Mathematics Curriculum: A Vehicle for School Improvement

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Over the past 20 years, curriculum has been a major lever for promoting change and improvement in the school mathematics program in the United States. By "curriculum" I refer to the broad construct of both what society values and expects in terms of mathematics content to be learned in the K-12 school system as well as the materials used by teachers to deliver mathematics instruction to students. This broad definition includes each of the following:

The Intended Curriculum – Specification of particular learning expectations, often delineated by grade, for school mathematics instruction. Often called "curriculum standards," these learning expectations provide guidance to teachers regarding what should be taught and when various mathematical content and processes should receive emphasis in the school program. They also guide the development of textbooks and assessments designed to monitor school programs.

The Textbook Curriculum – Publishers utilize curriculum standards to build textbooks and other instructional materials to implement the intended curriculum. These materials include textbooks typically designed to support the day-to-day teaching of mathematics over a school semester or academic year of study. They also include modules focusing on smaller amounts of mathematical content, workbooks, and computer software.

The Implemented Curriculum – Decisions made by U.S. teachers regarding what part of and how to use the district-adopted instructional materials differ. Therefore students using the same textbook may, in fact, have differing opportunities to learn mathematics. The implemented curriculum refers to the mathematics that students have an opportunity to learn which is often a function of the district-adopted textbook and/or the individual teachers' preferences.

The Assessed Curriculum – Increasingly, the mathematics that is emphasized on high-stakes assessments (e.g., mandated annual grade-level assessments or college admissions assessments such as ACT or SAT) receive primary emphasis in the classroom, sometimes over other priorities. The assessed curriculum refers to the content focus of accountability assessments designed to monitor student learning in relation to the intended curriculum.

Together, these forms of curriculum directly impact the "learned curriculum," that is, what students learn as a result of school mathematics instruction. Each has been used in the past two decades in the U.S. to promote change in teachers practice designed to enhance opportunities for students to learn important mathematics.

In this paper, I provide a summary of some work and ongoing discussions in the U.S. regarding each of these types of curriculum, particularly the intended and textbook curricula. The paper does not serve as a comprehensive analysis or status report regarding mathematics curriculum in the U.S. Rather, it provides some insight into the ongoing debates and tensions surrounding efforts to improve the school mathematics curriculum to serve all students.

The Intended Curriculum: Evolution of Curriculum Standards in the United States

Curriculum standards for school mathematics developed by the National Council of Teachers of Mathematics (NCTM) in the late 1980's (NCTM, 1989) and refined over the past two decades (NCTM, 2000, 2006) launched a broad and far-reaching standards movement in the U.S. In 2001 the *No Child Left Behind* (NCLB) federal legislation advanced this movement, requiring states to articulate curriculum standards for mathematics learning and to regularly (annually in grades 3-8) assess the extent to which students are learning the mathematics outlined in the standards.

Unlike most countries, the U.S. has always placed governance for educational decisions at the state level. Historically, the locus of authority for establishing the intended curriculum has resided at either the state or local level, depending on the governance structure of the state. For example, some states such as North Carolina, Texas and California exert control at the state level for aspects of curriculum regulation such as standard setting, textbook review, and assessment of student learning. Other states such as Nebraska, Colorado and Minnesota have deferred control for these decisions to the local school district level. However, since the passage of NCLB in 2001, authority for curriculum and assessment has migrated to more central or state control.

Since 2001, many states have developed new, more specific mathematics curriculum standards outlining the intended curriculum, K–12. Changes in the degree of specificity of standards is driven by the NCLB requirement for states to adopt “challenging academic content standards” in mathematics and to annually assess the progress of schools in supporting student learning matched to these standards (NCLB, 2001).

In some states the standards are intended to be “models” for school districts to utilize in shaping local curriculum specifications. In others, they are mandatory, specifying the mathematics all students within the state are expected to learn at particular grades. In addition to establishing different standards for school mathematics, states also vary in their cycle of review and revision of standards. Therefore, in any given year 6-8 states typically review and/or revise their standards. For example, at least three states (WA, MO, and MN) currently have new draft standards under development or review.

