STUDYING ARGUMENTS IN MATHEMATICS CLASSROOM. A CASE STUDY

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ABSTRACT. *This contribution presents a study of argumentation schemes developed by mathematics students during an integral calculus lesson and the role of the professor in the classroom interactions. The study was based on the adaptation of a design used in Cabañas and Cantoral (2006) whose activities center on the use of notions of comparison, conservation, measure and quantification of area at different levels. The arguments were analyzed whenever the students justified an answer, made transformations, built representations or determined magnitudes. It was also interesting to see whether, through characterized argumentation schemes, the students perceived that the area is compared, conserved, measured and quantified and the role of the professor in this process.*

1. Introduction

This contribution presents part of the preliminary analysis of a research on the reproduction of an engineering design relating to the definite integral, based on the school explanation of the "area under the curve". In this research we are dealing with two types of questions: one relating to the search for empirical evidence in relation to the reproductibility of didactic situations[1] in integral calculus, and another relating to the role that the notion of conservation of area plays in the explanations of teachers when explaining the concept of definite integral (Cabañas and Cantoral, 2006a). Thus, the studies carried out by Piaget J., Inhelder, B., Szeminska, A. (1970) and Freudenthal (1983) about the notion of the area become fundamental in our study . Piaget and colleagues identified the type of notions that stand out among 8 to 11 year old children when dealing with the notions of conservation and measurement of areas using concrete materials. They declare that the notion of "conservation of area" is preliminary and fundamental to understanding the concept of "measurement of area", in other words conservation precedes measurement. Freudenthal for his part considers different ways for approaching to the notion of area: *sharing*, *comparing and reproducing, measuring and quantifying, and conserving* through different methods. Prior to our research we were interested in having a first approach about how university students perceive the notion of the area related to planar regions. Specially we investigated: *How mathematics students perceive that area is conserved, compared, measured and quantified through their arguments while solving tasks concerning the notion of area;* and b). *The role of professor's interactions in the arguments developed by the students.*

We have framed the study in the socio-epistemological approach to research in Mathematics Education (Cantoral and Farfán, 2003) and the argumentation scheme proposed by Toulmin (1958) which is looked at in more detail in sections 3 and 4.

2. Some studies on the arguments

The study of arguments given by mathematics students has been a central theme of research in our discipline, Mathematics Education. Inglis, M., Mejía-Ramos, J.P. and Simpson, A. (2007) for example, are interested in the study of how, during the mathematical demonstrations construction process, students and mathematicians alike build arguments around the veracity of a given observation with no absolute conclusions, aimed at reducing their level of uncertainty rather than eliminating it altogether. In another study, Inglis, M. and Mejía-Ramos, J.P., (2005) describe an argumentation structure in postgraduate mathematics students, identifying different forms of argumentation used in realistic tests. The study presented by León, O.L. and Calderón, D.I., (2001), looks at the argumentation resources used to validate solutions to mathematical problems by students of a degree program in Mathematics Teaching. They identify components of a socio-cultural order which, in the words of the authors, although not traditionally part of doing mathematics, determine specific standards of argumentation interaction in a mathematical context. Cabañas and Cantoral (2006b) studied how university students perceive that area is conserved through the arguments they give when engaged in tasks associated with the notion of area and the integral. They identify seven types of argumentation models: algorithmic, congruent, similarity, parallel, dynamic, graphic and analytic (and combinations of these). It was observed that when using algorithmic and dynamic models students were able to prove that the area is conserved.

3. The Socio-epistemological Approach to Research in Mathematics Education

The study is based in the socio-epistemological approach to research in Mathematics Education, a systemic approach that incorporates aspects of social order and centers its attention on the study of practices, uses, contexts and procedures prior to the concepts. Thus, the *uses, contexts and procedures*[2] in which the notions are developed are studied from a socio-cultural point of view. Looking at the study of the integral from this perspective, then, we ask ourselves what are the uses and contexts of the notion of area prior to their Cauchy definition and of the contexts and procedures in which the definite integral is introduced since Cauchy's work. The analysis is made for didactic reasons, specifically for the redesign of activities for use in the study of student arguments.

Three *types of use* of area can be identified: area can be *compared, conserved and measured. Contexts* are characterized as follows: *Static.* Where we found the exhaustive method – used by the Greeks in the Classic period; the method of indivisibles – used by Cavalieri; the method based on the property of geometric progression - used by Fermat; and the transformation method – used by Leibniz. *Dynamic.* Quantities which vary with respect to each other and, eventually, with respect to the universal variable: time.

