DECA IS TEN

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This paper focuses on characteristics of activities that support students in relating length and area and help them to explore the metric system (especially in relating units of length and area). The developmental research presented here shows two examples that illustrate that a developed poster with prefixes (like 'milli', 'centi', etc.) can support discussion leading to constructing metric units and relations between these units. Some critical remarks are also made. For example we see that, when the prefixes call for 'fraction language' new problems are introduced and the poster presented here is not very helpful in constructing and relating metric units.

INTRODUCTION

In daily life people have to deal with measuring situations every day. They need to interpret numbers that result from measurements, carry out measurements and communicate the results of these to others. This means that competence in measuring is considered an important aim in primary education (e.g. National Council of Teachers of Mathematics, 2000). For example, US students in grade 3-5 – similar to students in many other countries, including the Netherlands – have to understand the need for measuring with standard units and become familiar with standard units in the metric system. Further, they should be able to carry out simple unit conversions, such as from centimeters to meters (National Council of Teachers of Mathematics, 2006).

Here we focus on learning and teaching measuring area. Many researchers show that understanding area measurement is difficult. There is no obvious instrument for measuring area (Battista, 1982). When students want to determine a rectangle's area they need to convert a length measure to an area measure (Zacharos, 2006; Kamii & Kysh, 2006). Students experience difficulty in perceiving area (in a rectangular structure) as a multiplicative relation of the lengths involved (Simon & Blume, 1994; Kidman & Cooper, 1997). They often have problems recognizing rectangular area as a row-by-column structure (Battista, Clements, Arnhoff, Battista, Van Auken Borrow, 1998).

RESEARCH QUESTION

Metric unit conversion is an aim in primary education. Moreover, this forms an obvious intermediate stage, when we want students to gain insight in the metric system. However, there is very little research available that focuses on how students can best be helped to construct relations between metric units of length and area. There is a small amount of research that focuses on how students can gain insight in

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the metric system as a system that is closely related to our number system. We observed that many Dutch teachers – usually after focusing on benchmarks for most common metric units – turn to rote learning when they want students to learn relations between metric units, in many cases leading to poor results. We therefore formulated the following research question:

> What are the characteristics of activities that support students in relating length and area and help them to explore the metric system (especially in relating units of length and area)?

We noticed that an omnipresent poster with the metric system encourages teachers in many classrooms to teach relations between metric units in a form of rote learning. We, therefore, decided to develop a new poster (figure 1), that on the one hand supports students to really explore the metric system, and on the other hand encourages teachers to support students' reinventing relations between metric units (cf. Freudenthal, 1983).

