Modern tools in mathematical education

Bashmakov Mark, member of Russian Academy of Education,
Chief of Institute of productive learning RAE
Pozdnyakov Sergej, Saint- Petersburg state university, professor of mathematics
Entina Sophia, Saint- Petersburg electro technical state university, assistant professor of mathematics

Mathematics is one of the central disciplines in modern education. This role of mathematics is determined by the wealth of ideas it had been accumulating for many centuries. These ideas make up an essential part of our cultural heritage and a good textbook in mathematics should popularize them.

In XXI century many of the information technologies that belonged to science fiction only few years ago became part of our daily lives. Unfortunately, all these innovations have very small effect on educational process. Modern effective and technologically advanced educational methods are not in great demand from teachers, while traditional educational process does not meet the needs of modern society. There is a need for new educational tools that would stimulate constructive creative activities of students and would be accepted by both teachers and students.

1. A Modern Textbook

Its main characteristics should be a modular structure rather than a linear presentation of the material. The text should be well structured and divided into small portions. The material can be made more accessibly by including a wealth of examples, illustrations and hints throughout the text. The textbook should contain a problem set for every module. Each problem set should include some research problems that would introduce the student to the real Mathematics.

Before the discussion of the new tendencies in creation of a school mathematics textbook, we shall briefly review the goals of mathematical education.

In our view these goals are:

- forming a notion of mathematics as a universal language of science and a means of modeling of various processes, a notion of the ideas and methods of mathematics;

- development of the logical reasoning and the ability to think algorithmically, of the spatial imagination and of the critical thinking necessary for success in the students' future professional life and higher education;
- acquirement of the knowledge and the skills necessary in everyday life;
- cultural and personal enrichment by means of mathematics, by teaching the significance of mathematics for the progress in science and technology, by emphasizing the place of mathematics in the human culture, by learning the history of mathematics and the evolution of its ideas. Along with the three main directions – psychological (cultural and personal enrichment), pragmatic (learning specific skills), and educational – we emphasize a "philosophical" direction aimed at forming a notion about the ideas and methods of mathematics.

To achieve these goals the teacher's work should be taken to a different level. We will call this a **modular** level.

Module is a single topic or section of the subject that is a part of the curriculum of the given school. Module must be associated with a didactic system that describes the interaction of various educational techniques and methods and makes it an integral part of the overall educational system.

Based on the chosen didactic system for the given module, the teacher develops the didactic complex that will be used in the class.

Parameters that describe the level of teaching mathematics can be divided into three groups.

The first group has to do with general development of the student. Here we included algorithmic, deductive and logical thinking; development of geometric and spatial concepts, mathematical language; ability to perform complex reasoning (analysis, synthesis, generalization etc.).

The second group includes more traditional criteria of the effectiveness of education.

The third group is associated with constructive activity of the student. It is closely related to the two preceding groups and therefore its structure is more complex.

This group can be divided in two subgroups. The first one contains parameters related to the applied aspect of teaching, that is constructing of mathematical models, computing, analysis of the result etc. The second one has to do with development of creative abilities and individuality of the student. For example, the ability to learn independently, self-control, creativity, etc.

We will provide some brief clarifications.

Algorithmic (reproductive) style is the most commonly used style of student activity in modern schools. Students perform precisely formulated standard tasks and are usually asked to follow a given model. Based on our research, such tasks make up over 80 percent of assignments in modern textbooks. However, we are not saying that this style has no value.

The term *visual style* needs some clarification. It is based on translation of information from one language into another, on mastering various languages especially the visual language.

According to a generally accepted point of view, there are three main informational languages that are used in teaching mathematics – verbal, symbolic and visual. Mastering these languages is necessary. It is very important **to be able to choose an appropriate language and to be able to translate from one language into another. We chose to call this style *visual* because of the growing importance of the visual language.**

The *Applied* style of teaching has long-standing traditions in Russia. Here we include ability to organize computations, solving verbal problems and constructing and exploring of mathematical models. On the other hand, realization of this style often confronts significant difficulties. Construction of models takes so much time that very often it does not justify the

results. Besides, the teacher has to spend too much time to prepare for the lesson. As a result, we usually do not go beyond several standard models or situations.

At the same time introduction of new situations is becoming necessary in view of the new applications of mathematics in economics, in the humanities, and in certain topics in physics that are new for the school curriculum.

Deductive style is traditionally considered to be the leading style in teaching mathematics. Mastering this style is usually associated with geometry. However, recently, the use of this style has changed significantly. First of all the formerly undisputable axiomatic approach to teaching geometry is now open to doubt. Authors of many textbooks believe that using a deductive approach in teaching geometry is neither necessary nor possible. Besides, the role of proofs in solving geometrical problems is becoming secondary.

