DESCRIBING EFFECTIVE TEACHING OF NUMERACY: LINKS BETWEEN PRINCIPLES OF PRACTICE AND TEACHER ACTIONS

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Studies such as the Effective Teachers of Numeracy Study (Askew, Brown, Rhodes, Johnson & Wiliam (1997) have contributed much to our understanding as to what constitutes effective teaching of numeracy. This paper aims to build on these findings and to contribute a model devised by the author to interpret teachers' numeracy practices. Findings from case studies conducted with three teachers indicated that the model provided a useful framework for observing and understanding classroom numeracy practices.

BACKGROUND TO THE STUDY

In Australia, the term numeracy refers to one's ability to "use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (AAMT, 1997, p. 15). Mathematical knowledge, understanding and skills must be translatable and applied in a variety of everyday contexts and situations for a person to be considered numerate. According to Askew (2005), effective teaching of numeracy involves helping students acquire knowledge of and facility with numbers, number relations and number operations, and assisting them with building an integrated network of understandings, techniques, strategies and application skills.

There has been considerable research into what constitutes effective teaching of numeracy (Groves, Mousley & Forgasz, 2006), including Askew, Brown, Rhodes, Johnson, & William's (1997) seminal study, which identified effective teachers of numeracy based on rigorous evidence of increases in pupil attainment, not on presumptions of 'good practice' (p. 8). Although Askew et al.'s (1997) definition of numeracy differed slightly from that used in Australia, in that it included number understanding and skills out of context as well as applications, they were able to identify a number of characteristics which distinguished highly effective teachers of numeracy from other teachers. Among other characteristics, they found that highly effective numeracy teachers held a particular set of coherent beliefs and understandings which underpinned their teaching of numeracy, used a variety of teaching approaches which fostered connections between different areas of mathematics, encouraged purposeful discussion and challenged pupils' thinking.

The findings from these studies are valuable in that they provide insight into the mathematical and pedagogical practices that teachers use to achieve effective teaching of numeracy, and a review of the literature has revealed that these two

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studies have been responsible for contributing in a major way to what is perceived to be effective teaching of numeracy. Askew et al's (1997) study in particular is widely cited in the literature (e.g., Anthony & Walshaw, 2007) and their findings have been endorsed by similar research (e.g., Smith & Geller, 2004).

PRINCIPLES OF PRACTICE

The term 'principles of practice' was adopted to identify the effective characteristics common to both studies. These were: make connections, challenge all pupils, teach for conceptual understanding, purposeful discussion, focus on mathematics and positive attitudes. As an illustrative example of how the principles were derived from the two studies, 'make connections' was derived from "used teaching approaches which connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols and diagrams" (Askew et al., 1997, p. 1) and "used teachable moments as they occurred and made connections to previous mathematical experiences" (Clarke, Cheeseman, Gervasoni, Gronn, Horne & McDonough, 2002, p. 13). The other principles were derived in a similar way. Although assessment was mentioned in both studies, it was not incorporated as a principle of practice, partly because at the time of the study, assessment was a contentious issue in the state in which the study was conducted, and partly because the author believed that if the other principles were incorporated into teachers' practices, then assessment techniques could be developed to reflect this (rather than being a separate consideration). The principles are important because, according to the literature, they encapsulate sound pedagogical principles that are required if effective teaching of numeracy is to occur. It is the researcher's contention that the principles interact with each other to contribute overall to teaching effectiveness and that all principles need to be present to achieve maximum effectiveness.