Although NCTM offers a national model and vision for mathematics curriculum standards, this vision has been interpreted individually by the states. While some argue that the U.S. has a national curriculum, namely the NCTM *Standards*, others believe that interpretation at the state level has introduced variation and therefore continued and advanced the historical “local-control” mentality of curriculum articulation in the U.S.

While local control of educational decisions, including curriculum standards, is a hallmark of American education, increased accountability has focused more attention on state-level curriculum decisions. A recent survey indicates that state-level curriculum standards are receiving as much, if not more, attention by school administrators and teachers as the textbooks purchased to support curriculum implementation (Reys, Dingman, Sutter, & Teuscher, 2005).

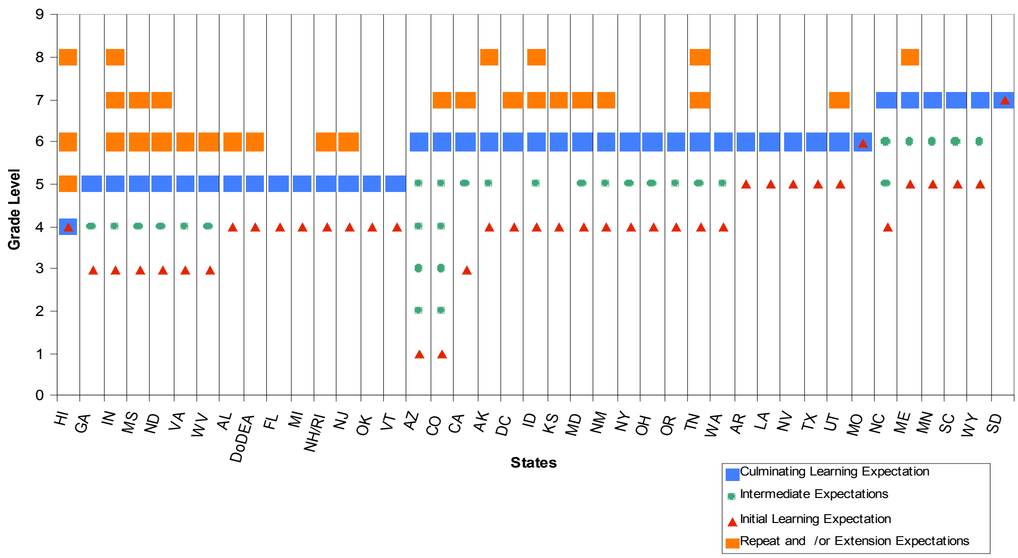
A review of state-level mathematics curriculum standards by the Center for the Study of Mathematics Curriculum (Reys, et al., 2006) confirms that mathematics learning expectations vary along several dimensions including grain size (e.g., level of specificity), language used to convey learning goals (e.g., understand, explore, memorize, etc.), and the grade placement of specific learning expectations. In particular, when mathematics topics are introduced, their trajectory of development across grades and the grade at which students are expected to know and apply particular mathematical content differs dramatically across the states.

For example, ability to compute with fractions is a core idea for the elementary and middle grade mathematics curriculum. Some state standards introduce computation with fractions (with common fractions such as 1/2) as early as grade 1 while others begin instruction on the topic in grade 3 or 4. Some states standards include an expectation for students to be fluent computing with fractions by the end of grade 5 and other state standards include this expectation at grade 8. Figure 1 provides a summary of the grade at which addition and subtraction of fractions is introduced and when proficiency is expected as outlined in 42 state standards. As noted, the states differ in when addition and subtraction of fractions is to be introduced (ranging from grade 1 to grade 7), the number of years this topic is developed (ranging from 1 to 6 years), and the grade level at which students are expected to be proficient with addition and subtraction of fractions (ranging from grade 4 to grade 7). The variation noted here is not unique to this one topic (computation with fractions) but is evident in all of the topics we analyzed (see Reys, 2006 for a full report of findings).