The research style is now becoming the centre of attention of the teachers due to the widespread use of parametrical problems and coordinate method. Instead of many small problems and exercises a student is offered a research project that requires a lot of work within the framework of a given mathematical situation. However the research style is not just a tasty new addition to the traditional dish. It may be given the leading role on all stages of education and this style may become accessible for an average student.

Combinatorial style means a wide use of notions and methods of discrete mathematics – natural and integer numbers, inductive processes and constructions, sequences and finite series and, finally, combinatorics and elements of probability theory. There is a growing need to introduce combinatorial style at all stages of mathematical education because of the growing use of digital technologies in industry and in everyday life.

The potential of the game style has been explored by many enthusiasts for a long time. The position of psychologists that games could be the main incentive in the development of interest and therefore of success in learning is shared by everybody. However there is

insufficient number of specific methodologies that would help to introduce the game style into educational process along with other styles.

2. A Digital Textbook.

Important characteristics of the modern textbook are *differentiation and individualization* of teaching. There is a conflict between the many requirements that a textbook has to satisfy and the limited nature of a conventional textbook. This conflict can be resolved by passing some of the functions of a textbook to other tools of the methodological complex. A new group of resources that have been developing very actively in the recent decades allows an integration of traditional hard-copy resources with modern digital and internet resources.

- **A digital textbook**

One of the most important examples is a digital textbook. It contains a very concise presentation of the main ideas along with dynamic examples. The material for a digital textbook is selected based on how well it lends itself to analysis, experimentation, and development of one's own understanding of the notions in hand. The use of dynamic models allows to perform some relevant exercises and to test the mastery of the theory.

- **Digital Interactive Mini-problem sets**

Among the new digital resources there are various means of testing and verification of the solutions. Along with simple interactive tests of the multiple-choice type, there are new programs that can verify hypotheses offered by a student. These programs do not use just a simple comparison of the hypothesis with preset correct answers but can interpret the proposed answer. Their responses are not limited to **true** or **false**; they provide examples that illustrate why the proposed answer is impossible or insufficient. The responses contribute to a deeper understanding of the problem and stimulate further analysis.

Labs and Research Projects

Wide Use of Manipulators

Manipulators are special tools designed to emphasize the meaning or the structure of a notion or a method. They automate the purely technical steps that are not relevant to the essence of the material, while providing the freedom of action that is necessary to master a new method. The role of the blackboard as the main tool in teaching will be diminishing thanks to the availability of dynamic illustrations, to the possibility to repeat the demonstration and to draw attention of the student to the more difficult aspects of the problem.

Digital resources are very useful in more complicated research tasks, such as an exploration of a scenario, which require student's initiative. Digital resources provide special tools or manipulators that can help perform the research.

Tools to Monitor Students' Progress

Computer programs that can check students' homework enable teachers to spend their time more efficiently while providing full report on the students' activity thanks to the ability program to provide a protocol of the student's work.

Testing software can check solutions without teacher's participation and free the teacher from grading a substantial part of students' homework. And yet, the report from the testing software that outlines all the steps taken by the student to obtain the solution will give the teacher full information about the student's.

Other Means of Teaching Mathematics

Resources designed for students should be supplemented by important tools and resources for the teacher. Among these resources could be a conventional teacher's guide with recommendations for organization of the teaching process, problem generators, and a portfolio to store information about the achieved results. The question of an effective use of

the Internet potential and of incorporating information search into the classroom activities has not been sufficiently worked out and needs further development.

The authors of this article are drawing upon their own extensive experience. M.I. Bashmakov is the author of many textbooks for regular primary and secondary schools, as well as schools with concentration in specific subjects. In particular, his textbooks serve as a basis for a project conducted by the Institute of Productive Learning of the Russian Academy of Education. The goal of this project is to develop an innovative methodological complex. The textbook serves as a fairly concise guide to the complex as a whole. Its characteristic propriety is its modular structure. The elements or modules of this structure are two-page spreads containing all key definitions, elements of proofs, examples, comments and references to other relevant parts of the complex.

All authors of the present article have participated in creation of other innovative methodological complexes, such as «Mathematics in school – XXI century», «Verification of constructive hypothesis and solutions of difficult logical problems», «Tools supporting research activity», «Deductive games in math lesson», and others.

The possibility of quick and easy inclusion of dynamic tools when and where they are needed allows to efficiently fulfill all functions of a methodological complex.

The most important new element of this complex is classification of the problems not just by their level of complexity, but also by the cognitive type of the problems. The digital components of the complex includes several types of modern resources. The simplest one is a system of presentations containing slides, texts, and examples that illustrate the main ideas of the textbook. Such presentations help the teacher to formulate the task, the goal of the lesson etc.