Influencing factors on the principles

According to Askew et al. (1997), understanding why some teachers were more effective than others required an examination of the relationship between teachers' knowledge, beliefs and classroom practice. Shulman (1987) proposed that a teacher's knowledge base was comprised of seven types of knowledge. Although acknowledging that all knowledge types are important and interact with each other to impact on effective teaching of numeracy, teachers' content knowledge and pedagogical content knowledge (PCK) are particularly applicable to the study reported on in this paper. According to Shulman (1987) the teacher has a special responsibility in relation to content knowledge and should possess depth of understanding in order to communicate what is essential about a subject and be able to provide alternative explanations of the same concepts or principles. Having a well-developed content knowledge, however, does not necessarily result in effective teaching. Teachers also require well developed pedagogical content knowledge (PCK) which entails "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8). The study of teachers' PCK has been the focus of recent research in mathematics education (e.g., Baker & Chick, 2006) and the highly successful Cognitively Guided Instruction (CGI) program (Carpenter, Fennema & Franke, 1996) focused on developing teachers' PCK through the provision of a framework that teachers could use to represent and explain a subject to make it comprehensible.

Together with teacher knowledge, teacher beliefs can also significantly influence classroom practice. While a concise definition of what constitutes beliefs cannot be agreed upon, Thompson (1992) distinguishes beliefs from knowledge in that they can be held with varying degrees of conviction and although independent of their validity, are valid for the individual who holds them. Askew et al. (1997) found that the teachers in their study differed in their beliefs about what it is to be a numerate pupil, how pupils learn to become numerate, and how best to teach pupils to become numerate. The more highly effective teachers held beliefs which were consistent with the 'connectionist' orientation, in that their beliefs were based around both valuing pupils' methods and using teaching strategies which emphasised establishing connections within mathematics. According to Askew et al. (1997), the implicit beliefs or theories that teachers have, combined with their knowledge, influenced the way that teachers interpreted classroom events. For example, if a teacher believed that being numerate involves "the ability to perform standard procedures or routines" (p. 31), then pupil errors were more likely to be interpreted as the result of pupil carelessness or lack of attention (a transmission belief). If however, a teacher believed that pupils were trying to make sense of information, then errors may be interpreted as arising from misunderstanding, rather than carelessness (a connectionist belief).

TEACHER ACTIONS

The literature consulted to this point was useful in establishing general principles of practice that indicated effective teaching of numeracy, together with the importance of considering the influence of teachers' knowledge and beliefs on classroom practice. Although both Askew et al.'s (1997) and Clarke et al.'s (2002) studies included illustrative examples from classroom lessons to portray particular orientations or practices, these excerpts were generally brief in nature and did not detail specific teaching behaviours that could be observed. As the study discussed in this paper involved documentation of teaching practices through observation of numeracy lessons, it was necessary to identify specific teaching actions which could be observed, and to some extent, measured. While some of these teaching actions were identified following initial data collection, further review of the literature revealed that these teaching actions had been considered by other researchers and impacted upon teachers' effective teaching of numeracy. The teacher actions identified by the author from directly observing case study teachers' lessons and from the literature were: choice of examples, choice of task, questioning, use of representations, modelling and teachable moments.

METHODOLOGY

A case study approach (Stake, 1995) was used to document the numeracy practices of three experienced upper primary teachers. The teachers were selected using purposive and opportune sampling (Burns, 2000); their selection was not based upon assumption of effective numeracy practice, but they were considered by their Principals to be 'good practitioners'. The author observed and videotaped a total of 17 numeracy lessons. Parts of the lesson involving teacher led class discussions were transcribed within hours of observation and field notes were also used to document aspects of the lessons which were not captured on videotape. Work samples were also collected and following each lesson, the video footage was viewed together by the author and the teacher, with discussions audio taped and transcribed. The transcripts of the lessons were manually analysed to identify the principles of practice in action, through the teachers' use of choice of examples, choice of task, teachable moments, modelling, questioning, and use of representations. Each of these actions were then further analysed and their effectiveness evaluated, sometimes through the use of specific criteria. A model devised by the author was integral to interpreting the data collected (see Figure 1).

