ORGANIZATION OF THE STUDY IN 10th GRADE CLASSES ANALYSIS OF THE DIDACTICAL CONTRATS[1]

Floria Arias
Universidad de Costa Rica
fariast@cariari.ucr.ac.cr

Andrea Araya
Universidad de Costa Rica
andreaaraya@racsa.co.cr

Abstract
In this paper are presented the different "periods of study" that describe the progression of the teaching strategies applied by two teachers of tenth grade, in math classes during the study of the polynomial factorization. The first identified periods of study are analyzed, in terms of the didactic contracts that regulate the interactions of the actors in class.

During 2006, as part of the field study of a doctoral thesis, one of the authors of the article performed a cycle of observations in five months, in four tenth year math classes. Some of the analysis presented here, are part of the results of such investigation. This article emphasizes the study of the didactic contracts identified in the classes of the participant teachers (named Sam and Ron, each of them in charge of two classes). It's described and analyzed the interactions that characterize two "strategies" of teaching of the topic of polynomial factorization through common factor and cross multiplication method.

The article is structured in three parts. The first one describes the principal notions of reference that guide the performed analysis: the notion of periods of study and the didactic contract. Following we describe and analyze, for the classes of each teacher, the didactic contracts identified in the first periods of the study. Finally, refer to some of the conclusions of the performed analysis.

Reference Notions

We call the 'organization of the study' the structure determined by the 'periods' of the lessons characterized by the actions that regularly performs the teacher in the class. That is, from a simple way, refer to the phases that students frequently follow to study a specific topic. In this sense, the "period" term is used in a chronological way. The identified periods have been analyzed in terms of the didactic contracts.

The didactic contract described by Brousseau (1998) as the set of the rules that determine what pupils and teacher "have the responsibility to carry on, and of what each one is

[1] Contribution to the second "type of information": "*Identification and analysis of classroom practices and their conditions…*"

responsible in some way”[2]. (Brousseau, 1998, p. 61). Extending this definition, Sadovsky (2005) describes this contract as a keen game in which the teacher communicates “sometimes explicit and many other times implicitly, through words and also through gestures, attitudes and silences, aspects linked to the functioning of the mathematical affair that is treated in the class” (p. 37). In such a way that during the process, “meanings are negotiated, mutual expectatives are transmitted, methods of performing are suggested or infered, mathematical norms are communicated or interpretated (in an explicit or implicit manner)” (p. 38).

During the *VIII Ecole d’été de Didactique des Mathématique* in 1995 in France, Brousseau presented in the course 2: *Les stratégies de l’enseignant et les phénomènes typiques de l’activité didactique*[3], a typology of possible contracts in the class. According to the author, such contracts

> concern, first, the emission of knowledge – communication, validity, novelty, value, interest or cultural status – and the conditions in which these could manifest, be received, learned, reproduced, etc. (Brousseau, 1996, p. 17).

Among the contracts exposed by the author, are of particular interest for the performed analysis, the ostension contrat, the “*mayeutic socratic*” and the ‘rappel’ contract. In the following section, such contracts are described during the analysis of the first periods of the class.

Analysis of didactic contracts

The analysis of the didactic contracts present in the observed classes begins from the identification of the periods of study. These periods were identified based on the structure recognized in these classes of each teacher.

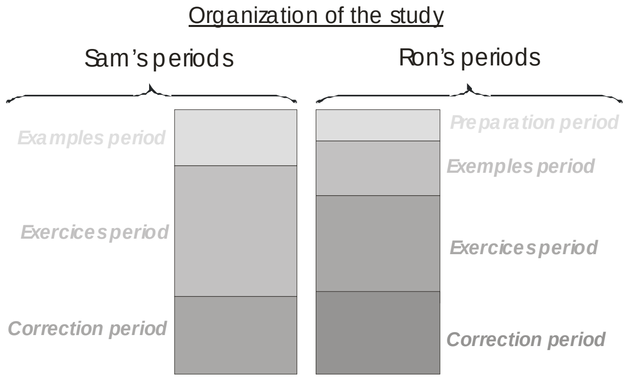


Figure 1: *Sam’s and Ron’s study periods*

Sam’s classes can be described from three periods of study: examples period, the ones dedicated to the exercises and

[2] The translations in the paper are free, performed by the authors.
[3] “Teachers strategies and typical phenomena of didactical activities”.