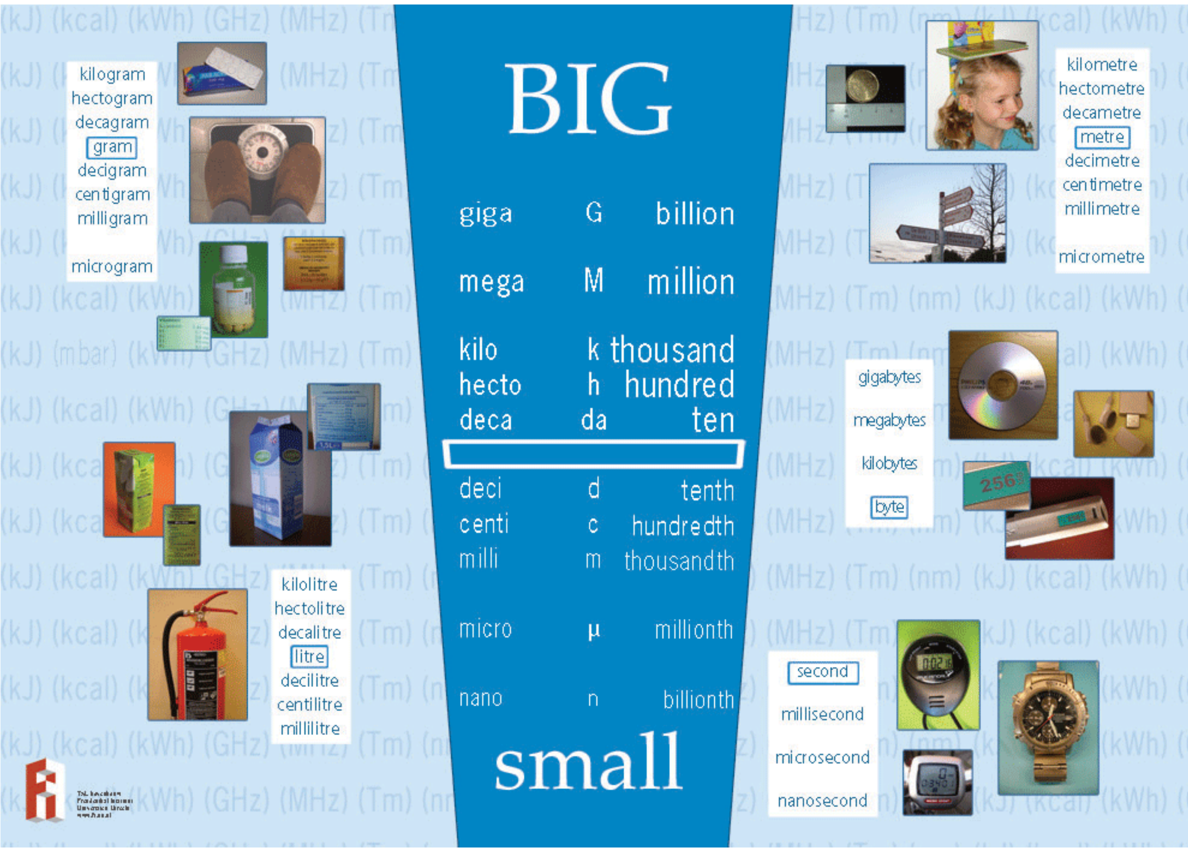

Figure 1: Prefix poster.

DEVELOPMENTAL RESEARCH

Freudenthal (1991) proposed developmental research as a means to develop conscientiously and well considered mathematics teaching:

> ‘*developmental research* means: experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience.’ (p. 161)

Here we follow Gravemeijer’s (1994) ideas on developmental research as a ‘cyclic’ process of program improvement, based on theoretical notions, student observations and teaching experiences. In this process the researcher is a developer. He or she develops an idea for a teaching approach or new teaching tool. The researcher/developer next constructs ideas on how the new approach or tool could work out in teaching. In the following stage the ideas are tested in a classroom setting. This try-out in the classroom leads to reflection on teaching outcomes and student reactions, and these are related to theoretical notions on learning and teaching mathematics. This forms the basis for adjusting and refining the initial scheme.

In the next sections I will describe key elements in such a cyclic developmental process on measuring area, where teaching is aimed at understanding relationships between metric units, especially those for length and area.

EXPLORING BENCHMARK UNITS

We experienced that most students in primary education are able to use units for length and volume meaningfully. They will know for example that 1 meter is the length of a long step and that 1 litre is the volume of most milk cartons sold in Dutch supermarkets. Dutch ten to twelve year olds generally develop several benchmarks. These include for example the student’s height (about 150 cm or 1 m and 50 cm), the volume of a tin can (about 33 cl), the weight of a pack of sugar (1 kg), the volume of a bottle of shampoo and a carton of drink (both about 200 ml). We however noticed that many students were unable to connect their knowledge of measuring length to measuring area. More generally, many students have problems in reconstructing relations between metric units (Janssen, Van der Schoot, & Hemker, 2005).

We formulated several arguments to develop a prefix poster as shown in figure 1. We would provide teachers with a means to construct metric relations with the students. The new poster should appeal to teachers. We therefore chose to make the poster ‘look like’ the metric system poster that was available in many classrooms and which was used to teach the metric system by rote learning.

Our main objective was to stimulate teachers to talk with their students about relations between metric units. We therefore suggested a familiar first activity with the poster. Exploring benchmark units is done in many grade 6 (nine year olds) classrooms. In a typical activity the teacher shows bottles and cartons and asks the students to tell about the volume or weight. Next the students may try to give their answer using a different unit. For example students say that a bottle of shampoo can be 200 ml and that the first mentioned volume can also be denoted as 0,2 litre, 2 dl or 20 cl. At this point the poster is introduced. The prefixes used are on the poster: deci, centi and milli. The meanings of these prefixes are discussed in relation to how they constructed, for example, 2 dl from 200 ml.

These considerations were followed by a first teaching experiment. As expected students were able to name the volume of well known items. When next the poster was introduced, they recognised the prefixes and the poster helped them to even more easily move from one unit to the other. However, the students were still unfamiliar with fractions and therefore expressing the meaning of prefixes in terms of fractions (e.g. 'centi' is one hundredth) did not help them.

We interpreted this observation as follows. In exploring benchmarks, the pattern in prefixes (meaning you know how they are ordered) is all that is needed, at least if you have some idea about the decimal structure of metric units. Put somewhat differently, if you know that cl is followed by dl and that the last one is the biggest, there are 10 cl in one dl. The poster was somewhat helpful here, as it showed a sequence in prefixes. However, this limited support in exploring the metric system and relations between metric units was not what we had in mind. We therefore developed a second activity where the decimal meaning of the prefix should be considered and where no fractions were involved.

