THE REPRODUCIBILITY PHENOMENON IN THE CONTEXT OF TEACHER-STUDENT INTERACTIONS

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ABSTRACT

In this work we report some aspects of an investigation centered on the phenomenon of reproducibility in didactic situations. Our focus lies particularly in the interaction between teachers and students as they work on a stage of the didactic situation corresponding to didactic engineering (*A didactic study of the function 2^x).* This engineering was done in such a way so that students would build the notion of exponential function (Appendix). The interactions reported here come only from the second phase which we considered appropriate for this study group. In this report we evidence the phenomenon of how students, when faced with the difficult task of interacting with the mathematical content of the situation, direct their interactions towards the teacher confronting complex communication situations such as those we have named ambiguous interactions.

Key Words: Reproducibility, interactions, ambiguous interactions.

The didactic engineering situation *A didactic study of the function* 2^x (Un estudio didáctico de la función 2^x) suggested two objectives:
To provide a geometric construction process of points on the graph of the function 2^x, and analyze its regularities and confront the spontaneous conception of students that 2^x can only be evaluated when x is a whole number.
To do this, students would evaluate the expression 2^x for some non-integer values, associating it with magnitudes of straight line segments using a geometric strategy (the geometric mean on a semicircumference and similarities of triangles) making use of compass and set square. The students would then be more likely to identify the growing nature of the function when comparing the straight line segments obtained.

The didactic situation in question (Appendix) was widely endorsed when it was staged in January of 1997 and 1998 (Lezama, 1999, 2001). In 2000 it was staged once again with the help of a new group of teachers. These experiences are reported in (Lezama, 2003, 2005).

Based on previously obtained results, the staging in 2000 centered its attention on three key aspects: the structure of the didactic situation, the mathematical activity of the students and the activity of the teachers. There was a general hypothesis that the teachers were a determining factor in the achievement of the didactic purpose of the engineering and for that reason we decided, for this experience, to observe three aspects in the teacher: the way he received and worked with the activity (the tasks were oriented to developing the aspect of communication in the setting, the contents and sense of the situation), the way they reformulated the activity, if that were the case (thus building what we call the teacher's intervention space, looking for a better level of ownership of the activity) and lastly, the interactions with the students when the group gets to work on the situation.

An Analysis Strategy

In this report we deal with the pole of the didactic system relating to the students by analyzing the mathematical activity they develop as they work in the didactic situation. This analysis will show how they experience the situation, how they put the design to the test, how they express their thoughts as they work through each stage, how they put their prior knowledge into action and how they acquire new knowledge. We will also be able to observe their discussions with classmates and teacher as they encounter problems they are unable to resolve.

When the students' activity is analyzed an expected phenomenon is observed in the majority of cases: although the students' activity begins by interacting with scenarios given through written instructions, as time goes on the interaction gravitates significantly toward the teacher. We will show that in many cases the student's mathematical activity is mediated by the teacher. We will attempt to characterize this mediation and, where relevant, identify its function. We will approach these phenomena by analyzing the interactions produced between students and teacher within the work groups.

We maintain that students' mathematical activity is strongly influenced by the variety of roles the teacher plays.

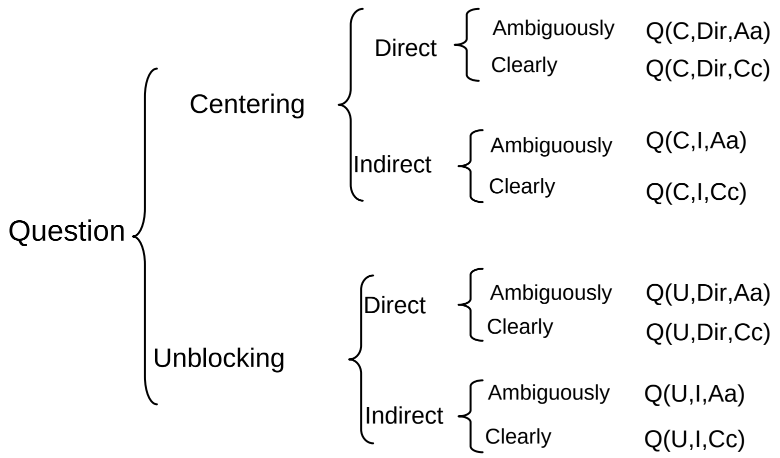
The Didactic System in Action: Student-Teacher Interactions

We will give a brief overview of what happened in the groups in order to show how they evolved within the activity and how they approached the didactic purpose of the situation.