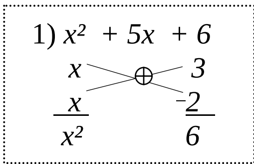
the correction period[4]. From Ron lessons, four periods of study were determined: the period of preparation, of examples, of exercises and correction (see Figure 1).

Following, we analyze the first periods of study identified in the classes of each teacher, in terms of the didactic contracts that regulate the interactions of the actors in the class.

Sam's classes

During the first period, examples period, students are occasionally interviewed about the knowledge (the *savoir-faire*, the vocabulary or some notions) the teacher supposes as necessary for the study of the new knowledge. So, once the topic to study is announced, the teacher performs in the blackboard an example of the kind of exercises proposed in the class.

In general, the knowledge responsibility is always in charge of the teacher. The teacher shows an object (property, technique, example, etc.) and the students accept 'to watch it "as a representative of a class, about which, may recognize their elements in other circumstances." (Brousseau, 1996, p. 46) or objects. In other words, the interchanges between actors are ruled by the ostension contract. As the author indicates, the contract permits the teacher to communicate knowledge, avoiding the action and formulation situations[5]. Let see an example extracted from the first class about the cross-multiplication method to factorize polynomials. At the beginning of the lesson, Sam announces the new topic. Then, he writes on the board the polynomial $x^2 + 5x + 6$, and asks if we can factorize through one of the up to know studied methods[6]. Since none is possible to be used, he indicates that the cross-multiplication method must be used.

PROF=Sam	[…] let use the cross-multiplication method. Consists as follows. Will seek two expressions that multiplied result in this x^2, two numbers or two expressions, right? And two that multiplied result in a 6.
S=F	$x \bullet x$.
PROF=Sam	Well, we can write $x \bullet x$. Also could be written $x^2 \bullet 1$ right, result in x^2. Well… decided to write $x \bullet x$. Then write two that result in, 6.
Ss	$3 \bullet 2$
PROF=Sam	Then think in all factors that has 6. 6 × 1, 1 × 6, 3 × 2, 2 × 3. Now, which of those are helpful to me, in what should I be based to choose the pair. Then, numbers I write here, may be combined with those I wrote here, or expressions, in such a way that when I 

4 Due to space, and as mentioned, will expose the contracts set during the first periods: period of examples in Sam's classes, and period of preparation and examples in Ron's classes.

5 "The sequence of the "situations of actions" constitutes the process by which the student builds strategies, that is, "teaches himself" a resolution method of his problem" (Brousseau 1998, p. 33). The formulation situations, "allow to progressively set a language that everyone understand and that considers the objects and the pertinent relations of the situation in an adequate manner (that is, allowing the useful reasoning and actions)" (Idem, p. 36).

6 In other words, factorisation using the common factor, by grouping "two and two", or using any "notable product": difference of two squares, perfect squares, etc.

	cross or multiply them obtain as result that 5 *x*, or the center one say. I cross multiply and add it; so let say that if I write here, 6 • 1. 6 • 1, 6. Right, it is correct. Now, *x* • 1, how much will it result in? *x*
Ss	*X*

Observe that in the example, Sam's affirmation: "Well, we can write *x* • *x*. Also could be written x^2 • 1 right, result in x^2. Well… decided to write *x* • *x*." illustrates another clause of the contract identified in this first period. It is not about a students' decision, since those lack of the necessary criteria to make a proposal, according to the demands required by the technique. However, the teacher not only validates it given that it is the factorization needed to illustrate the technique, but also with the purpose to evidentiate that it is a product of the collective work between the teacher and the students.

