# Motion sensor: a learning tool for reading function graphs

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**ABSTRACT**
The aim of the lead experimental research was to underline improvements about the reading and the understanding of function graphs, through didactic activities in the laboratory where a sensor motion was used. Particularly, there were improvements in the reading of maximums of functions and of interval widths on 10 year-old students; while the activities developed with students of the 4th year of high school (17 years) conducted satisfying results about the reading of relative maximums and growth of function and about forming hypothesis on the base of experimental data.
Such research, born from a historical-epistemological analysis of the function concept, finds its basis in the *Embodiment Theory and Metaphorical Thought* of Lakoff e Nuñez and in the numerous works about the use of Microcomputers-Based Laboratory (MBL).
The experimental methodology references *Brousseau's Theory of Didactic Situations*; it is based on the qualitative analysis of a lesson developed in the laboratory and on the use of the software *Chic* of implicative statistics (Gras, 1979, 2000), as for the interpretation of the pre-test and the post-test.

**Key Words:** function, graph, sensor motion, learning.

## 1. INTRODUCTION

The research work consists of the analysis of didactical activities, conducted in a 5th class of Primary School (10 years) and in a 4th class of High School (17 years). These activities were based on the study of space-time graphs and velocity-time, representative motions of bodies, produced in real time through the use of a motion sensor interfaced with a computer. This instrument is utilized in Physics laboratories to study rectilinear motion and its relative graphic representations. It reveals, instant by instant, through the emission and the reception of ultrasounds, the distance of bodies put in front of it and it transmits the measurements to the computer that, making use of an opportune software (called *Data Logger*), visualizes the data in tabular and graphic way.
By achieved results it was observed that not only the students improved reading and understanding of motion graphs (Thornton & Sokoloff, 1990) but they improved these skills with function graphs in general.
This research is part of a wider context of doctoral working theses, regarding *The Acquisition of the Concept of the Function and of its Representations.* The idea is that one of the approaches to introduce the concept of function could be the graphical one, using graph representing of real physics phenomena. This proposal comes from some considerations of a historical-epistemological analysis of the concept of function, which finds its historical and contextual origins in the ambit of kinematics. Considering this analysis and taking into account the fact that history, «as a source of problems, from the teacher's point of view can become an instrument of projection and for the student an occasion to give meaning to mathematical concepts» (Spagnolo, 1998, p.76), we propose introducing the concept of function through a physical approach and graphical representation.
Nevertheless, in this paper the obtained results are reported relative to the reading and comprehension of graphs and to the conjectures made about such representations. These skills are part of the prerequisites necessary for the introduction of the concept of function towards a graphical representation.

## 1.1 SOME DIDACTICAL CONSIDERATIONS

To clarify the connection between the skills with regard to the reading and the understanding of motion graphs and of functional graphs in general, we made a comparison between didactic objectives. The table below reports the results of this analysis:

| Mathematical Objectives | Physics Objectives |
|---|---|
| 1. To read the coordinates of graphic points of a function, or rather the correspondence between the values of the independent and dependent variables<br>2. To read size extremes and the size of intervals<br>3. To read the correspondence between intervals of the independent and dependent variables<br>4. To distinguish among increase, decrease and constancy of a function<br>5. To individuate maximums and minimums of a function<br>6. To confront the different degrees of rapidity of increase or decrease of tracts of a curve<br>7. To form hypothesis and conjecture | 1. To read the values of a spatial variable in relation to the values of the temporal variable<br>2. To read space and time of departure and arrival, the length of space and the time spent<br>3. To read the correspondence between spatial intervals and temporal intervals<br>4. To distinguish between motion of approach, motion of separation and still bodies<br>5. To read the maximum and minimum distance reached with respect to the system of reference<br>6. To confront the velocity of differing tracts of motion<br>7. To form hypothesis and predictions based on experimental data |

## 2. THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

The motion sensor is one of the MBL tools (Microcomputer Based Laboratory), introduced into physics teaching in the 90's to improve the students' learning and comprehension of physics concepts (Thornton & Sokoloff, 1990).