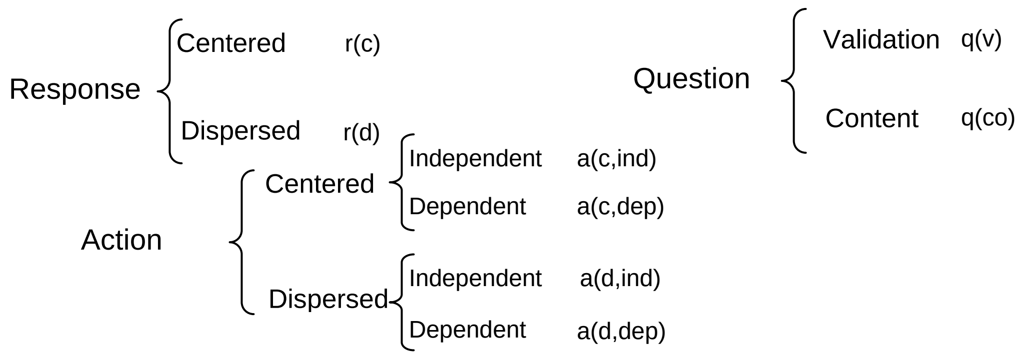
We were able to observe the groups' progress, some of them laboriously investing significant amounts of time in discussing each element of the activities which made up the sequence. In this part of our work we will establish, albeit roughly, the nature and outcome of interactions between teachers and students. Interactions were taken into consideration when planning the staging and it was evident that they had to be teacher controlled. We knew the student would consult the teacher to ask for orientation, to validate results, etc., and it was agreed that the teacher would always generate an interaction policy but subject to two basic criteria. A *centering*, aimed at returning the group to the subject of the situation if it digressed, and an *unblocking* if for any reason the group were stuck for some time on a problem or showed signs that they would not be able to get around it. There are, in fact, many reasons why a group gets blocked or digresses, and we decided to focus on those relating to the mathematical content of the situation, the mathematical background of the students and the working dynamics of the groups.

In an effort to simplify the description of the interactions observed we have created a descriptive device explained as follows:

"←" This symbol indicates interactions which are initiated or brought about in a Teacher to Student direction. These interactions may be Questions (**Q**), Observations (**O**) (comments on the students' work), Indications (**I**) (for example, pointing out errors, suggesting activities) and Actions (**A**) (activities done by the teacher for or with the group). The function of **Q**, **O**, **I** and **A** may be centering (**C**) or unblocking (**U**). They may be of a direct (**Dir**) or indirect (**I**) nature. The direct or indirect nature may be formulated ambiguously (**Aa**) (incomprehensible) or clearly (**Cc**) to the students. The descriptive device is used in the following scheme for questions and could be repeated analogously for observations, indications and actions.



“↱” This symbol indicates interactions which are initiated or brought about in a Student to Teacher direction. These interactions could be: Responses (**r**), Questions (**q**) and Actions (**a**). Responses **“r**" may be centered (**c**) in nature or dispersed (**d**) in relation to the question asked. Questions, “**q**”, may be aimed at getting validation from the teacher (**v**) or asking for information on mathematical content (**co**). Finally, actions “**a**” may be centered (**c**) or dispersed (**d**) in relation to the activity being performed. It can be said that the responses, questions and actions can be carried out independently (**ind**) or dependently (**dep**) of the teacher. The schemes for responses, questions and actions follow:



If the classification of each interaction in the above categories seems arbitrary, it should be said that our interest is in a precise interpretation of the student-teacher dialogue in order to illustrate how the said interaction can determine the group’s trajectory and finally be able to pinpoint the factors which lead to the didactic achievement or not of the situation.