The ostension contract in Sam's lessons appears to be articulated with the "*mayeutic socratic*". Thus, in several occasions, the teacher modifies its questions, taking any rhetoric way (analogy, metaphor, etc.), to obtain the expected response. The more characterized style of the *mayeutic* from this teacher, consist in enunciate certain proposition, finishing the intervention with one of these expressions: "yes?", "no?", "yes or no?" In this way, the role to affirm or negate is assigned to the student. For example, let see certain extracts from the introductory class to factorization[7], the class about the common factor:

PROF=Sam	Why this last one is called complete factorization and the other two no, if equally they are factorized, that is they are multiplying…	$12 = 3 \cdot 4$ $= 2 \cdot 6$ $= 2 \cdot 2 \cdot 3$
S=F	Cannot factorize another term.	
PROF=Sam	Because I cannot factorize any other term, that is it cannot represent As product any other term? **Or yes?**	
Ss	No, no.	
PROF=Sam	Well I can write, for example this 2, written as 2 by 1. **Or no?**	
Ss	Yes.	
PROF=Sam	And I'm representing it as a product. Then what you are maybe telling me is I cannot factorize in other way different by 1. Then itself by one. Why? For example, say here, 2 by 6, the 6 can be factorized even more, **or no?**	

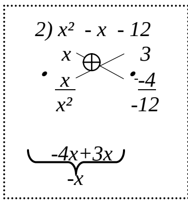
When the formulated questions by Sam are not of the type *mayeutic socratic*, the time he offers the students to answer is short. In general, after two non expected answers, Sam replies. Students know in this way, that it is not necessary to effort to deliver the expected response; if they not know the answer, following, the teacher tells what should be know of the studied topic.

[7] During this lesson, factorization by common factor and grouping method were reviewed.

In addition to the identified clauses defined among the contracts mentioned before, we point out other two clauses observed in the Sam's interactions with his students.

When the students are able to elaborate a question, in the best case, the answer is in hands of the teacher. These answers are always affirmations and not other questions that guide the overcoming of obstacles that the students have. That is, the teacher do not delegates the responsibility in the students to generate possible answers to their questions. For example, in the following extract, after factorizing $x^2 - x - 12$ as $(x + 3)(x - 4)$ the following interaction occurs:

S=1	What happens if we put it inverted is it important?
PROF=Sam	Something occurs… what you are asking me is if something intervenes?
S=1	Yes, if we write it inverted.
PROF=Sam	It is not important. Because I imagine that you first wrote here -4 and then 3.
S=1	Right
PROF=Sam	-4 • 3, results in -12. And when performs the cross multiplication, the product will result in -1, equal. There is no problem, right? Ok. Are we up to here? Yes? Ok, next example.



In the second observed clause, the responsibility to justify or explain the used techniques to perform a task is not assigned to the students. That is, it is enough knowing how to apply it and to obtain the result expected by the teacher. Thus, Sam, after factorizing the trinomial expression $x^2 + 5x + 6$, performs the multiplication of the obtained factors, $x + 3$ and $x + 2$, with the purpose of showing that he obtains the original polynomial expression. However, finishes with the following comment:

PROF=Sam	Very good, then these factors are serving me perfectly. Of course, it is not necessary that you corroborate it. Fulfilling by yourself the conditions we are requesting for the factorization is enough. Correct? We agree up to here?

In general terms, the presence of clauses as the last one, do not permits that the student builds basis that help him the verification of a used technique in a determined task. But also, blocks the development of skills that leave him recognize the scope and field of the technique; this enables a vision of mathematics as a finished and unquestionable product.

