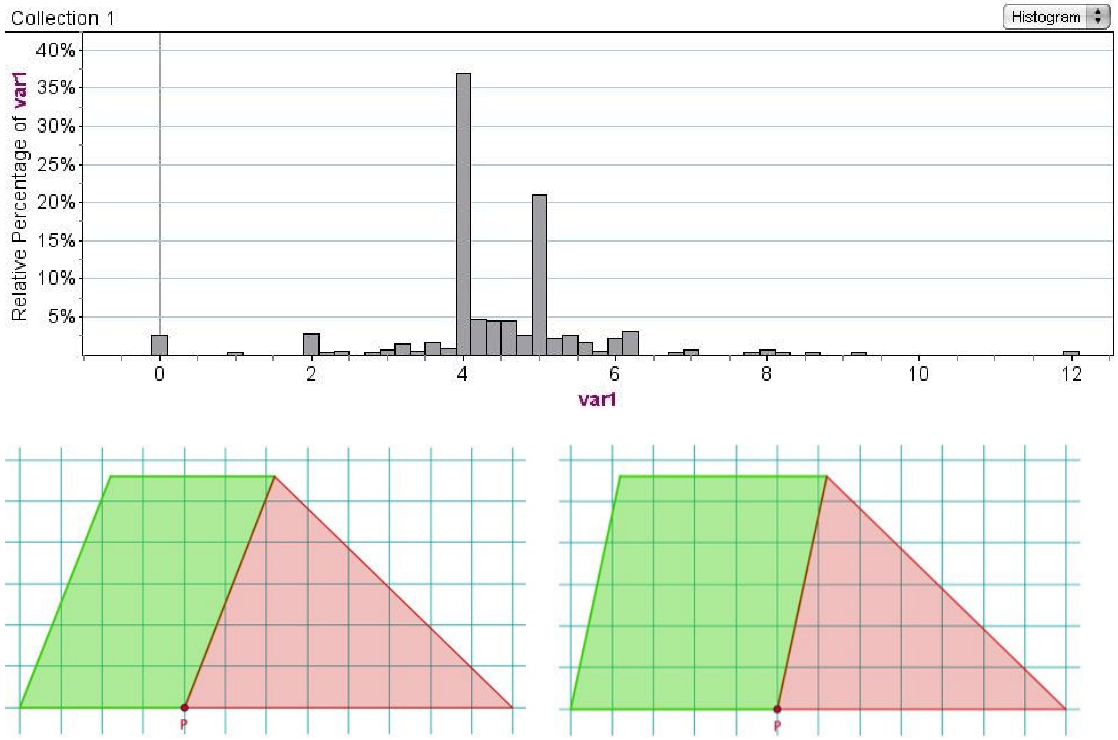


Figure 7

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