We will use these interaction schemes to analyze two episodes of interaction produced during the 2000 staging looking closely at how they evolved and thus identify elements of the students’ activity in order to determine the didactic success of the experiences.

The episodes and their classifications in terms of the descriptive device referred to above are presented in the following table.

Group 1 b	
TEACHER • **T:** Asks *What phenomenon do you observe with points* 2^0, 2^1, 2^2*?* **S**: They reply that they are talking about it.	←Q(C, Dir, Aa) ↱ r(d)

• **T:** *We are studying other properties. What does $2^{1/2}$ mean?* **S:** *It's half of a whole.*	← Q(U, Dir, Cc) ↱ r(d)
• **T:** *That's how you interpret it, but there is a mathematical condition. You looked at the laws of exponents, didn't you?* **$S_{1,2,3}$:** Yes, yes. **T:** *And then radicals, right?* **$S_{1,2,3}$:** *Yes.*	← O(U, I, Aa) ← Q(C, Dir, Aa)
• **T:** *Can you see the relationship?* **T:** Saying to S_1: *You seem to be close. With what I said, can you make an interpretation?* **S_1:** No response.	← O(U, Ind, Aa)
• **T:** *Don't you remember? It's like a circumference, you reviewed two geometric strategies. Each one has a meaning. Why don't you try to do one? What did you use it for?* **S_2:** *To find the root.*	← Q(U, I, Aa) ← I(U, I, Cc)
• **T:** *Do it now.*	↱ r(c)
• **T:** In this part of the participation the teacher tries to get the group to go back to the geometric algorithm for the square root (insisting a lot to get them to achieve the objective).	← I(C, Dir, Cc)
• **S_2:** Marks a segment that divides into four as he did on the "X" axis. He draws the supposed root and asks the teacher.	← I(U, Dir, Cc) ↱ a(d, ind)
• The **Teacher** attempts to show that the line drawn by the student doesn't take into account the unitary segment given on the worksheet. **S_2**: Draws a new segment and divides it again.	↱ q(v) ← O(C, Dir, Cc)
• **S_1**: Makes a new suggestion, according to his interpretation.	
• The group takes out the notes they made in the corresponding activity in stage one of the sequence which they did before the staging; they discuss the two proposals already made; they still don't get back to the algorithm in spite of having consulted their notes.	↱ a(d, dep)
• **T:** Insists by going as far as showing them the document they worked on in previous sessions, where the elements for developing the algorithm are given.	↱ a(d, dep) ← A(U, Dir, Cc)
• Unable to recover the algorithm, the group goes back to the corresponding notes and with the teacher's help discusses the procedure to obtain $\sqrt{2}$.	↱ a(d, dep) ← A(U, Dir, Cc)

Comments

Group "1b" has a weak mathematical background and is in no condition to rebuild the geometric algorithm.

The teacher's action stems from a committment to the group but realizing the students' situation he finds it difficult to generate a helping strategy to enable them to advance more quickly. The teacher knew of the characteristics of the students but in practice found it difficult to adjust to their needs.

The teacher's ambiguous responses can be interpreted as follows: Either the teacher does not understand what the students are doing or, being unable to identify the errors in what the students say, does not take them into account and performs interventions which in his judgment will introduce the students to the problem.
In his eagerness to lead the students in the right direction, he looks at what they say and do without stopping to explore why. He is focused on the purpose of the situation and tries to fill the enormous voids of knowledge in the students by adopting the role of instructor. This fact can be attributed to the limited time the group had and the teacher's awareness of this.
The teacher starts by questioning or making ambiguous observations, moves on to giving clarifying indications and ends with direct illustrative actions. In other words, he becomes involved in the group's activity.
The questions and observations classified as ambiguous are highly interesting: they were considered ambiguous because they are not relevant to the dialog either in their sense or in the language used by the teacher and are of little use to the students. What the teacher says, instead of putting them on the right track, serves only to distract them further. In almost every case where a question or observation was classified as ambiguous, the response or action it prompted in the students was of the same ilk.
Students faced with ambiguous questions generate unfocused answers, move on to asking questions in an attempt to validate their activity and conclude with actions dependent on the teacher.
What characterizes this episode is the ambiguity of the teacher's interactions; not until the teacher takes control or hold over the group do his interactions become clear both in his language and objectives.