For many states, grade-level learning expectations represent a new level of state leadership for curriculum articulation. Although individual state standards may provide increased clarity and coherence within their respective state, as a collection they highlight a consistent lack of national consensus regarding common learning expectations in mathematics at particular grade levels.

As states look ahead to the next cycle of review and revision of mathematics standards, it is likely that they will continue to look to national groups for curriculum leadership. In fact, 85 percent of respondents to a recent survey of staff at state Departments of Education indicated that national leadership is needed to assist in future articulation of curriculum standards in mathematics, particularly from national professional organizations of mathematics teachers (K–12 and university) and mathematicians (Reys et al., 2005).

Figure 1. Progression of initial to culminating learning expectations for addition and subtraction of fractions, by state.



The Textbook Curriculum: Mathematics Textbook Development in the U.S.

To be effective, curriculum standards whether developed at the national, state or local level, must be translated into materials that guide the day-to-day decisions of teachers and help them focus on the important mathematical learning goals in significant ways. Tyson-Bernstein and Woodward (1991) describe as ubiquitous the role of textbooks in American schools, and as a prominent, if not dominant, part of teaching and learning.

In spite of the acknowledged role of textbooks, only recently (c.f. Schoenfeld, 2002; Senk and Thompson, 2003) has there been research to extend our understanding of the effects of textbooks on students' learning of mathematics. Kilpatrick (2003) calls for additional scholarly work in this critical area and Pellegrino (2002) makes the case for establishing design principles for instructional materials that draw on research on how students learn (Donovan, Bransford, and Pellegrino, 1999).

One of the major obstacles to the development of high quality mathematics textbooks in the U.S. is the lack of national consensus on grade-level learning expectations. In order to meet the diverse standards of states, publishers of mathematics textbooks include a wide range of mathematics content in a single textbook (Seeley 2003). The result is a collection of concepts and skills that "align" with the standards of many states but may fail to focus on or develop in depth mathematical ideas across grades. Historically U.S. mathematics curricula have been characterized as superficial, highly-repetitive, and copious in the amount of content reviewed in any given year (Schmidt, McKnight and Raizen 1997). Reviews of textbooks conducted by the American Association for the Advancement of Science and the U.S. Department of Education characterize many commercially-developed middle school mathematics textbooks as "unacceptable" with regard to content emphasis (AAAS 2000). Furthermore, these researchers found that, "many textbooks provide little development in sophistication of mathematical ideas from grades 6 to 8" and "most of the textbooks are inconsistent and often weak in their coverage of conceptual benchmarks in mathematics" (AAAS, 2000, p. 1).

The NCTM *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) provided a new vision for K-12 mathematics. One of the challenges the NCTM *Standards* presented was the development of new curriculum materials to support the new focus on quantitative reasoning, problem solving and communication as well as increased engagement of students. Given the history of commercial publishers being unwilling to risk millions of dollars to develop a textbook series that is significantly different from the market leaders, it seemed unlikely that new mathematics textbooks reflecting the vision of the NCTM *Standards* would be forthcoming from the commercial sector.

The National Science Foundation, concerned with mathematics performance reported by National Assessment of Educational Progress and the consistently low performance on international assessments, began supporting the development of research-based K-12 mathematics textbooks. The NSF ultimately funded 13 different comprehensive curriculum development projects that spanned K-12 (Reys, et al., 1999). These textbooks were extensively field-tested in schools and then revised before becoming commercially available. The resulting mathematics curricula represent notable exceptions to traditional textbooks that typically lack a research and development phase prior to release (Trafton, et al., 2001).

The NSF supported mathematics textbooks have been reviewed by committees of the US Department of Education and AAAS (Kulm, et al., 1997??) and judged of exemplary quality compared to other commercially available textbooks. Studies have consistently reported positive growth in the mathematics learning, particularly related to reasoning and problem solving, as a result of use of the new curriculum materials (Senk & Thompson, 2003).