Contexts and procedures in which the integral is introduced are as follows:

Contexts: Conception of function and continuity. *Procedures:* Conception of primitive function and the distribution of points on the interval of integration where the function is continuous.

From the socioepistemological approach we analyzed the professor activities through the task he set to the students: how the professor interacts with the students, direct students activities and the effects of his interventions on the arguments developed by the students.

4. Toulmin's Argumentation Model

The argumentation model proposed by Toulmin (1958) is made up of six basic elements that we will call categories, each one playing a different role in the argument. These categories will not always be found explicitly in an argumentative text. The elements of this model are: The *claim* (C) is the thesis to be defended or debated by the argument either in oral or written form. The *data* (D) is the information on which the claim is based. The *warrant* (W) justifies the connection between the data and the claim whether by means of a rule, a definition or an analogy. The warrant is supported by the *backing* (B) of new evidence. The *modal qualifier*

(Q) specifies the degree of certainty, the strength of the claim, expressing the degree of confidence in the thesis; and the *rebuttal* (R) presents the exceptions to the claim.

The six categories of the model are connected in the structure shown in Figure 1.

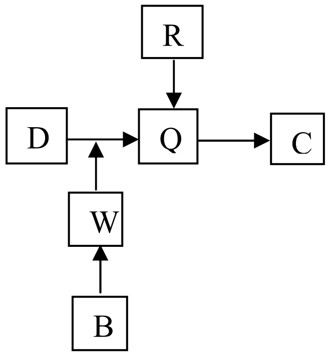

Figure 1. Toulmin's Model of general argument

5. Methods and participants

The data reported in this article comes from one of eight activities carried out by 13 mathematics students (19 to 22 years of age) during an integral calculus course, as well as an interview. The lessons and interviews were videotaped, transcribed and analyzed.

The research activities

The activities used in the study were based on the adaptation of a design used by Cabañas and Cantoral (2006b) and were done in three two-hour sessions during the calculus lesson. In the adaptation of the design the activities centered on, for our purposes, the use of notions of comparison, conservation, measurement and quantification of area to different levels on planar regions. The activities were structured in three series. In the first series the students were put to work on convex and non-convex polygons. In the second they were required to work on graphic and analytical models of lineal and non-lineal functions. In the third they had to determine the value of parameters in algebraic expressions of constant, lineal and quadratic functions, and work on definite integrals.

The professor

The professor, who agreed to collaborate in this activity, was developing an integral calculus course with the students who participated in the study. He is a mathematician by profession and with eight years' experience developing elemental calculus and analysis courses. He normally develops his calculus lessons on formal explanations: definitions, theorems and examples of application, etc. He stated that this way of working contributed to better mathematics training for the students as it covers the topics in the syllabus within the time frame so that students possess the knowledge they require for subsequent courses.

To the professor was asked to develop his activities as follows:

- Organize to the students in small groups (one team was made up by four members and the others by three);
- Give the research activities to the students by series (each students had the research activities);
- Ask the students to work on the activities individually at the beginning, write their procedures, reflections, reasoning, etc. They then had to explain their procedures to the team members;

- Ask that the team write the final procedures and the reasoning they used in the solution process.
- Provide additional information during the lessons, for example, to explain terminology used in the formulation of the activities which the students did not understand.

Students´ activity:

- Worked on the research activities individually and wrote their procedures;
- Shared the procedures with their companions and discussed the methods used in the solution process of the activities and their relevance.

In the individual work stage the students were allowed 15 to 20 minutes to analyze the activities and work on the solution process. For team work the time depended on the type of activity. In the activity analyzed here the students had around 20 minutes. After each series the team members were interviewed in the classroom in an attempt to reconstruct the arguments that evolved as they completed the tasks as a team. The students were interviewed two days after each series were done. Before this, both the professor and researcher analyzed the students' productions and the videotapes of the individual and team work in order to plan the interview questions.

The researchers

Prior to developing the research activities, we worked with the professor to explain him the objective of the design application and of each activity, the dynamics of the work as the activity developed and the role he would play during the interactions with the students. In this respect, the professor was briefed that his intervention should be kept to a minimum when students asked questions; that the interventions should in no way lead to ways of solving the activities. At the end of each session and the interview we analyzed with the professor both interactions among students and professor between students. We asked him some questions about his interventions during students work. The first author was present as an observer during the calculus lessons and the interviews.