Group 5 b	
TEACHER (45)	
• The teacher approaches to review the work.	
• **S_1:** Asks him "*Are we doing alright?*" **T:** *You're doing okay.*	↦ q(v) ← O(C, Dir, Aa)
• **$S_{1,2,3}$**: They explain in parts the scheme suggested in the solution up to now.	↦ a(d,ind)
• **S_1**: Says: *"In a number raised to the power of n, the number is multiplied by itself n times"*, he explains how: $2^{1/2}= (2)\ \frac{1}{2} = \frac{2}{2}$. *No?*	↦ a(d,ind) ↦ q(v)
• **T:** *No, $2^{1/2}$ is not $\frac{2}{2}$.* **S_1**: *Why not?*	← O(C, Dir, Cc) ↦ q(c)
• **T:** *How many times are you going to multiply 2 by 0.5?* **S:** *Ah, by 0.5?*	← Q(C, Ind, Aa) ↦ r(d)
• **T:** *By 0.5. It's about locating segments first and then points. You have already located some segments, for example: Where is $2^{1/2}$?.*	← I(C, Dir, Aa) ← Q(C, Dir, Cc)
• **S_1**: *Responds (2) by (0.5)= 1*	↦ r(d)

• **T:** Where is $2^{1/4}$? How much is $\frac{1}{4}$? **S_1:** 0.25, *right?*	← Q(C, Dir, Aa) ↱ r(d) ↱ p(v)
• **T:** *So it\s about half way, according to the drawing.*	← O(C, Dir, Aa)
• **S:** *Okay, okay, we'll erase it* (he erases it). **S_3:** Why are you going to erase it all?	↱ a(d,dep)

Comments

This episode illustrates a case of ambiguous responses and indications that keep both teacher and students engaged in a dialog which fails to probe into the meaning of the teacher's or the students' statements, thus maintaining the dialog at a surprisingly ambiguous level. This renders the group incapable of escape from the erroneous vision in which they find themselves.

The interaction is so vague that in the end the students believe they are wrong because they cannot justify their statement but, nevertheless, something makes things correct because the result was validated by the teacher. As if it were an acceptable mistake.

Comments on the Interactions

Each group generated its own history of interaction from the characteristics of the group members and the teacher who accompanied them in their work.

We cannot speak of ideal configurations. Interaction is highly complex and seems as unique and unrepeatable as any human experience.

It can be said, very generally, that observations, indications and actions are of a far more interventive nature than questions, but observation remains partial.

Given the breadth of the problem, students had difficulty in getting the sense of the activity or in some cases purposefully resolved each point at a time without being able to see the "big picture" of the situation.

This fact prepared the teacher to be ready to intervene whenever necessary but the analysis has served to illustrate the nature of the interventions. As can be seen in the episodes described above, the course of an interaction may be very varied, but what is relevant is perhaps that the analysis shows us that these interactions are not uniform. In other words, while they are all aimed at centering and unblocking, the ways of achieving that are far from appropriate. We discovered that some had to be classified as ambiguous because of the language they used or because a clear purpose could not be identified from what they said.

It also showed us that, in many cases, unfocused or senseless actions were related to ambiguous responses, observations or indications. Similarly, systematic actions or interventions by the teacher to the students' work resulted in dependent actions on their part.

In the context of the staging, interactions were fundamental to the development of the activity. If these were suppressed the experience would change radically and we cannot say if this would be for the better. The groups accepted the teacher's presence as an important, natural element to the development of their work.

Conclusion

This report, gives only a sample of the third aspect we set out to observe in our investigation: to see how student-teacher interactions affected the didactic outcome of the engineering.

How, then, to optimize interactions and activity design that succeed in overcoming the complexity of interactions and prepare teachers and students to study them in more detail and benefit from them. From the examples discussed above, we can identify a number of factors to consider. It is useless and impossible to attempt to compare people. Once again, it is far too complex. What we can allow is the comparison of a process and a dynamic built around a well defined object of school knowledge in a specific design.