Estimates of the market share of NSF-funded textbooks range from 10-20 percent of students and teachers at the secondary level and from 20-30 percent at the elementary level (Education-Market, 2006). Use of these textbooks by a significant segment of the school population is evidence that NSF's effort to stimulate new models of textbooks has been successful. In addition, commercially developed textbooks are beginning to incorporate some of the features of the NSF materials (e.g., extended investigations, focus on multiple strategies, and encouragement of students to discuss, explain and defend their thinking).

The NSF funded mathematics textbooks provide the basis for enacting a different vision for teaching and learning – one that emphasizes student exploration over teacher lecture; conceptual understanding over rote skill development; contextualized problems over "naked number" problems. Thus, the textbooks themselves are often the impetus for philosophical clashes between reform and anti-reform groups. For example, at the state level it was reported that “California's mathematics policy followed a persuasive (albeit deceptive) campaign alleging the failure of the current reform movement in mathematics education” and the NSF supported mathematics curricula were at the epicenter of these discussions (Becker and Jacobs, 2000). The story of one school textbook adoption committee was recently chronicled and illustrates the range of issues and personal biases that surfaced, how opinions were persuaded, the value attached to research evidence and ultimately how decisions were made (Newman, 2004). The story is a reminder that “decisions about educational reform are driven far more by political considerations, such as the prevailing public mood, than they are by any systematic effort to improve instruction” or learning (Dow, 1991).

The Implemented Curriculum: Teacher’s Use of Textbooks

Research demonstrates that, while teachers rely heavily on textbooks, they often make major alterations to the textbook lessons resulting in an *implemented curriculum* that looks very different from the intended curriculum (Ball, 1996; Cohen, 1990; Lloyd, 1999; Remillard, 2000). Kilpatrick explains,

> Two classrooms in which the same curriculum is supposedly being “implemented” may look very different; the activities of teacher and students in each room may be quite dissimilar, with different learning opportunities available, different mathematical ideas under consideration, and different outcomes achieved. (p. 473)

It is essential to keep in mind the way teachers use curriculum materials and the factors that influence this use. Specifically, we need to learn a great deal more about what curricula and associated instructional practices work for which children in what kinds of circumstances with what level of support for teachers. Progress on understanding the complex role that curricula play in the learning of both teachers and students will help advance the development of future generations of mathematics curricula, of professional development strategies for supporting teacher learning around curricula, and of mathematics teacher education practices.

The Assessed Curriculum: Influence of NCLB-mandated Annual Assessments

As noted, NCLB has prompted major changes in attention to the *assessed curriculum*. That is, accountability measures mandated by NCLB legislation have prompted increased attention to test preparation and significant changes in school schedules and time spent on mathematics. The writing and selection of test items and the analysis of results play increasingly important roles in making policy decisions concerning school mathematics. It is critical, therefore, that increased attention be given to the creation of tools and methodologies for evaluating the impact of curriculum on teacher behavior and student learning. Additionally, we need to monitor the influence of high stakes assessments on decisions teachers make regarding the implemented curriculum. That is, to what extent and in what ways is the intended curriculum modified in response to pressures to perform well on tests that may not emphasize the full set of curriculum standards?

Policies Regarding School Mathematics Curriculum and Course-Taking

States have initiated major changes since the passage of NCLB with regard to specifying standards for school mathematics, increasing mathematics requirements for high school graduation, and developing assessments for accountability purposes. Although standards, course requirements, and assessments differ across states, efforts are directed at a common goal, namely to strengthen mathematics programs in order to provide opportunities for all students to learn important mathematics and to be prepared for continued study of mathematics as it relates to their future endeavors. Some policies directed at secondary school mathematics are summarized below.

<u>Mathematics Course Requirements for Graduation</u>. States not only govern curriculum standards but also the requirements for high school graduation. Increasing the number of years of mathematics required to graduate from high school is a recent strategy employed by states to strengthen the preparation of students for college and work (Achieve, 2004). Current credit requirements vary across states (see Table 1). As noted, five states do not mandate a minimum number of mathematics credits for high school graduation. Rather, course requirements for graduation in these states are made at the district level. Seven states require 2 years of high school mathematics and the majority of states (24) require 3 years of mathematics for high school graduation. As of 2006, 11 states require students to take 4 years of mathematics.