The advantages of the use of the MBL tools are multiple as they allow to visualize in tables and graphs the experimental results in real time and to analyze of such data. This facilitates the comprehension of abstract representations as the data are revealed and represented in real time and the students can make observations on the physical phenomenon and can interpret, discuss and analyse the data as they see fit (Tinker 1996, Thornton 1997).

In particular, the possibility for the students to visualise and analyse the graphs of the bodies in movement or of objects physically perceptible at a sensory level, finds strong support in the cognitive theories *of the Embodiment of the mind*, for which «the detailed nature of our bodies, of our brains, and of our daily functioning in the world structures human concepts and reasoning», and from *Metaphorical Thought*, according to which «for the most part, human being conceptualize abstract concepts in concrete terms, utilising ideas and models of reasoning founded on a sensor-motor system» (Lakoff & Núñez, 2005, p.27). Moreover, «the functions on the Cartesian plane are often conceptualized in terms of motion on a route» (Lakoff & Núñez, 2005, p.70) and the motion sensor induces this type of conceptualisation so that the student sees the graph constructed under his own eyes as "motion of a point that leaves a wake". This process facilitates the acquisition of the ability to read graphs of function and the learning of such a concept so that «revealing the cognitive structure of mathematics, renders it decisively more accessible and understandable» (Lakoff & Núñez, 2005, p.30).

As above, under the hypothesis that *the motion sensor is a good instrument to learn to read graphs of function*, we put forth our self the following research question: *Using the motion sensor to learn to read graphs of motion, will the student acquire ability and knowledge relative to the reading, comprehension and conjectures about graphs of any function?*

## 3. EXPERIMENTAL WORK AND RESEARCH METHOD

The experimentation consisted of the carrying out of a laboratorial lesson[1] of two hours, where the students compared the readings and the predictions of the graphs realised with the motion sensor and the software *Logger Lite*[2], It was leaded in two classes:

[1] Lessons was conducted by the teacher-researcher M. L. Lo Cicero

1. May 2007, 5th class of Primary School (10 year), *School: II° Circolo of Villabate (PA) "G. Rodari", Italy;*
2. December 2007, 4th class of High School (17 years) of type classical lycée, *School: Liceo Classico "Scaduto", Bagheria (PA), Italy*.

It was decided to perform the research activity on students of different grades to confront the obtained results, which were thought to be different because of motives linked to the age of the students and their courses of study. In fact, before the experimental work the students had got different skills, showed in succession:

| *Skills possessed by students before the experimental work:* | |
|---|---|
| **5th class of Primary School** | **4th class of High School (**classical lycée) |
| 1. To know, to represent on a straight line and to operate with positive decimal numbers<br>2. To know and to operate with the Cartesian plane (it was utilised, for instance, to study and to represent translations and similarities of images)<br>3. To know, from experiences lived in physical education and in every day life, motion of his own body and of other bodies<br>4. Any systematic knowledge of kinematics | 1. To know, to represent on a straight line and to operate with real numbers<br>2. To know and to operate with the Cartesian plane (studied and visualized various times during the scholastic career)<br>3. To know, by experiences lived in physical education and in the life of every day, motion of his own body and of other bodies<br>4. To know analytical and graphical (space-time) representations of the rectilinear uniform motion and of the rectilinear uniformly accelerated motion |

The research methodology adopted is *Theory of Didactic Situations* by Brousseau (Brousseau, 1997). The laboratorial lesson was preceded and followed by the administration of a test, with the aim of evaluating the a-priori and a-posteriori state of the students. The lessons were conducted in a constructivist way, positing question-stimulus to the students so as to solicit observations and discussions between pairs and with the teacher, allowing for the active construction of knowledge (Kilpatrick, 1987).

### 3.1 PHASES OF THE DIDACTICAL ACTIVITY

The phases of the didactical activity for both the classes were the following:

1. Discovery of how the motion sensor works, by the observation of graphs and tables representing a student's motion, and reflections on the variable studied by the sensor
2. Prediction and reading of the graphs of rectilinear student's motion of three types:
    a. Leaving motion from the sensor
    b. Approach motion to the sensor
    c. Still body with respect to the sensor
3. Prediction and reading of the graphs of various rectilinear student's motion, with leaving and approach with respect to the sensor.

