

THE EXPLANATION OF THE TEACHERS. AN EXPERIENCE STUDY OF THE NOTION OF SIMILARITY IN UPPER MIDDLE LEVEL.

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ABSTRACT. *This report centers its attention on the explanations in upper middle level geometry class, when it tries to teach concepts and mathematical processes linked to the notion of similarity. One of the obstacles in the evolution of this concept has been the relation between the figurative aspects and numerical aspects. Here we will explain a segment of an analysis that illustrates those aspects in the development of one class in particular. We are considering a qualitative investigation model based on the ethnographic method. The episode in this report I present is a joining of observations from the classes and interviews as part of the interpretive work.*

INTRODUCTION

In these last years, there has been a noticeable increase in the number of investigations that have attempted to understand the practice of math teacher. Some are orientated on identifying the influences of different knowledge authorities of the teacher in relation to the practice (Aubrey, 1996; Escudero and Sánchez, 1999). Other work adopts more of a sociocultural character, coming from a teaching perspective that “implies understanding and negotiating meanings through communication” (Herbst, 2006, 2002; Martin et al., 2005). These investigations have tried to describe and interpret the activities of the teacher, looking for regularities in the interactions that teachers and students develop in daily practice.

Also, since the theory of didactical situations, some investigators (Hersant and Perrin-Glorian, 2005; Laborde and Perrin-Glorian, 2005), analyze the practices of the teacher in ordinary classes. Where the researcher intervenes neither in the preparation nor in the management of the lessons. In this approach, the teaching techniques employed by the teacher help understand his actions, and they are objects of investigation.

Other perspectives support the empirical analysis of the interaction of the classroom and of the complex joining of relations that generates between teacher-student-content, placing emphasis on the teacher-content relation (Bromme and Steinbring, 1994; Steinbring, 2005).

Recently the area of the didactics of mathematics, from different theoretical approaches, has proposed new theoretical tools to analyze the teacher-student interactions with regard to mathematical work (Cobb and Bauersfeld, 1995; Voigt 1994; Brousseau, 1986; Escudero, 1999). Constructs as <<patterns of interaction>>, <<negotiating of meanings>>, <<norms sociomathematical>>, <<didactical contract>>, are all useful tools that permit analyzing the teacher practice.

Diverse investigations have identified interaction patterns in the classroom (Voigt, 1994; Herbst, 2006; McNeal and Simon, 2000). The system of routines and obligations can be described as patterns of interaction. The patterns of interactions are considered as regularities interactively constituted between the teacher and the students.

The investigation that I now report is not related to investigations oriented to discovering the thoughts of the professor. Our work focus, it studies an aspect of the teacher's practice that brings students interests alive in a certain mathematical notion, subject to their own restrictions of functionality of the didactic system.

Different lines of studies have supporting evidence about the difficulties to adapt the notion of similarities. One of the obstacles in the evolution of this concept has been the relation between the figurative aspects and the numerical aspects (Escudero, 1999). The articulation of both registers and the weight of each one of them in the treatment of the theme (Lemonidis, 1991), is one of the important components that should be present at the time of considering the similarity as an object of the teaching-learning.

In this report, we want to understand: What is the role that explanations play in the development of the notion of similarity in a particular teaching situation? By explanation, it is understood those parts of the discourse that are made to understand the notion, idea, fact, object or phenomenon (Reséndiz, 2006).

In our investigation, by explanation, we understand those direct actions to establish the relationship between dates, phenomenon, processes or events, or those actions directed so that the students construct conceptual webs. This way, there is intentionality, not only to offer new information, but that this could also be united to previous information and incorporated as a scheme of thought. Therefore, the explanation is part of one of the means that the teacher utilizes to make an understanding or "give sense". It turns into the object of communication or argumentation.

The theoretical perspective that will permit the study of explanations in the class is the emerging perspective described by Yackel and Cobb (1996), that contributes to our interpretation of the events in the classroom, because this model recognizes reciprocally the influences of the professor and students, individually and collectively in the teachings of the social context of the classroom.

In the same sense, the notion of didactical contract will allow us to analyze the decisions of the teachers, as this notion regulates the relationship between teacher and student's that corresponds to the knowledge, establishing rights and obligations between both, in relation with a scholarly content (Brousseau, 1986). The assumption that a didactical contract exists, establishing global responsibilities among teacher, student, and subject of study, helps one understand why teacher and students act as if they have permission but are also under the obligation to work together, dividing labor and trading on subject matter. This postulate invites inquiry into negotiation of the terms of this contract between participants (Herbs, 2006).

METHODOLOGY

The investigation is marked by the qualitative paradigm, based on the ethnographical method (Erickson, 1986). The ethnographic focus permits us to obtain relevant information about the context of the class that is relevant for its interpretation. This methodology permits one sequential study of the situations of teaching (Reséndiz, 2006). The ethnographic perspective that consists in discovering and rebuilding analytically the scenes and groups that are the protagonist, and participate in the educative practice, putting them in a linguistic register that allows their readers to represent them, like seen before the eyes of the investigator.