Five states offer different graduation diplomas that are dependent, in part, on the number of years of mathematics students complete. For example, high school students in the state of Georgia who earn a "Technology/Career-Preparatory" diploma are required to complete 3 years of mathematics, while students receiving a "College Preparatory" diploma are required to complete 4 years of mathematics.

Table 1. *Number of Years of High School Mathematics Courses/Credits Required for Graduation**

Specified at Local Level	1 year	2 years	3 years	4 years	Varies by Diploma
CO, IA, ME, MA, NE		AK, AZ, CA, ID, MT, ND, WI	CT, DC, DoDEA*, HI, IL, KS, KY, LA, MD, MN, MO*, NH*, NM, NJ, NV, NY, OH, OK, OR*, PA, TN, UT*, VT, WY	AL, AR*, DE*, FL*, MI, MS*, RI, SC, TX*, WA, WV	IN (2-4 yrs) GA (3-4 yrs) NC (3-4 yrs) SD (3-4 yrs) VA (3-4 yrs)
5	0	7	24	11	5

* This information includes requirements that have been approved and are being phased in with a particular freshman class.

Requiring Specific Mathematics Courses. About half of the states currently require students to complete at least Algebra I for high school graduation (see Table 2). Four of these states (DE, LA, MI, TN) specify Integrated Mathematics I as an appropriate substitute for Algebra I. Six states (AR, DE, MI, MN, TX, VA) require courses equivalent in rigor to Algebra II for graduation. In addition to specifying a required number of high school mathematics courses, two states (DE and MI) require that students take a mathematics course in the senior year of high school.

Table 2. *Mathematics Courses Required for High School Graduation/Diploma*

Required Courses	States	Number of States
Algebra I	CA, DC*, FL*, GA*, IN*, LA, MS, NC*, ND, NH, NM*, OK*, SD	13
Algebra I and Geometry	AL, DoDEA, IL, KY, MD, TN*	6
Algebra I and II	MN	1
Algebra I, Geometry and Algebra II	AR, DE*, MI, TX, VA	5

* An equivalent course is permissible.

Passage of Exam Required for Graduation. *NCLB* requires that every student participate in assessments based upon state standards at least once during high school; however, federal legislation does not mandate a single test, the year during which the test is taken, or the stakes/consequences of the exam score. In fact, the state assessments vary as to the level of consequence for individual students. In some states, students are required to pass the assessment(s) in order to receive a high school diploma. In other states, while students are required to complete the state assessment, if they do not pass the exam they can receive a high school diploma by meeting alternative requirements. In some of these cases, the assessment score is factored into a course grade. Finally, some states require students to complete the assessment, but the score does not have consequences for the student in their course grade and/or graduation eligibility (Reys, Dingman, Nevels & Teuscher, 2007).

The type of exam also varies by state. Some states require an "end-of-course" exam (e.g., to follow Algebra I) while others require all 10th graders to take an exam that covers a broad spectrum of content from multiple content strands.

Summary

The past 20 years have witnessed an unprecedented focus on school mathematics in the U.S. Prompted by national reports and international assessments, attention has focused on the need to raise the quality of school mathematics programs for K-12 students to be more successful in order for the U.S. to continue to compete in the global economy. Curriculum has been central to many of the recent school mathematics improvement efforts. In fact, K-12 students are studying more mathematics, often at an early grade, with a focus on conceptual understanding as well as skill development and problem solving. Many students and teachers are using new kinds of textbooks that are based on investigation and discussion that place greater demands on them. More students in the U.S. are also studying mathematics through high school.

One of the unresolved educational issues in the U.S. revolves around governance of curriculum. That is, should there be more central authority for specifying mathematics learning goals and for measuring student learning or will the tradition of local control for these decisions continue? This debate will likely continue, although it appears for the first time in a long while that more scholars and educators are speaking out for centralized authority. Stay tuned.

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