CHILDREN REFLECT ON THEIR MATHEMATICAL THINKING

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Children were interviewed about their mathematical thinking and asked to reflect on their learning. This was part of a larger study exploring teacher behaviours that challenge children to probe their mathematical understandings. Fifty-three interviews were conducted in 4 schools with 5- to 7-year-old Australian children. The subjects were involved in conversation with their teachers during the mathematics lesson. Video-stimulated recall was used with a conversational interview to prompt children's recollections and reflections. Findings indicate that young children in the first years of schooling are able to recall events in their mathematics lessons, to reconstruct their thinking and reflect on their mathematical learning.

This paper reports an aspect of a larger study of teacher behaviours that challenged children to probe their mathematical thinking in naturalistic classroom settings. Children's accounts of their mathematical thinking show that young children are able to recall events in their mathematics lessons, to reconstruct their thinking and reflect on their mathematical learning (Cheeseman & Clarke, 2007).

BACKGROUND

The theory of social constructivism underpins this research. Cobb, Wood, Yackel and McNeal (1992) argued that the construction of knowledge occurs within a social and cultural context where discourse is a vital component in establishing an effective learning context. The focus of this research is the meaning constructed between the teachers and children in classrooms.

There has been a long history of interviewing young children to describe their mathematical thinking (e.g., Donaldson, 1978; Gelman & Gallistel, 1978; Hughes, 1986). These interviews often involved children performing mathematical tasks to demonstrate their thinking or development. Task-based interviews have also been used to assess and plot the growth of the mathematical thinking of children over time (Clarke & Cheeseman, 2000). However there appears to be little research that reports young children's reflections on their thinking in post-lesson interviews.

Franke and Carey (1997) conducted interviews to research first-grade children's views about what it means to do mathematics in problem solving classrooms. They found that young children were in fact able to reflect on classroom events.

McDonough (2002) reported procedures that prompted eight to nine year-old children to articulate their beliefs about mathematics. Children found it a difficult to talk abstractly about learning, however, they "held beliefs about mathematics, learning and helping factors and could articulate beliefs when prompted" (p. 270).

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To capture some of the complexities of classrooms settings and to collect rich data, the approach termed *complementary accounts methodology* was used for the larger study (Clarke, 2001). While the methodology used differed from that of Clarke, similar fundamental techniques were used: videotaping the whole mathematics lesson, audio taping participants' reconstructions of classroom events, and an analysis of the multiple data sets. The children's accounts of their classroom interactions with the teacher are reported here.

METHOD

In total, 53 children were interviewed on the day their mathematics lesson was conducted. The children were aged five to seven years from four classes, each in a different Australian school. The selection of students was based on classroom observation notes of the researcher.

The interviews were audio taped for transcription and analysis. A video of the lesson was used as a stimulus to recall sections of the lesson directly involving each child. Children were asked to recount events where they were in conversation with the teacher, to say what they were thinking at the time, and to reflect on what they had learned in the mathematics lesson. The interview was conversational in style. While there was an interview script, it was adapted in order to elicit responses from each child. The scripted questions were:

1. I am interested in the times when teachers talk to children in maths lessons. I noticed that your teacher had a talk with you in that maths lesson. Can you remember that? Can you tell me what happened?
2. I think that we got that on video. Would you like to see it?
3. What were you thinking about? (Maybe just watch it at first.)
4. Can you say what was happening?
5. What did you learn in maths today? Was there anything else?

These questions are modelled on those used by Clarke (2001, pp. 13-32) however these questions have been simplified for young children.

Video-stimulus recall

There appears to be scant literature describing the use of stimulated recall using video with young children to investigate their perspective of mathematics lessons. There are reports of Year 8 children, using video-stimulated interviews to reconstruct the learner's perspective (e.g., Williams, 2003) and reports of teachers video-stimulated recall of the events in their classrooms (e.g., Ainley & Luntley, 2005) but there seems to be no use of this methodology in mathematics education with young children.

Because little was known about how young children would respond to video-stimulated interviews, some piloting was undertaken. It became clear that the best way to prompt recall was to play a little of the beginning of an incident of interest to set the scene for the child then to pause the video and to ask, "Do you remember that?

What was happening there?” If a child had no recollection of the event, the entire video episode involving them in conversation with the teacher was played and used as a stimulus to describe their thinking or reflect on their learning. In general, the video was used as a starting point only.