6. Interactions and argumentation schemes in the classroom

The concepts, properties, relations and processes associated with the tasks constituted a fundamental unit of analysis in the study of the arguments given by the students during the sessions and the interview. They were, therefore, asked to write down their reasoning as they worked through each task both individually and by team (see figure 2). It was interesting, then, to analyze the activity solution process from their writings as well as the written and verbal arguments they gave.

a. Students working individually on the activities | b. Students working by team | c. Team member explaining procedures

Figure 2. Students working on the activities

We chose two episodes to show the interactions between the professor and the members of one of the teams as well as one argumentation schemes which came out of the solution process they developed. The analysis is given on activity I.1. The team members that we chosen to show the dialog are: Abraham, Jose and Jorge.

Activity I.1.

For each of the following polygons, make at least one transformation so that the area of the new figure is equal to that of the original.

The first episode shows the interaction among the team members during the first calculus lesson. That is, when they were working by team on the research activities. In some moments the professor asked questions to the students.

[…]

[11] Jorge: We need to transform the polygons in others ... We can use parallels lines to reduce the numbers of the angles of the polygons . . .

[12] Jose: As you . . . I thought in the same method to transform de figures

[13] Abraham: Do you think that we can to reduce the numbers of the angles of the triangle too?

[14] Jorge: Humm . . . Not really . . . only on the second polygon.

[15] Jose: What about the first one? . . . You said that is not possible to reduce the number of angles . . . what are you doing to transform it?

[16] Abraham: We can use the parallels lines too . . . by increasing the numbers of the angles . . . in this case

[17] Professor: Is it the only method you can use to transform the polygons in order to conserve the measure of the area?

[18] Abraham: We can cut some parts . . . and to paste it . . . in another side of the figures

[19] Professor: What do you understand by transformation?

[20] Jorge: The figures have to change its shape . . .

[21] Professor: Always? . . . The figures always change its shape through a transformation?

[22] Abraham: Hum . . . I think so

[...]

Second episode shows the dialog between the professor and the team members during the interview

[1] Professor: Team . . .please . . . could you explain to me what you were asked in this activity?

[2] Abraham: That. . . Given these polygons. . .we had to make a transformation. . .

whatever we wanted ... but the new figure had to have the same area as this. . . as the given polygon . . . In this case . . . this triangle

[4] Professor: What did you understand by transformation?

[5] Abraham: By transformation . . . that . . . we had to change the shape. . . in the given polygon.

[6] Professor: To change the shape . . . Do you think always a transformation means to change the shape in the polygons?

[7] Abraham: Well . . . only when we use the method to eliminate angles in polygons . . . through parallel lines . . . then . . . the shape changes.

[8] Professor: What did you do?

[9] Jose: In the first polygon. . . We drew this line parallel to the base . . . of the given triangle. So from any point on this parallel we can draw a triangle . . . another

[10] Professor: Why did you draw a parallel line?

[11] Jorge and Abraham: To conserve the height . . . of the given triangle . . .
[12] Jose: And then the area wouldn't be affected . . . because they would have the same base and the same height . . . the two triangles
[13] Professor: Why did you think about the base and height?
[14] The team members: Because the area of a triangle . . . is found by the formula base times height over two.
[15] Professor: What can you say about the given polygon, with the new figure?
[16] Abraham: Well … the shape changed . . . although it still conserved the base and the height . . . and the area
[18] Professor: Can the new figure be any polygon or just a triangle?

[19] Abraham: Yes . . . any polygon.

[20] Professor: Is it possible to conserve the area in any polygon?

[21] Abraham: Yes . . . as long as we draw a parallel to one of the sides of the given polygon . . . to conserve the heights . . . in triangles.

[22] Professor: You can only do a transformation conserving the area by drawing parallels to one of the sides?

[23] Jorge and Jose: The method we use . . .to transform . . . is to eliminate angles in polygons

[24] Jose: Yes. . . using parallels . . . like in the second figure to form triangles . . . and drawing parallels to the bases so that the angles are eliminated one by one and at the end you get a triangle with the same base and the same height. . . we don't remember another.

[25] Professor: In which cases don't you get the conservation of area when you do transformations?

[26] Abraham: Well . . . we think in all of them.

[27] Professor: Are you sure?

[28] The team: Yes, we are

We identified that the four teams used the same method in the transformation processes of the activity I.1. That is, through the parallelism relations. The structure of their argument is shown in Figure 3.