DECA IS TEN

We developed the second activity 'Deca is ten' where the prefix deca is discussed with students. We chose this unfamiliar prefix so that students still needed to construct its meaning and could use the poster for doing so. Moreover, as 'deca' is ten and ten is not a fraction, students should not be hindered by a lack in fractions knowledge. Since all students are acquainted with the number '10', discussing the prefix may be used to go beyond units of length. That is, there seems to be a basis to discuss the meaning of square decametre.

It is June and the end of the school year. Grade 6 students of a school in Hoofddorp (near Amsterdam) did several activities with the prefix poster. In all these activities they used the sequence in prefixes to determine relations between metric units. That is where the present discussion starts.

I tell the students that we stop looking at the prefixes 'milli', 'centi' and 'deci', which were looked at previously. I ask them instead to look at the prefix 'deca' at the poster. I ask Mark to tell me what 'deca' is. Mark reads the answer from the poster: 'Deca is 10.' He spontaneously adds that a decametre is 10 metres. I discuss this meaning a little further with the students. I ask them to tell me what a decalitre, a decagram and a decasecond is. That isn't difficult. A decalitre is 10 litres, a decagram 10 grams and a decasecond is 10 seconds.

I choose another perspective to look at the decametre unit and ask the students to tell me where we would be if we walked a decametre from where we are now. The students immediately know how to use their benchmark for a metre here. One metre is a long step, so a decametre is 10 long steps. If you take 10 long steps, they know, you end up halfway along the school's main hall. In a similar way we look at the unit 'hectometre'. Not hindered by fractions, the poster also supports students in finding the unfamiliar unit ‘hectometre’. According to the students a hectometre is 100 metres, which is the distance you cover when you walk around the school.

Square decametre

After having explored metric units of length, we turn to units for area, where the first is derived from the poster and the second is to become a mental construction. I ask the students to show me what a square metre is. Megan takes a metre ruler to explain. She points the ruler in two directions. Using her hands she shows that the space that arises between these two directions is the square metre. As this appears to be clear for the students, I focus on the next problem. I introduce this problem as a difficult one: can we find out what a square decametre is? The students need some time to think about this problem. Finally Douwe explains that this is an area of 10 m by 10 m. Mark adds his own interpretation. He apparently walks mentally around the square decametre and answers 10 metre by 10 metre by 10 metre by 10 metre. Megan further clarifies: ‘10 metre in a square.’ Douwe interprets and concludes that we are talking about 40 metres.

To focus the students on area instead of perimeter, I ask them if the classroom is smaller or bigger than a square decametre. Megan thinks you cannot tell, since the classroom’s corners aren’t right angled. I say that we are making an estimation and that they therefore may assume the corners have right angles. She then looks in two directions and sees that you cannot take ten big steps in either direction. We therefore cannot construct a square decametre in the classroom, but we can in the schoolyard; Mark knows because he once measured the place. The square decametre fits in the yard and I use this image to present the next problem. I tell the students that I have a lot of square metre tiles, enough to tile the square decametre in the schoolyard. How many do I need to do so?

We now arrive at the most difficult question. I am aiming for a mental representation of the situation and therefore remain in the classroom. Moreover, talking about the problem, I get the impression that the students have indeed imagined the square decametre in the schoolyard. They think about the problem for some time and then make initial guesses about the number of tiles: 10, 20, 40. Douwe is not sure. He asks if the square decametre should be fully filled with tiles. I tell him that that is what I mean, and realise that the students are not filling the square decametre, but are mentally tiling the edges.

As I want to check these ideas, I ask the students how they imagined 10, 20 or 40 tiles. Unfortunately they can’t explain and I decide to help a little. I ask them if we can draw the situation. I make a start and draw a line divided in ten. I tell the students that this is a decametre in the schoolyard. Mark knows how to finish the square and does so. He also divides the new sides in ten parts. I ask Megan to put a square metre in the drawing, which she does correctly.

I take the drawing to reflect on answers the students formulated earlier. I ask them if there are really 10, 20 or 40 square metre in a square decametre. The students observe the drawing to adjust their answer. Mark and Megan think it should be 100 square metres. Megan recognises the situation and knows one has to do 10 times 10 here. However, this is not clear for all students. Moreover, Megan is unable to explain her answer, probably because she just mentioned a meaningless strategy she once learned to determine a rectangular area.

Mark starts to count the tiles. He follows the squares edge from the outside to the inside. He counts 10 tiles, 9 tiles, one more time 9 tiles and next 8 tiles (figure 2). 'But in this way can you come to 100 tiles?' I ask surprised. Mark admits he guessed his answer. He only counted up to 72.