A more complex type of digital resource is a demonstration, which is usually a certain dynamic model. The most successful among them are those related to the history of

mathematics. Another cycle of demonstrations show a sequence of operations and in that way help students learn new algorithms.

The following projects of innovative methodological complexes are now being completed:

“Mathematics in school – XXI century” (fig.1). The main menu of the digital textbook is shown in figure 1:

	Lesson plans and teacher’s guides
Concise textbook with dynamic models	Problems with virtual elements
Interactive mini-problem sets	Labs
Research problems	Individual homework assignments
Independent projects	Exams and self-tests Students’ work

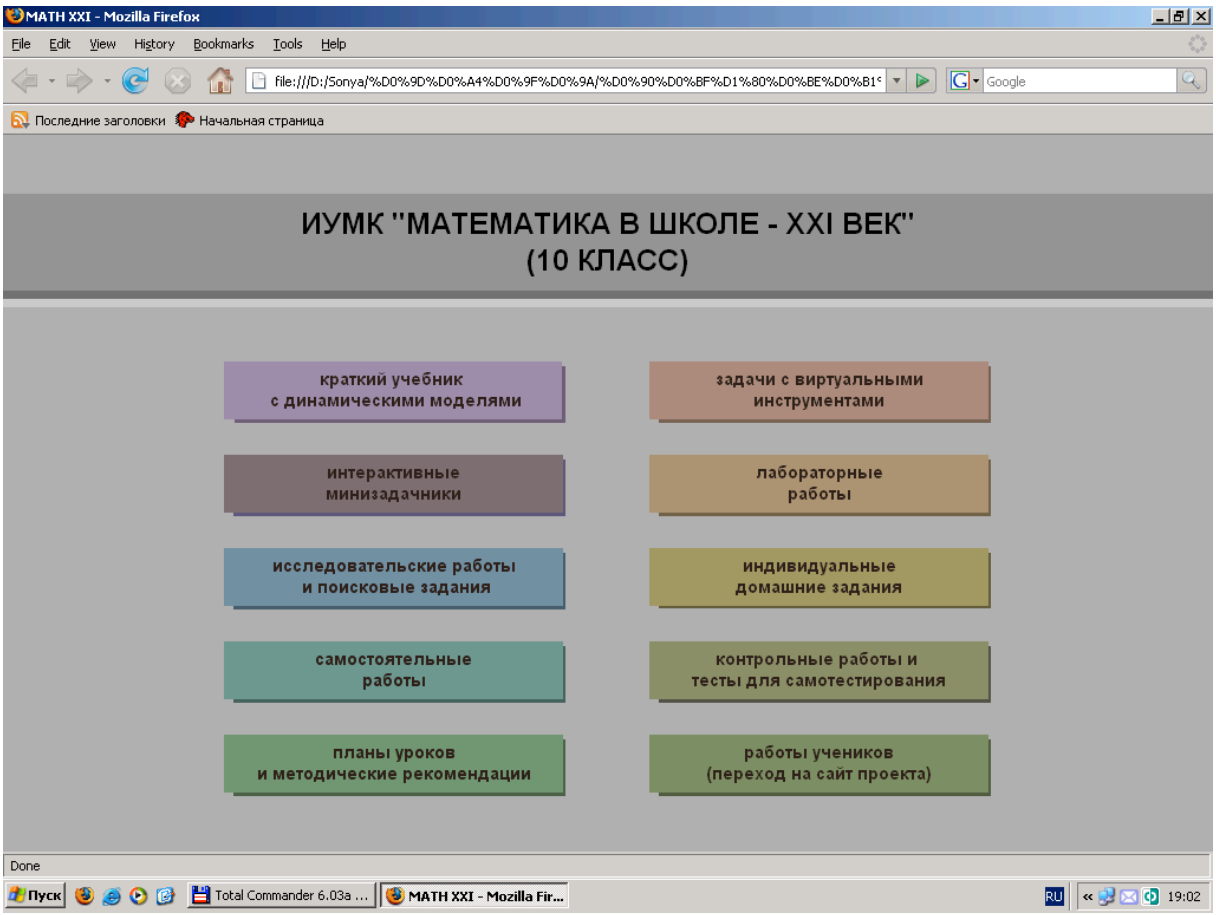

Figure 1.

"Verification of constructive hypothesis and solutions of difficult logical problems" (fig. 2) The student sees the statement of the problem and a virtual keyboard to enter the reply. Reaction of the environment to the input of parameters is displayed in three windows (from left to right).