In this direction, the episode presented in this report forms part of a group of observations of a class and interviews, as part of interpretative work. Audio and video tapings were used in its design when the teacher undertook the notion of similarity. These recordings and interviews were transcribed in their totality.

For the analysis of the obtained transcripts, were developed in two stages. In the first part, recordings of the videos and notes taken during observations of the class were used. The second part started from the transcriptions and the visions of the videos identified segments of interaction and are characterized by: the explanations given by the professor about the notion of similarities and the concrete actions that are put at play at the moment of undertaking this concept.

The analytic scheme generated permits us to identify the following general categories of analysis that we dominate as: *The explanation of an example to arrive at a definition*, the enunciation of a property, with minimum intervention or none at all from the students and the constant participation from the part of the teacher; *the explanation to complete, previous to the enunciation, definition, theorem, etc.*, in which the teacher previously adds what he considers is lacking in order to introduce the enunciation, definition, theorem, etc; *the explanation starting from the development of a learning situation*, by means of developing a situation from learning, the professor introduces a notion, a concept, a theorem; *The explanation by means of a solution to the problems*, by means of a series of concrete problems, the teacher tries to get the student to identify the mathematic notion by games. *The explanation by a series of exercises*, The teacher by means of a series of classic textbooks exercises tries to introduce the notion of similarity.

In this article we are going to explain the development of a particular case. Due to the question of space, here we are going to describe and characterize the structure of an analysis segment; the explanation by the resolution of problems.

RESULTS AND DISCUSSION.

The Professor Alfonso case.

The teacher Alfonso, participant in the following episode, has six years of teaching experience in this educational level. He manifests certain concern in bettering his own formation, participating in different courses and diplomas offered by the Faculty of Mathematics, University Autonomous of Guerrero and the Local Association of Mathematic Teacher in México. His predisposition to participate in initiatives that he is able to gives some improvement in his professional work. This was the primary reason he decided to participate in this study.

The teacher Alfonso sees the concept of similarity as a method of connecting numeric visions with figures, the similarity for him is that the students distinguish inside the figures that something is proportional. This form of conceiving the similarity emphasizes the numeric-algebraic relationship, which is demonstrated in the choice of exercises that are given to the students. The uses and the different roles that the exercises play in the teaching, requires their consideration of the similarities, like a context to visualize the proportionality. Amongst the ideas that Alfonso wants to strengthen in his explanations, is to highlight the importance that students recognize similarities in figure:

Teacher: If we have any kind of triangle and we pass one straight segment parallel to any one of its sides, we are going to call it “*m*” or “*l*”, any letter that works to distinguish this straight segment. In this case, if the line cuts through the sides *AB* and *BC* in the points *D* and *E*, in a way that $BE : EC$ and $BD : DA$. Where, the $\angle D$ is going to be the same as the angle $\angle A$, and the $\angle E$ is going to be equal to $\angle C$.

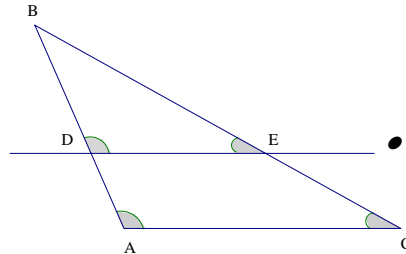


Figure 1

We can observe that teacher Alfonso, although he does not make a reference in respect to the Tales Theorem, considers that as a starting point the students should arrive at a formalization of the theorem by means of the division of proportional segments. For this, it is necessary to identify these distances as sides of similar triangles, being the approximation to the concept inside the *interfigure* relation (Lermonides, 1991). In which, the properties of two configurations are studied, and which, although highlights the correspondence between elements of a figure in the correspondence of the other, the idea of transforming one figure into another is totally absent.

Considering this problem, and the others that Alfonso has prepared, presents the necessary aspects to establish the connection between the numeric aspects and proportionality of the geometric aspects. In this second problem, he now incorporates another aspect of knowing the similarities as an object of teaching-learning: the idea of the similarities of figures in homothetic dispositions.

Teacher: Well, now we're going to resolve the following exercise, number two. The sides of a triangle are 7 cm, 8 cm and 10 cm, respectively, to find the sides of a similar triangle, whose perimeter is 75 cm. I am going to draw this moment the triangle while you all reflect, analyze and understand the problem.