During phase **2** the students observed and calculated space and time of departure and arrival, the length of space and the time spent. In phase **3** these values were observed and calculated for every tract of leaving, approach or stilling of the curve of the motion. Also the maximum and minimum distance reached with respect to sensor was read. The students noted that the slope of every tract of the curve depended on the corresponding velocity of the student. Then the students were asked to make a relationship between spatial intervals and temporal intervals about tracts of a curve and to make comparisons.

Moreover the Secondary school students calculated the mean velocities and compared them and the observations about the slopes of the tracts of the curve with the graphs velocity-time. This was followed by phase **4**, in which the students observed graphs of rectilinear uniform

[2] In the research we used instruments and *Data Logger* from the Venier Software & Technologies. In the same way we could have used instruments and software from other equally valid companies.

motions and uniformly accelerated motions of a train on tracks, developing the activity described above.
The Primary school students met difficulties in calculating the length of space and the time spent. To get over these difficulties, they had to put up a metric scale on the pavement to compare the space physically run with the graphic representation.
The motion sensor induced curiosity and desire to learn in the students. They were encouraged to experiment with typologies of motion from these same suggestions and to compare the graphics produced with their own predictions. It be noted that the process of prediction is important to acquire the skill of *forming hypothesis on the base of experimental data.*
A qualitative analysis of the laboratory lessons was developed but it is not decrypted in this paper. Only the results of the quantitative analysis of the tests are shown here.

**3.2 TEST**

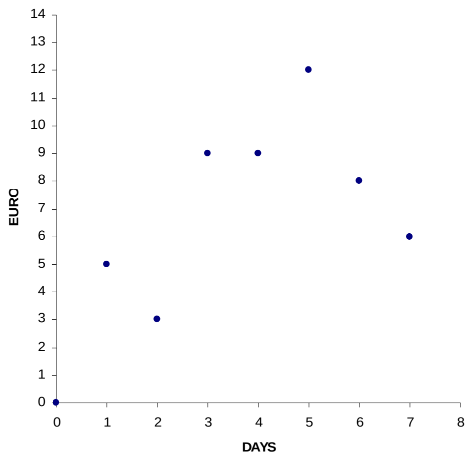
A test was administered before and after the laboratorial lesson to evaluate the a-priori and a-posteriori state of the students. In each class the pre-test was equal to the post-test. The students worked individually, they were not allowed to consult books or notes. They were given sixty minutes.
In the tests of both classes there were exercises concerning reading of space-time graphs but the typology of graphs proposed and of questions were different in the two classes. In fact, the description of the tests schematized below shows that, concerning kinematics part, in the test of the Primary school students the questions were open and for the other students closed. It is due to the fact that the Primary school students did not study kinematics in systematic way before, so it was intended to make the cognitive resources emerge liberally and this process was stimulated by the comparison among graphs. Instead, in the case of students of Secondary school it was intended to verify the acquisition of specific skills, through questions expressed in scientific language, familiar to the students jet.

| Test for students of Primary school | Test for students of Secondary school |
|---|---|
| • Two exercise of reading and two of writing of points on a Cartesian plane<br>• <u>Two questions about reading of a graphs euro-days</u><br>• Request of commenting and comparing on the three graphs representing a rectilinear uniform motion of leaving, a rectilinear uniform motion of approach and a still body.<br>• Request of commenting on two graphs of rectilinear uniform motions, in which the time intervals were the same but the covered distances were different<br>• Request of giving a definition of velocity | • <u>Five questions about reading of a graphs euro-days</u><br>• Questions about four types of motion: constant motion, rectilinear uniform motion, accelerated rectilinear uniform motion and various motion. They were requested to explain the following characteristics: the space covered, the time spent, the space of departure and arrival, the maximum and minimal distance reached with respect to the system of reference, the type of motion, the mean velocity and the Law of the Time. |

In both cases, the obtained results have confirmed the effective validity of the utilization of motion sensors and how this instrument constitutes a valid support for learning of kinematics, corroborating our research hypothesis. However, in this paper we would like to focus the reader's attention on the obtained results regarding the euro-days graph.
The euro-days graph was the same in test of both classes but in the test of the Primary school students there were not some questions in comparison to the other one. The complete exercise is reported below:

*Sara's dad decided to reward his daughter every time she got a good grade at school by giving her five euros, which she could decide to spend or save as she pleased; but this would be her only source of income. The adjacent graph shows the money earned by Sara in a week. Observe it and answer the following questions:*

a. How many euros did Sara have on the 4th day?
b. On which days did Sara surely get a good grade?
c. Knowing that on the second day Sara didn't get a good grade, how much money did she spend that day?
d. Could she have gotten a good grade on the 6th day? (justify your answer.)
e. On which day did Sara have the most money?
f. On which day did Sara accumulate more money compared with the previous day?