DATA CODING AND ANALYSIS

Interviews were digitally recorded. Seventeen interviews were transcribed in full. The remaining 36 interviews were coded directly from the audio files. Data were considered in terms of the children’s recall of an incident or task, description of events, explanation of their thinking, and description of their learning. Categories of response emerged from the data. Descriptors of response were listed in increasing levels of sophistication, with 1 being the least and 5 or 6 as the most sophisticated responses as follows:

Recall of the incident/task

1. no recall
2. child could recall the event only after of the entire video excerpt was replayed
3. recall with the video paused just before the event of interest or with the video playing in the background with no audible sound
4. recall spontaneously with little or no assistance of the video extract

Description of events

1. no description of interaction with teacher
2. describe actions
3. describe outcomes only, e.g., a work sample
4. describe the event from their perspective
5. describe reasoning and/or justify their thinking

Explanation of their thinking

1. no explanation
2. “account for” the videotape e.g., make up a “story”
3. explicit description of thinking
4. explain/reconstruct thinking, reasoning, justifying, evaluating thinking

Description of learning

1. unable to specify learning
2. learned “nothing”
3. learned a behaviour not mathematics e.g., “to share”
4. remembered factual information e.g., number facts
5. learned how to do something e.g., “to count by 6s”
6. described learning at a conceptual level, expressed as a mathematical principle or an insight, e.g., “I can count by 1s, 2s, 3s, 4s, 5s, 6s, 7s, 10s, and 100s and 1000s

> …once I can count by ten I can count by all the rest. Like 10, 20, 30, 40, 50, and it always has a zero on the end."

In general, the highest level of descriptor was coded when evidenced anywhere in the interview. Codes were then entered into a statistical analysis program (SPSS) to produced descriptive statistics.

Reliability of coding

To improve internal reliability, interviews were re-coded. An independent person, skilled at listening to young children describe their mathematical thinking coded a 20% sample of the audio data. All points of difference were discussed and an agreed understanding of the data was reached. Based on the combined critical analysis, 6 further interviews were transcribed in full (17 in total) and category descriptions were refined. The entire data set was recoded applying the new protocols without any reference to the previous coding. The results of this second coding form the data reported here.

RESULTS

Recall of events

Using videotape of events involving each child in the mathematics lesson of the day to stimulate the recall and an account of the episode from the view of the child was largely successful. This is evident from Table 1, which summarises the categories of responses of children's recall of events, where only 2% of children were unable to recall the events of the lesson. Some children needed to watch the entire replay of the videotape where they were in conversation with the teacher in order to talk about it (23%). Many children, having watched the video of the lesson leading up to the event, could recount their version of what had unfolded after the videotape was paused (30%). In addition almost half of those interviewed could recall a conversation with the teacher before the video was replayed.

Category of response	Frequency as a percent (n = 53)
1. No recall	2
2. Recall with video replay of the event	23
3. Recall with video paused or with no audible sound	30
4. Recall spontaneously	45

Table 1: Categories of response of children's recall of an event.

Description of event

An analysis of the children's descriptions of events revealed an interesting three-way split of responses (see Table 2). Some children described only what they did (23%).

The following example illustrates this category of response. James could be seen on the video interlocking blocks but saying nothing:

Interviewer: So what was happening here?

James: My brain was counting and I wasn't. [James, J2.3:25]

Other children offered a description from their point of view (36%). For example, Ali explained his counting of 5 groups of 5 teddies saying, "It goes 10, 20, 30, 40, 50. You have to count the ears" [Ali, G1, 7:30]. It is hardly surprising that 36% of children who could remember the event described it from their point of view. In fact what was interesting was that such a large proportion described the event with some reconstruction of their reasoning at the time (28%). This was perhaps the most interesting group of responses. For example, Jessica was explaining how to weigh a dog, Joey, who would not stand on bathroom scales:

Interviewer: Can you tell me about your good idea for maths today please?

Jessica: I thought of holding Joey on the scales. I would know how much Joey weighed. So I hopped on the scales with him and I holded him. And then we took away 19 [from 28] because I was 19 and he was 9 and so that was 9 kilograms and that's what he weighed [Jessica, J3, 0:35].

Category of response	Frequency as a percent (n = 53)
1. No description of interaction with teacher	4
2. Describe actions	23
3. Describe outcomes only, e.g., a work sample	8
4. Describe the event from their perspective	36
5. Describe reasoning and/or justify thinking	28
6. Missing	2

Table 2: Children's descriptions of events.

Explaining thinking

Table 3 shows the number of children who could explicitly describe their mathematical thinking was high (85%). Expecting children to be able to communicate their thinking has been an element of Australian mathematics curriculum definition for years (Australian Education Council, 1991). Certainly based on classroom observational data from the classrooms of the children interviewed here, it is a clear expectation of their teachers that they explain their reasoning.