Ron's classes

During the first period, Ron tries to "make to remember" the mathematics objects (definitions, techniques, reasoning, etc.), he considers necessary to board the new knowledge. According to the teacher, such mobilized objects may be conceived as elements from the rational discourse that justify and explain the factorization technique to be studied. The mobilization of these objects implies to evocate, formulate, rebuild, rationalize or justify, in a didactic particular situation, the main facts and past actions. In this way, this first period is characterized by the transformation of previous knowledge, that is, the class interactions are regulated by the 'le rappel' contract according to Brousseau (1996).

The dynamics established in Ron's class to evocate previous knowledge are characterized principally by interactions "question-answer".

PROF=Ron	[...] **What is to factorize?**
S=S	To reduce [...]
PROF=Ron	**To reduce how?**
S=S	Well…do not know
S=Se	Express more little
PROF=Ron	**Hmm?** [...] If I tell you to factorize 80, **what do you do? Hmm?**
S=Se	Divide it.
S=V	Simplify it, no? [...]
PROF=Ron	And, **what is to simplify it?**
S=V	The terms that deliver the result. Search the terms that result in 80
PROF=Ron	**How are the terms that result in 80?**
S=V	The numbers that multiplied or added result in 80

Note for example, that rarely Ron formulates a question to which he follows a response. The silence he does after a question (up to 40 seconds), indicate the students their responsibility to answer.

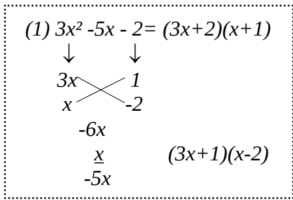
The dynamic of questioning that Ron sets in his lessons, describes one of the more latent rules in this first period: the teacher is conditioned to "not say directly all"; the didactic interaction supposes, justly, that the student makes his own what is learned. In that sense, even when students do not answer as expected, Ron do not responds, but formulate other questions delimiting even more the "cognitive place"[8] (Araya, 2007) in which the students may be placed to "mobilize" the expected knowledge. Pupils have the responsibility to answer and the teacher to validate the questions, and not deliver the answers.

Remark that, during the preparation periods, it's always the teacher whom has the responsibility to articulate the responses and comments to the students. Even though, and different from the Sam's lessons, the students acquire other responsibilities: must be "carriers

[8] Delimit the « cognitive place » of the class refers to delimiting the cognitive universe of this one; universe determined by the objects and relationships to the objects the teacher assumes the students have (Chevallard, 2003).

of the knowledge" enunciating technical and technologic elements used to perform types of tasks. Thus, during factorization of $3x^2 - 5x - 2$, and once identified the terms 6*x* and *x* (resulted from 3*x* • 2 and *x* • 1), Ron pretends that pupils decide in which expression to "place the minus symbol" so that when you add them, you get -5 *x*.

PROF=Ron	In order to obtain -5 *x*, what do I've to write?
S	One positive and the other one negative
PROF=Ron	Which one has to be negative?
Ss	-6
PROF=Ron	In order to obtain -6, what do I've do/ who receive the minus?
Ss	The 2
PROF=Ron	The 2. And now, -2 • 1, would be -2, right? Ok? So, I found a factorization of this one and a factorization of this one, when I multiply and add like terms, you also get the one in the middle



In this way, two rules characterize this period: one is verbalization of the employed techniques to perform a type of tasks and the other is to make explicit the justification of the given answers. Even though, subject to the topics to study, the responsibilities of the actors may vary. In some occasions, the technological elements are shown by the teacher, who expects to be "observed" by the students.

During the second period dedicated to examples, Ron works a transition between what has been set during the preparation and the new knowledge. That is, the possible problematic character that could generate the new knowledge and that should become the "reason to be" in front of the lesson (Chevallard, Bosch & Gascón, 1997), is neutralized. In this way, Ron looks to make evident a transparent transition, not problematic, between the previous knowledge and the new one.