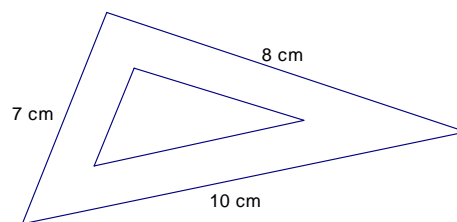


Figure 2

The teacher seeks to facilitate by means of explanations and justifications, which are linked with the content over proportions, with the intention to promote the student’s reflection about the problem to solve. The potential conflicts during the negotiation are minimized through routines and obligations, as you can appreciate and the following dialogue:

Teacher: Here is the figure, right? How much is “*x*”? How much is “*y*”? -He asks the group, while he erases the chalk board-. Which is the ratio? Remember that the ratio is the

quotient of the two quantities. What happened? Is solved already? -he says to a student coming towards him showing his book, and then asks-. How much is the ratio?

Student: I don't understand,-out loud-

Teacher: ssshhhh!! Do you have the results?

Students: No

Teacher: What? You don't understand?

Student: I don't understand

Teacher: Here it is, -showing the pages that he gave out-.

Teacher: Wait a second, -the teacher realizes that the students have not understood the problem- let's see my friends! Let's see now! I feel like it's going to help you very much the explanation that I'm going to give about the proportions of a triangle. We are going to see, look, this is going to resolve everything.....

The explanations manifest how the teacher acquires different ways of representation, to achieve the articulation between the numeric and the graphic. That permits him, the visualization in the geometric context of proportionality and the similarities of triangles.

In the third problem, the teacher explains that he has the finality, that the student resolves applying the similarities of triangles (figure 3). In this problem, some students justify that the triangles ABC and ADC , are not similar.

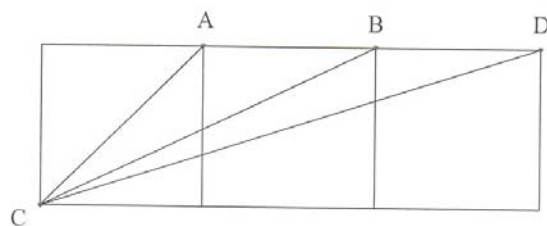


Figure 3

Teacher: Let's see friends, this is a figure that you all are analyzing, right? The triangle CBA , and the triangle CDA , are they similar? Let's begin, who wants to participate? Let's see Jonathan.

Jonathan: They are not similar.

Teacher: Why aren't these triangles similar? Jonathan, your argument?

Jonathan: Tales of Mileto says that in his theorem, in order for a triangle to be similar, it should cross a parallel line in one of its sides, and there is no parallel line here.

Teacher: Can you trace it? Come up here and trace it. Well, he says he cannot by his argument that he just gave, let's see here friend. Are they similar, or are they not similar?... and why?

Students: Because of their angles.

The negotiation in the classroom, can be first off covered by an asymmetric power in relation between the teacher and the students. Surely, the adaptations of the teacher and the students are motivations of different intentions and take diverse forms. The students try to respond correctly to the professor's questions, adapting their answers based on the intentions of the professor, they try to identify the expectations of the professor that postpone their previous knowledge, in order to adjust them to the conditions of the mathematic class. Even though the professor tries consciously to influence the mathematic content in the game, he is interested in the adaptation of the development process in the student's answers.

The connection of knowing obligates adapting it, to modify it, to cut it, to reorganize it. That process, called transposition, is necessary, but only in a certain sense. The relationship game and obligations that are established in the didactic relation, produced diverse effects, and in few occasions favorable for whom are in position to learn. Even the effects of the didactical contract deteriorate and finally substitute learning (Brousseau; 1986).

In this teaching-learning process, the teacher and the students have very different experiences and knowledge (Ponte, 1994). For the teacher, the mathematic concepts have a rich sense, full of relationships with other concepts and mathematical processes. The negotiation of mathematical meanings in class, implies that each one of the participants, teacher and students, form their own meanings in the process. By means of explanations, each one can come to know better regarding each of their and their relationships with the mathematical knowledge. In this sense, "the ambiguities and the negotiation of meanings in the classrooms is fragile... they are at a permanent risk of collapsing, disorganizing the interactive process" (Voigt, 1995, p. 178).

CONCLUSIONS

In this report, we have tried to show, by means of segment analysis, the explanations of the teacher through a series of problems as a form of introducing similarities as an object of teaching-learning. In this investigation verifies, as the teacher explained, and justifies the different ways of representation to get an articulation between the numeric and the graphic, that permits the visualization in a geometric context of proportionality and the similarities of triangles. It can be appreciated that the teacher intent to relate the generated activity in the register of graphic representation with the numeric-algebraic, appreciating in his explanations the intent to elevate both registers.

But, besides the related aspects with different domains and the form of integrating them, these characteristics permit us to establish inferences about the action of the teacher that will permit us, in some form, to affect an immediate future, about his practice. We want to finish, highlighting that micro-ethnographic investigations are impossible to accomplish without the disposition and the help of the teacher. They can help us to study in depth bringing information that contributes from the same practice and into the professional development.

We hope that a deeper discussion of this issue will be possible in further research.

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