It should be said that these children had been learning mathematics in the classrooms of "highly effective" teachers of mathematics (McDonough & Clarke, 2003) for 8 months. Perhaps this would account for their readiness to describe their mathematical thinking. Whether children in other classrooms can explain their thinking with this frequency is a question that might be explored by further research.

An example of the type of response that shows a child reconstructing and evaluating his thinking is when Tom offered a thinking strategy for his classmates who could not count by 4. His idea was to use a count by 2.

Interviewer: Now Mrs A says that's a really complicated way to work it out I can't really hear what you were saying. She was looking at a page that had 8 legs and 4 things on each leg. How were you trying to work that one out?

Tom: Oh a different way. You know, when there's 8 legs and I was thinking if people didn't know how to count by 4, I was splitting 4 in half to make two on each side. Then I did 2 X 8 equals 16 then I have to count by 2s up to 32 what it equals. I have to count by 2s 16 times [Tom, G1, 1:00].

A few children could not explain their thinking and another few gave an explanation of their thinking as if telling a story. In examining the knowledge that experienced mathematics teachers access to operate effectively, Ainley and Luntley (2005, p.78) made a distinction that may be pertinent here. Teachers were shown episodes of videotapes of their classrooms and in these interviews some teachers gave an "account for" rather than an "account of" their actions. The children who made up a story to suit the occasion may be doing the same thing or perhaps there is a different mechanism at work. No definitive statements could be made based on the evidence collected here all that can be said is that 3 (6%) children made up a fiction to match the video.

Category of response	Frequency as a percent (n = 53)
1. None	6
2. "Account for" or gave an invented story	6
3. Explicit description of thinking	43
4. Reconstructs thinking, justifies, reasons, evaluates	42
5. Missing	4

Table 3: Children's explanation of their thinking.

Specify learning

Only 15% of children did not know what they learned in the mathematics lesson (see Table 4). The category of "nothing" proved unreliable because it became clear that young children translated "What did you learn today?" into "What new things did you learn today?" and these two questions are quite different. Therefore this category will not be discussed. Some children talked about behavioural learning, for example, "to share." Or they referred to non-mathematical things, for example the learning context, "talking about tools and building" [Michael, Jk2]. Totalling the first 3 categories of Table 5 shows that 30% of the children did not specify mathematical learning. The three categories of most interest were those that made distinctions between learning factual information (15%), learning how to do something (23%) and learning at a conceptual level (21%).

About one third of the children who remembered facts talked in terms of numbers. For example, Annie who had been talking about measuring with a piece of string when asked what she learned said, “I learned that 9 + 11= 20.” While it is not possible to be certain from these data, it raises a question as to what these young children think constitutes mathematics learning. Is learning mathematics equated to remembering numbers? What can be said is the children interviewed for this research described their learning in detail. For example, Tom talked about his learning saying,

Tom:	I think I might have leant some new times tables.
Interviewer:	In which times table?
Tom:	I think some were in the, I think some were like 9 X 6. I didn’t know that but then I knew it because I just counted by 6 nine times [G1: 6:36].

Some children learned how to do something, for example Jordan, who “learned how to count by nines.” Another substantial proportion of the children (21%) reflected on their learning at a conceptual level. For example, Tahani reflected on a lesson where the teacher intended to introduce multiplicative thinking, saying she learned “about groups, to make groups and to count them altogether and I learned to count by 6s.”

Category of response	Frequency as a percent (n = 53)
1. Unable to specify learning	15
2. Nothing “new”	9
3. Learned behaviour/ not mathematics	6
4. Remembered factual information	15
5. Learned how to do something	23
6. Specified a conceptual level of understanding	21
7. Missing	11

Table 4: Children’s learning.

DISCUSSION AND IMPLICATIONS

Children could recall at least part of their conversations with the teacher during the day’s mathematics lesson. These interactions appear to have some lasting effects. This is an important finding because I believe that interactions that challenge children to think about their mathematical understandings are a critical factor in their learning. Therefore knowing that many young children spontaneously remember these conversations and can reconstruct their thinking is an important finding.

The sophistication of their descriptions of events in the classroom were fairly evenly split between recounts of actions, descriptions of the event from the child’s perspective and a description that involved some recount of their reasoning. It was surprising and impressive that such a large proportion of five- to seven-year-old children (42%) could reconstruct their thinking and justify it. It is assumed that the experiences offered to children in mathematics classrooms contribute to their learning. These data indicate that 59% of children could talk about their learning as a result of the lesson—some at a factual level, some at a procedural level, and some at a conceptual level. Further research might investigate factors that influence different levels of understanding reported by young children.

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