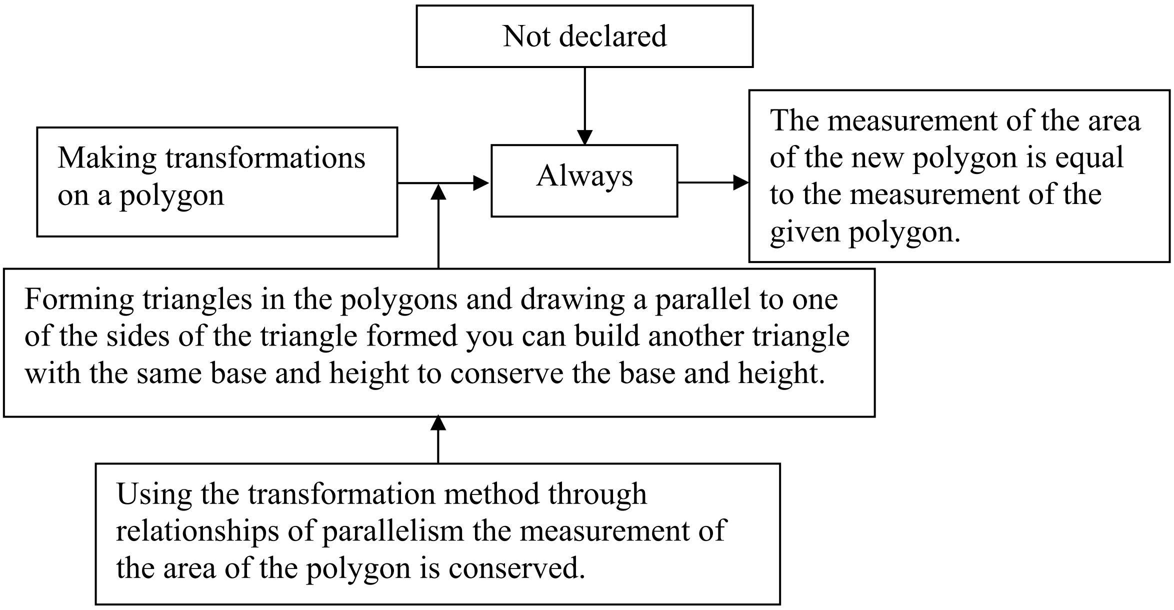

Figure 3. Part of the answers of the team members to the activity I.1.

7. Discussion

On the basis of this study we can draw the following conclusions:

The professor played the role of organizing student's mathematical activities and the communication in the mathematics classroom. Thus, at the beginning of each series professor's interventions consisted in setting research activities for the students and gave additional information about some meanings related with the terminology used in the formulation of the activities. In the interactions with the students there were observed that professor's questions lie stress on the method chose by them in solving some activities but not about the reason of the choice or for analyzing how through their arguments perceive the area is conserved, measured and quantified. He justified this type of actions arguing that he noticed during the team work the students did not think about performing plane transformations in the activity analyzed, because they associated the word "transform" with a change in shape; a change in position did not occur to them and the professor corroborated that in the interview. Obviously the professor tried to direct the students' activities toward other method to transform the figures ─ based in the movements on the plane─.

In this activity it was observed that the arguments developed by the students can be inferred from their individual action with the tasks and by team, but not directly from the professor's interactions. The teacher's interactions were minimal.

The arguments developed by the mathematics students in the described activity were based on a formal demonstration structure to do the transformation, using the relationships between parallel lines. Their arguments centers on the conservation of the area by their reducing the number of sides of a polygon to one with fewer sides: a triangle. They alluded to the formula for calculating the area of a triangle in a transformation process as a way of demonstrating that the area of the polygon is conserved, since the base and the height of the triangles that the shape is conserved. Their transformation method using the relationships of parallelism are the *warrant* and *backing* of their arguments to conserve the area. We identified that the students perceived the area is conserved from the formula for calculating the area of a triangle —by using the relationships between parallel lines to transform the polygons— or cutting out and pasting some parts of the polygons —some of them referred to this method during interventions of the professor, but no used by them—. Comparison is perceived when the students explained the relationship between the original polygon and the transformed polygon. That is, when they explained that each polygon changed its shape but the measure of the area is the same to the original. Comparison and quantification of the area is identified when curves were studied; the value of the parameters is determined on functions and represented graphically its results.

We also identified some difficulties in the students observed during the interaction processes: related with the meaning of transformation and the concept of no linear function. For instance, students consider that in any transformation the shape of the polygons "change" but they are not considering that the polygons can "change its position". Some students understand that not linear function is always related to the graph of a curve.

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NOTES

1. In relation to a school activity carried out by teachers, i.e. repeating "the same class" in different settings.
2. By *uses* we understand the ways in which a determined notion is employed or adopted (Cabañas and Cantoral, 2006). *Contexts* will be the situational surroundings in which a fact is considered and *procedures* the ways of organization (Cordero, 2005).