THE PROPOSED MODEL

Figure 1 shows a model devised by the author which was used to interpret and understand the numeracy practices enacted by the teachers in the study. It shows the relationship between teacher actions, teacher knowledge and beliefs, and the principles of practice. The principles of practice have been placed at the top of the model to indicate that the achievement of these principles is the goal, and that this goal is influenced by the circled factors below. Teacher knowledge and beliefs have been separated, but the two-way arrow indicates that, like the other factors, these two factors are also inextricably linked. Knowledge and beliefs influence teachers' actions; for example, a teachers' choice of examples is influenced in particular by their content knowledge and PCK and by their belief in how best to teach students to become numerate. In order to understand a teacher's classroom practice, one could select any of the circles as a starting point. For example, if a particular teacher's practice was characterised by *making connections*, then the teachers' actions could be examined to determine which ones were particularly helpful in establishing these connections. Similarly, teacher knowledge and beliefs could be examined to determine what types of knowledge and beliefs led to the teacher's ability to *make connections*. In order to demonstrate how the selection of a particular action (in this case 'choice of task') could be used as a starting point to determine the presence of one or more of the principles of practice, this action will now be discussed.

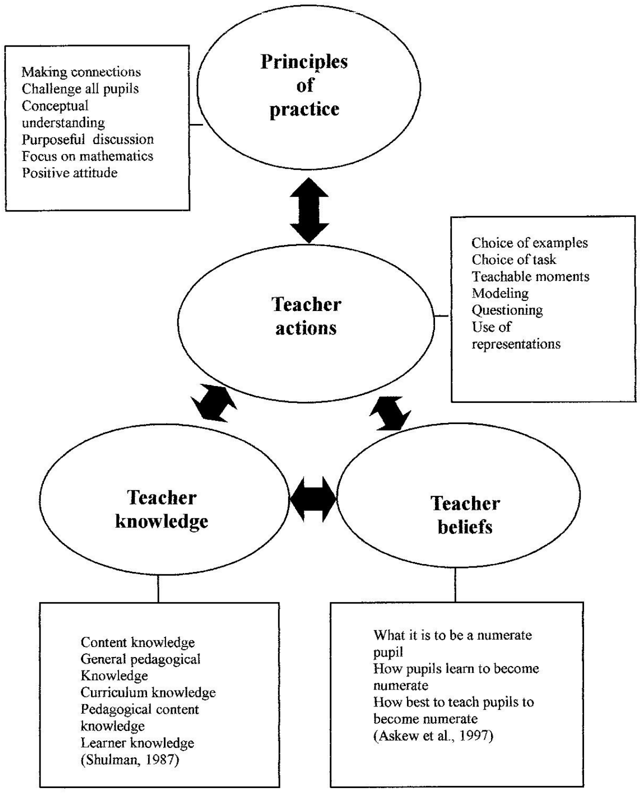

Figure 1: Teacher actions and their relationship to teacher knowledge, teacher beliefs and principles of practice.

Choice of task

Choice of task was found to be an indicator of effective teaching of numeracy. Clarke et al. (2002) identified that the effective teachers in their study structured purposeful tasks that enabled different possibilities, strategies and products to emerge, and that engaged children and maintained involvement. When selecting tasks, a balance must be maintained between supporting students' reasoning and thinking without reducing the complexity and cognitive demands of a task (Henningsen & Stein, 1997), making it particularly applicable to the principle of *challenging all pupils*. The criteria used to evaluate the case study teachers' choice of task involved rating the tasks according to the level of cognitive demands they required. The framework designed by Smith & Stein (1998, as cited in Arbaugh & Brown, 2005) provided a set of criteria to distinguish between tasks that involved higher-level demands (such as exploring and understanding the nature of mathematical concepts) and tasks with lower-level demands (such as tasks which involved memorization or procedures without connections to meaning).