This transition influences the contract set during this second period, declining into an "altered" ostension contract: Ron writes an algebraic expression on the board, and shows the employed technique to factorize it. In one side, as the ostension contract indicates, the students accept to "observe" such example as representative of one category of which they must know the elements in other circumstances or objects. In the other side, and is just what brings the "alteration", the teacher expects that recognize, during the development of the example, the mobilized knowledge in the prior period or during the exemplification, and that form the technological discourse. Let see a passage from the first class about the cross-multiplication method. Ron has written on the board $x^2 + 11x + 30$:

PROF=Ron	[...] This method is very simple. Inspection is like guessing. Is to seek, trying an intelligent guess, not a dumb guess, two binomials that multiplied result that trinomial found there. For example, in the first one, I need to find to binomials that multiplied result in, $x^2+11x+30$. Remember that, if I have the product of two binomials, if I got here x^2, there had to exist, well as the coefficients are integer, $x \bullet x$, right? Yes? Now here may exist a number. The numbers you put here, multiplied, in what may result?
S=L	30 Factorization of trinomials by the cross-multiplication method (1) $x^2+11x+30=(x\_\_)(x\_\_)$
PROF=Ron	Yes… must result in this 30 that stays here, right? And how to obtain the 11*x*?
S=L	Adding them
PROF=Ron	Adding what?
S=L	The numbers of these two.
PROF=Ron	Yes in this case, because here we have only a 1. But this *x* must be multiplied by this one right. And the number here must be multiplied by this *x*. And the sum of these two in what may result? 11 *x*. Ok? So, in place of putting them thus, will put them here x^2 separated as $x \bullet x$. I'll factorize it as $x \bullet x$. The 30, there are several possibilities to factorize with integers. Like which ones?

In Ron's lessons, he tries to show the mathematical discourse that justifies and explains the used techniques. However, and as we observe in the example, the "logos" of a mathematical organization[9] is always presented by the teacher, then fomenting the rigidity and low flexibility of the math organizations studied by the pupils.

Conclusions

The characterization of the study periods of two teachers' lessons has been seeked, using the notion of didactic contract. The "types of contracts" have been used as instruments of analysis of the practices, so they are not that are not examples to be adopted nor to rejected. As observed, the teaching strategies of each professor foment the setting of the ostension contract, the 'rappel' and the mayeutic socratic during the first periods. In the same way, the presence of other interaction clauses was identified between the actors in the class.

[9] In the study from which this work is obtained, the notion of mathematical organization is used to describe the math activities performed in the class. More precisely, an organization refers to two components of human activity: praxis, i.e. the practical part, and logos, on the other hand (Bosch & Gascón, 2006). *Praxis* can be described in terms of the types of tasks and the techniques employed to performed such tasks, while logos is characterized by the explicative discourse and justification of the techniques (Chevallard, 1999).

For both teachers, we recognized an interrogative dynamics that describes the way in which the teacher presents the knowledge to the class. During this communication dynamics, it is delegated to the students tasks as: complete with one or more words phrases enunciated by the teacher, enunciate the result of an indicated operation, reaffirm the teacher institutionalizations, etc. Let point out that this dynamic lacks a closure that takes and order the sequences of ideas that takes to the answer expected by the teacher.

Some elements of the contracts limit the students, in such a way that these can assume other mathematical responsibilities. For example, the students could assume responsibilities such as: elaborate questions, answer other's questions, explain their suggestions and contributions, rebuild the cognitive way that takes him to a result, justify a technique, propose verification strategies of the technique, etc. In this way also, we could think in other kind of responsibilities for the teacher, such as, pose the informative and formative objectives that looks to achieve developing a topic or determined strategy of teaching; for then to explicitly showing to the students the new roles they may assume.

The analysis and characterization of the different periods of study that can use two or more teachers developing the same topic, can constitute an important contribution to open thought spaces of the teaching practices. From them, we could pose proposals that, in a more controlled or regulated form, contemplate roles, tasks, knowledge or contracts that we may want to set in a mathematical class.

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