Work experience from previous engineering staged in 2000, showed stability in the results that were obtained. Modifications to the design and structure of the situation were minimal. The objective has remained unaltered in spite of the intervention of more than twenty teachers. The geometric content, despite the difficulty that students always displayed when working with it, has been kept intact as it represents a fundamental element in confronting the didactic and epistemological obstacles that emerge when passing from whole to non-integer exponents.

The student groups were kept within an age range of 16 to 22 years with an educational level between the last year of senior high school and the first undergraduate years. The mathematical content dealt with is entirely curricular.

The most novel element for students and teachers was the structure of the didactic situation, the way the problems were posed and the working dynamics this implied from both parties. Practices were imposed that contrasted with those usually followed in their everyday classes. This was a deciding factor and the examples discussed above show how teachers got stuck on very basic aspects and were without the resources to move the students on.

The activity modality that was imposed sought more independent work from the students. Nevertheless, as time elapsed teachers were put in critical condition and became disturbed by seeing their students stuck on aspects they (the teachers) considered elemental. This fact made them lose the direction and vision as to where the students should be headed. The teachers, therefore, are seen to be especially intervening; not being used to attending to students' progress in class and time pressures rendered them unable to understand the difficulties facing the students, producing ambiguous interventions. The interactions result in strange dialogs in which the teacher, as much as the students, speaks of unrelated matters. Naturally, the students seek instruction and approval from the teacher but are unable to understand his vague contributions.

Within the framework of our investigation this is paradoxical; we understand the teacher as a fundamental factor in the student's achievement, nevertheless, in trying to forward this achievement he unwillingly becomes one of its key obstacles.

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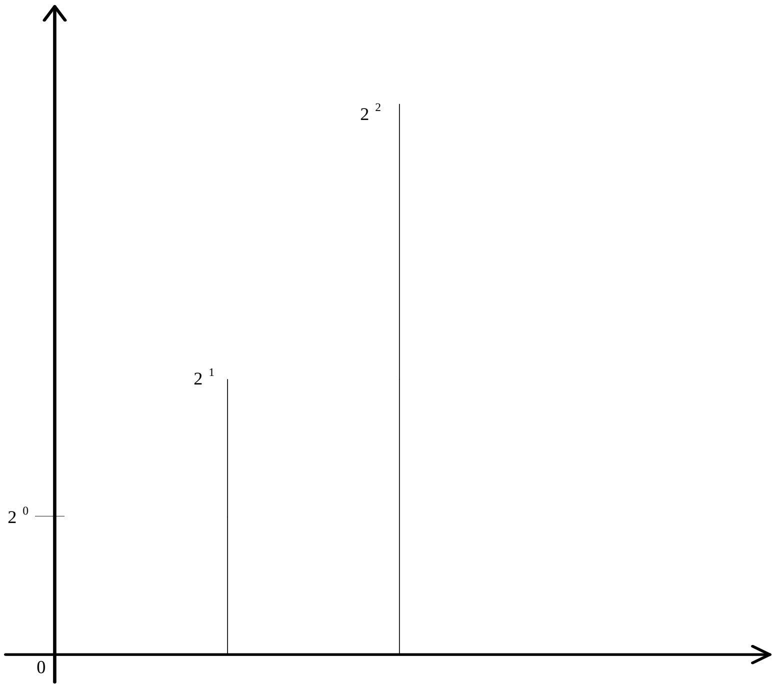
Appendix



GEOMETRICAL CONSTRUCTION

Activity: In the following drawing, the segments of magnitudes 2^0, 2^1 and 2^2 are shown which enable us to locate the points $(0,2^0)$, $(1, 2^1)$ and $(2, 2^2)$.

The problem consists of locating the points $(½, 2^{½})$, $(¼, 2^{¼})$, $(¾, 2^{¾})$, $(5/4, 2^{5/4})$, $(3/2, 2^{3/2})$ and $(7/4, 2^{7/4})$. To do this, you must locate the segments of magnitudes $2^{¼}$, $2^{½}$, $2^{¾}$, $2^{5/4}$, $2^{3/2}$ and $2^{7/4}$ using only geometrical procedures.