- An example when the given parameters meet the conditions of the Problem'.
- A counterexample when the given parameters do not meet the conditions of the problem.
- An example when the conditions of the problem are met with different parameters.

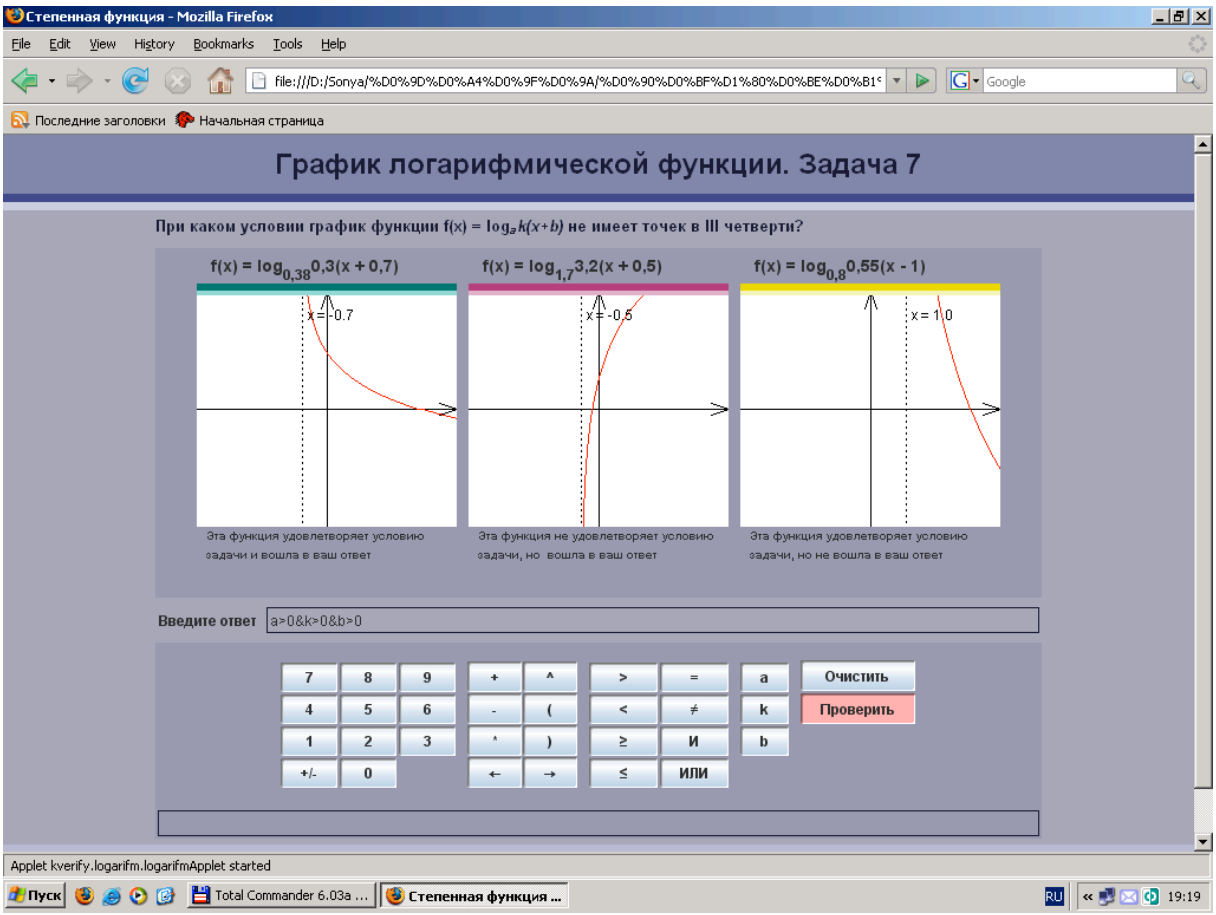

Figure 2

"Tools supporting research activity"

This system can parameterize the problems and thus generate any number of new problems and provide students with various variants of problems. Teachers and students can create problems and problem sets on their own and, what is important, all this can be done as on-line or distance communication.

Two topics taken from the 10-th grade mathematics curriculum are the basis of this methodological complex – "Integer Numbers" and "Combinatorics".

What is special about these two topics is that the most simple and the most complex problems can be formulated using the same language. Thus, using the same language one can construct problems accessible for average school students as well as problems that still remain unsolved.

"Didactic games in math lesson" and other projects

In conclusion we would like to mention the KIO competition (abbreviation of the words: **c**onstruct, **e**xplore, **o**ptimize in Russian). All contestants work with the software that allows them to experiment with the contest problems. All problems are of the research type. The winner must find the optimal solution. Information about the competition and past problems can be found at the website www.ipo.spb.ru