In the test for Primary school students there were only questions ***b*** and ***c***. This is due to the fact that the experimentation in the Primary school was previous to that of the High school and for the latter it was decided to amplify the questions to investigate more mathematical skills. However, question ***a*** was substituted in the test of the students of Primary school by two exercise of reading of the coordinates of five points depicted on a Cartesian plane. They were carried out correctly by 97% of students.
In succession, it is related the mathematical skills that we would have like to have emerge through the questions of the test euro-days:

| Questions | Skills | Primary S. | Secondary S. |
|---|---|---|---|
| a. | **R:** To read the value of the ordinate in correspondence with abscissa | X | X |
| b. | **Mr:** To identify the relative maximums of a function | X | X |
| c. | **Ma:** To identify the absolute maximum of a function | | X |
| d. | **I:** To measure the width of an interval | X | X |
| e. | **G:** To compare tracts of curves with respect to the rapidity of growth | | X |
| f. | **H:** To form hypothesis on the base of experimental data | | X |

## 3.3 A-PRIORI ANALYSIS OF STUDENTS' BEHAVIOUR

As it is indicated by *Theory of Didactic Situations*, we made an a-priori analysis of students' behaviour in working out the test. In succession, for reasons of space, only the students' behaviours extrapolated from protocols are reported. The letters with which the behaviours are called point out the connection to the skills that we wanted to investigate.

### Behaviours of Primary and Secondary school students:

R.1 Correct reading of the value of the ordinate in correspondence with abscissa;
Mr.1 Correct identification of the days corresponding to the relative maximums [days: 1,3,5]
Mr.2 Confusion between the concept of relative maximums and of absolute maximum [5]
Mr.3 Writing, other than the days which correspond to the relative maximums, also, of the 4th day, in which the euro remained constant with respect to the previous day [1,3,4,5]
I.1 Correct identification of the width of the interval
I.2 Confusion between the concept of interval and of value of the coordinate

### Other behaviours of Primary school students:

Mr.4 Confusion between the concept of relative maximums and of no-absolute minimum [1,3,4,5,6,7]
Mr.5 Confusion between the concept of relative maximums and of greater, (writing of the days which correspond to euro greater or equal to a certain value) [3,4,5, or, 5,3,4,6]
Mr.6 Writing of the days without apparent criterion [4, 6, 7, or, 1, 3, 4, 6, 8]

### Other behaviours of Secondary school students:

Ma.1 Correct identification of the absolute maximum [5]
G.1 Correct identification of the day corresponding to the relative maximum with greater degree of growth on left neighbourhood of width "a day"

G.2 Confusion between the day corresponding to the relative maximum with greater degree of growth on left neighbourhood of width "a day" and the absolute maximum [5]

G.3 Correct identification of the day corresponding to the relative with greater degree of growth on left neighbourhood of width "a day" but writing of the following day too, in which the euro remain constant compared to the preceding day [3,4]

H.1 Affirmative answered to the question *d*, justifying with the affirmation "she could have spent the earned money": forming correct hypotheses on the base of experimental data

H.2 Negative answer to the question *d*, justifying with the affirmation "she spent 4 euro" or "her budget would have become 13 euro: faithful reading of the graph

H.3 Affirmative answer to the question *d*, justifying with the affirmation "she earns 8 euro": no forming hypotheses based on experimental data and wrong reading of the graph with behaviour of type "I2"