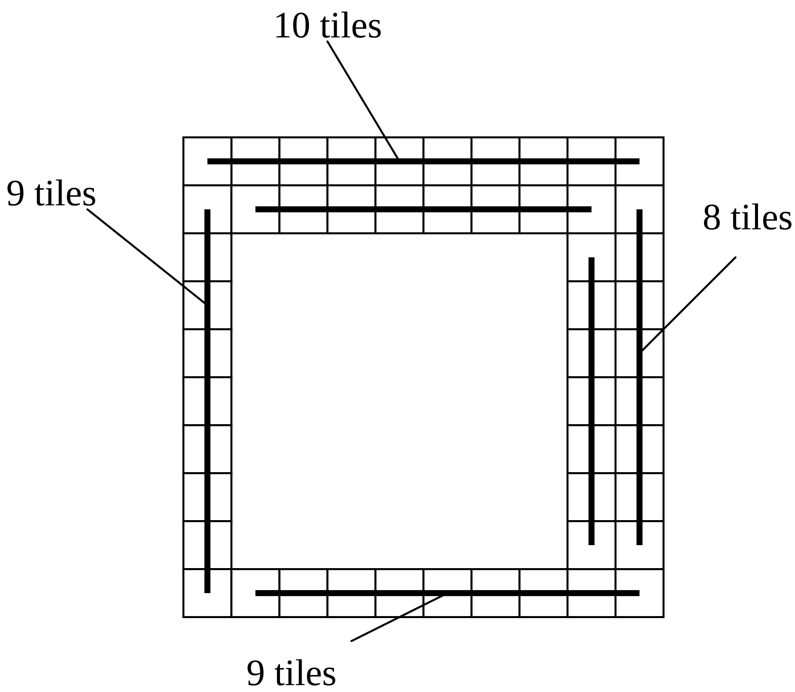

Figure 2: Mark's counting of tiles.

Mark's approach is different from Douwe's, who also suggested '10 by 10'. I realise that Mark choose his strategy to prevent double counting the vertexes, as is suggested by multiplying length and width. Not all students developed the idea of ten rows of ten. I therefore suggest drawing some of the tiles. The students start drawing the first row. Then a few students see how this pattern continues, but are unable to explain to the others. So we carry on drawing row after row. And when nearly all rows of tiles are put on paper everybody sees the 10 rows of 10; sees the 100 tiles as a result of a multiplication (Figure 3).

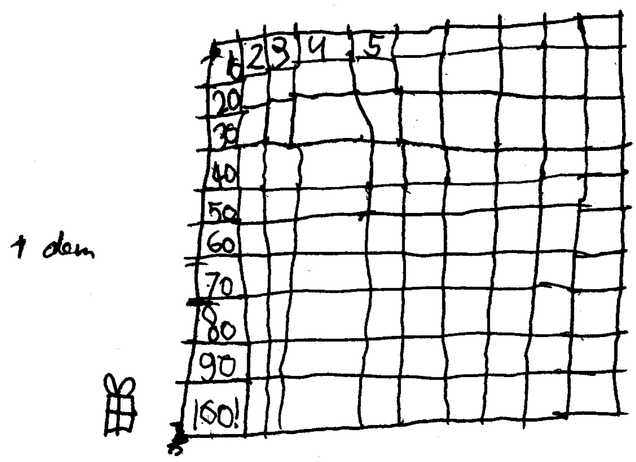

Figure 3: Ten rows of ten.

DISCUSSION AND CONCLUSION

In this paper we described a poster containing prefixes and two activities with this poster. We developed both to support students to explore relations between metric units, and thus get to know the metric system. We did so as in the Netherlands, like many other countries, the ability to relate metric units is an important aim in primary school. Moreover, there are many indications that students do not easily gain insight in the metric system. Especially relating length and area is difficult. This issue is expressed in the research question formulated in this paper. In this developmental research we sought for characteristics of activities that support students in relating length and area. We – in a sense – hypothesized that using the aforementioned poster could be helpful here, as the poster provides a language for metric units, which supports unit construction.

We saw that the poster can lead to discussing relations between metric units and that the 'metric language' provided on the poster indeed helped in constructing and relating metric units, but there are at least two important remarks to make here:

- If the 'metric language' is also 'fraction language' the 'metric language' forms an (additional) obstacle for (young) students.
- The poster gives a reason to discuss metric unit construction, and carefully aimed activities can help students to extend their notions on units of length to units of area.

There is only a limited amount of research done in the field of metric unit construction. The remarks made here therefore add to other issues that are still open for future (developmental) research. This also includes developing effective and efficient ways to help students relate exploring benchmarks and the mathematical structure underlying the metric system.

Over the course of history mankind developed a charming and simple mathematical system for measurement, the metric system. The system shows many analogies with the number system. The example presented here shows that this can support students. However, there is more work to be done. This charming and simple system now provides important challenges for developers and researchers of mathematics education.

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