The author observed a sequence of lessons based on problem solving conducted by Sue. Using the framework and its descriptors, the author classified all Sue's tasks as requiring higher-level demands. Example tasks involved solving a number of non-routine problems and carrying out investigations, such as identifying the total number of squares on a chessboard. The chessboard task was classified as requiring higher-level demands in that (potentially) it met the criteria of requiring students to use complex thinking, required considerable cognitive effort and demanded self-monitoring of one's own cognitive processes (Smith & Stein, 1998, as cited in Arbaugh & Brown, 2005). Observations and field notes taken by the author indicated that students used various strategies and their engagement with the task illustrated how tasks could be used to illustrate the principle of *challenge all pupils*. On other occasions, although the tasks met the criteria for demanding higher-level cognitive demand, the instructions that accompanied the task led to students accessing it at a lower level. For example, students were given a number of problems to solve, but were instructed to only use 'guess and check' to solve them. Furthermore, they were required to place their answers in a table, with alternative strategies not encouraged. This example highlights the limitations associated with examining tasks alone, without consideration of context or students' responses.

In the case of Ronald, his choice of tasks provided an example of how the model proposed in Figure 1 could be used to understand his teaching actions. An analysis of Ronald's tasks showed a balance between tasks that required a higher-level cognitive demand and tasks that required lower-level cognitive demands. In a sequence of lessons on percentages, Ronald provided opportunities for students to carry out investigations, conduct their own surveys and construct graphs. In one lesson, students constructed a 'placemat' to record their current understandings about percentages. The following excerpt details how the task was introduced to students:

> So on your place mat, what I want you to do is in your groups discuss what do percentages tell us? So you're thinking about in society and how they actually tell us. OK. Where are percentages used? That's where we start to think about sport, shops and things like that so I want you to find as many places as you can where they are used, and the top question is why and how do we use them? So when we're asking why and how we use them I want you to give me an explanation on why we use percentages instead of fractions sometimes. Why do we use percentages instead of decimals?

The task provided an avenue for students to *make connections* with real life and between areas of mathematics, such as the link between percentages, decimals and fractions. The task also allowed for various possibilities to emerge and provided Ronald with an insight into students' thinking that would shape future teaching directions. In this instance, the task required higher-level demands and also produced higher-level responses in students. There were a number of occasions, however, when students were required to complete questions or problems involving percentages and were instructed to use the standard percentage algorithm to solve them.

In order to understand the perceived inconsistencies in Ronald's teaching practice, the author used the model in Figure 1 to interpret Ronald's actions. Ronald consistently demonstrated a sound content knowledge of the mathematical topics he was observed teaching. The author was also confident that Ronald possessed sound PCK in relation to teaching the topic of percentages. This was evidenced for example, in his provision of examples that involved numbers greater than 100 (anticipating the tendency for students to think of percentages only in terms of up to 100%) and through his consistent emphasis on linking percentages with decimals and fractions. Ronald's teaching actions, however, could at least be partially explained through considering the influence of his beliefs. On the one hand Ronald could be said to have shared beliefs with the connectionist teachers (Askew et al. 1997) in that he was aware of the links between different aspects of the mathematics curriculum and often encouraged students to reason and justify their results. However, his predominant use of the percentage algorithm indicated an orientation towards the belief that being numerate involves primarily the ability to perform standard procedures or routines (Askew et al., 1997). Subsequent conversations with Ronald revealed that this latter belief was largely influenced by his perception that as a Grade 5/6 teacher, he had the responsibility of ensuring his students were "ready for high school", which he interpreted to mean that they needed to learn standard procedures.

CONCLUSION

The results from the study indicated that there are a number of teaching actions that are observable in the classroom. Within each of these actions, there are qualitative differences which can affect effective teaching of numeracy and impact on students' understanding. The model presented in Figure 1 was useful in examining what types of actions teachers used, the influences which impacted upon the relative effectiveness of these actions and how the actions themselves could be used to demonstrate the principles of practice associated with effective teaching. Teaching is a complex act and studying the interactions that occur in classrooms can be made more manageable through the use of such a model. "The key seems to be in paring the rich, interwoven, practical teaching experience into pieces that are small enough to be readily investigated, yet large enough so that their distinctively practical character is preserved" (Leinhardt, 1990, p. 20); linking the teaching actions with the practice enabled this to occur. It is hoped that the model, principles of practice and teaching actions described in this paper will be used by other researchers (and teachers) to interpret and understand effective teaching of numeracy.

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