## 3.4 STATISTICAL METHODOLOGY AND ANALYSES OF THE TESTS

The results of pre-test showed that before of the didactical activity the students of the Primary school already possessed the competence *R* and the students of the Secondary school possessed the skills *R*, *Ma* and *I*. For this reason, the behaviours related to these skills were not considered during the analysis. So, according with the research questions, using of motion sensor would have had to improve the remainders skills. To verify it, an analysis of the tests was effectuated with the implicative statistical software *Chic* and the *supplementary variables* method (Spagnolo, 2005) was utilized. A table with double input was filled in which the 1st column was constituted by the nickname of the students' behaviours. Instead the other columns represented the "student variables", called *Si* (i=1,..,n), with component *j* equals to "1" if the student assumed the *j*-behaviour, "0" otherwise. Also, on the right side of the "student variables" the following "supplementary variables" were added, which represent models of students' behaviour, relating to the analyzed skills:

| Supplementary variables of Primary school Students | | | |
|---|---|---|---|
| | **MaxR** | **Int** | **MaxR-Int** |
| **Mr1** | 1 | 0 | 1 |
| **Mr2** | 0 | 0 | 0 |
| **Mr3** | 0 | 0 | 0 |
| **Mr4** | 0 | 0 | 0 |
| **Mr5** | 0 | 0 | 0 |
| **Mr6** | 0 | 0 | 0 |
| **I1** | 0 | 1 | 1 |
| **I2** | 0 | 0 | 0 |

| Supplementary variables of Secondary school Students | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **MaxR** | **Grow** | **Hp** | **MaxR-Grow** | **Max-Hp** | **Grow-Hp** | **MaxR-Grow-Hp** |
| **Mr1** | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| **Mr2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Mr3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Ma1** | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **G1** | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| **G2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **G3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **H1** | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| **H2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **H3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

For instance, in the table of Primary school, **MaxR** represents the students who answer correctly to the questions about the reading of relative maximum and answer wrongly or do not answer to the question about the width of the interval.

The "student variables" and the "supplementary variables" are analyzed together in *trees of similarity* obtained with *Chic*. The purpose of this type of analysis is to visualize how the "student variables" are grouped in comparison to the "supplementary variables". So it is understood for each student if his behaviour is similar to a certain behavioural model rather than to another. Of course, seeing the choice of the "supplementary variables", the research question would have had an affirmative answer if there had been an improvement from the pre-test to the post-test in the similarity of the "student variables" in comparison to the "supplementary variables".

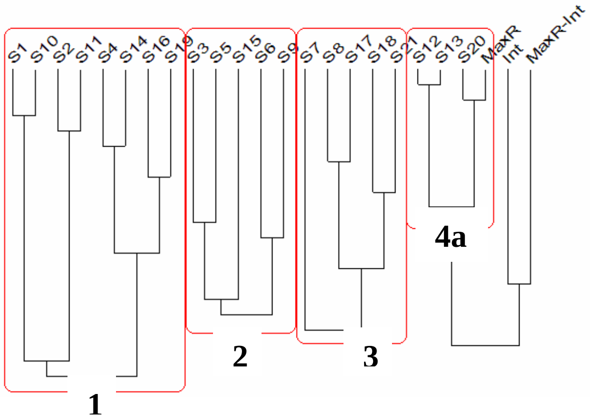
Moreover the tree of similarity can be read also in individual terms, observing the improvements of the single student in base to the group of which he takes part in the graphs of the pre-test and of the post test.

## 4. EXPERIMENTAL RESULTS AND CONCLUSIONS

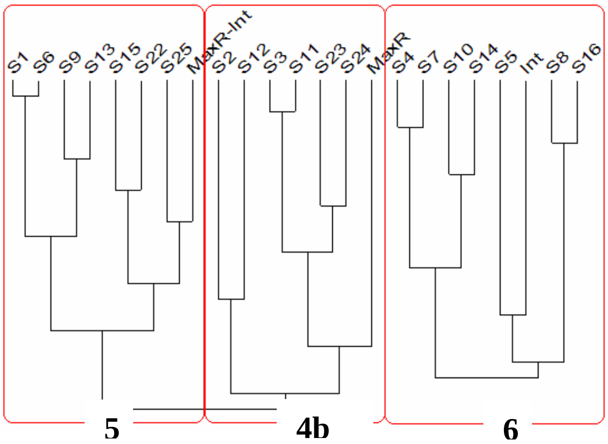
### 4.1 EXPERIMENTAL RESULTS FOR PRIMARY SCHOOL

Analysing the tests of the 5th class of Primary school with *Chic* the following graphs were obtained:

**PRE-TEST – PRIMARY SCHOOL**




**POST-TEST – PRIMARY SCHOOL**

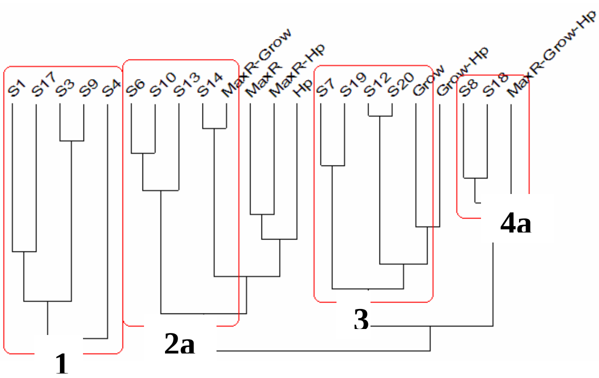



In the tree of similarity of the pre-test it is observed that the variables are divided in four groups. Only group **4a** is characterized by supplementary variables and the students who take part to it are who correctly read the relative maximums. The other students did not possess the skills of reading of relative maximum and width of the interval. In the graph of the post-test there are three groups of similar variables, each characterized by a supplementary variable: the group **4b** is analogous to **4a** of the pre-test but wider, in the group **5** there are the students who correctly read the relative maximums and the width of the intervals and the group **6** is that of the students who correctly identify the width of the interval. So we can affirm that all the students have shown improvement, some in one skill, others in both.
Also, the improvement of the single students can be observed. For example S1 acquired both the skills.

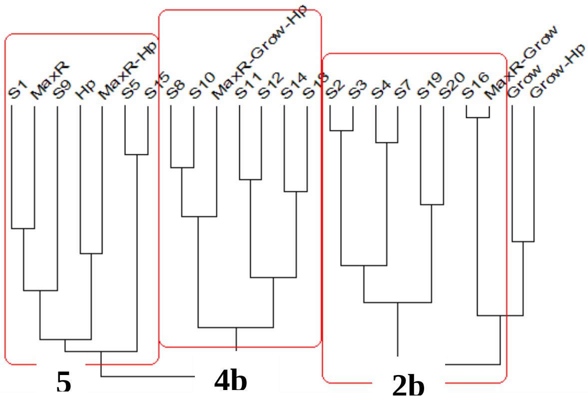
### 4.2 EXPERIMENTAL RESULTS FOR SECONDARY SCHOOL

Analysing the tests of the 4th class of Secondary school with *Chic* the following graphs were obtained:

**PRE-TEST – SECONDARY SCHOOL**




**POST-TEST SECONDARY SCHOOL:**




In the graph of the pre-test the students variables are divided into 4 groups. Group **1** has not got supplementary variables, so it is constituted by students who do not possess any of the required skills. In this group, only *S*1 is a student who reads the relative maximum but he is more similar to the students who do not correctly read the degree of growth and do not make hypotheses rather than to MaxR. Group **2a** is formed by students who correctly read the

relative maximum and the degree of growth. The students in group **3** correctly read the degree of growth. In group **4a** the students possess the three mentioned skills.
In the graph of the post-test the student variables are divided into three groups. Groups **2b** and **4b** are analogous to groups **2a** and **4a** of the pre-test but wider. Also in this graph there is a new group, **5,** constituted by students who correctly read the relative maximum.
The improvement of the single students can be noted from these graphs. For instance, *S*12 before the didactical activity, correctly read the degree of the growth, after he learnt to read the relative maximum and to form hypotheses as well.
Moreover it is observed by both the graphs that skill *forming hypotheses on the experimental data base* is subordinate to the skills of reading of the maximum relative and of the degree of growth.

## 4.3 CONCLUSIONS

Further to a historical-epistemological analysis of the concept of function, in two classes of different grades, various activities were conducted on predictions and reading graphs produced in real time with a motion sensor. The quantitative analysis of the pre and post-tests has shown that the laboratory activities, even if short, conducted the students to total or partial acquisition of preset skills of reading and understanding of graph of functions.
So it can be affirmed that the research question admits affirmative answer: *the motion sensor can be useful for learning to read function graphs*.

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