Visualisation: From sports training to school Teachings practices

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This paper puts forward the theoretical results of the visualization-related spatial sense teaching and learning process in elementary and secondary school. To better counterbalance the shortcomings observed in math classrooms, this field of study takes advantage of sports research results and experiences to map out the genesis of our activity-generating architect (AGA). The main goals here are to operate the architect as a lesson planning analysis tool and, most importantly, as an activity-generating tool to create specific activities as part of a progressive teaching sequence based on didactics, mathematics, psychology and sports.

CONTEXT

This study originated from the interaction of two specific activities: mathematics teaching and figure skating coaching. The antithetical development of spatial sense in these fields fathered my inquiries. From a mathematics teaching standpoint, pupils face several difficulties (Bessot, 1994; CREM, 2001) and teachers appear resourceless. During our study, we discovered major shortcomings in the development of spatial sense, specifically of visualization-related knowledge, throughout the mathematics curriculum (both before and after Quebec's educational reform); we can observe both a lack of continuity from one grade to another as well as strong similarities in spatial knowledge taught in elementary and secondary school (Marchand, 2006a).

Spatial sense, and the underlying use of both concrete and mental visualization, is an important part of the geometry chapter of our schools' mathematics curriculum. From first grade, this curriculum emphasizes various concepts closely related to spatial sense; starting with the position of objects and the pupil's body in space (up / down and front / back), it carries on throughout elementary school to the end of high school. Recent studies link these concepts and visualization to the development of geometry (NCTM, 2005) and more concretely to the development of mathematical reasoning (Whiteley, 2002; 2004). But, even with the curriculum's clear intention towards the growth of spatial sense, it conveys little information on a concrete approach to adopt and tools to use in the classroom. Furthermore, school mathematics textbooks do not provide adequate tools for this specific subject; hence, teachers are without any concrete resources to deal effectively with this teaching and learning process.

However, our school system does not hold a monopoly on spatial sense development; other human activities can contribute to its improvement. In sports, several tools have been created to better an athlete's performance. In the last 20 years, sports studies have concentrated on improving the psychological aspects of an athlete's training to reduce the occurrence of poor performances (Duda, 1995; Nideffer, 1985; Pellissier & Billouin, 1989). And out of all the training tools developed, visualization, which is a process that requires the athlete to mentally "see" himself successfully producing various technical components, is the most effective. For example, a figure skater will "see" himself doing a double axel -a jump of two and a half turn- which requires him to mentally achieve two geometrical transformations, a rotation and a translation. Sports coaches also use visualization during the learning process as a correction tool.

On onehand, school teachers lack proper tools to handle the development of spatial sense and its closely linked tool, visualization, thus generating learning difficulties for pupils, and, on the other hand, sports studies and experiences have created several effective tools to teach and learn visualization. Would it be possible to use the various tools created in sports to improve the teaching and learning of spatial sense in a school setting? Our results indicate that it is possible and we now have all the theoretical components to design several activities for the elementary and secondary school levels.

The present paper presents the genesis of our activity generating architect (AGA), based on sports experiences and designed to create a series of mathematical activities. From a basic activity, our AGA will allow us to gradually add difficulty to create a sequence of activities, effectively covering all school levels between first grade (5-6 years old) and secondary grade 5 (16-17 years old), which promote the development of visualization through spatial sense-related activities.

THEORETICAL FRAMEWORK

Spatial sense-related activities proposed by school textbooks usually require pupils to perform very simple tasks. Piaget (1970) identified several tasks that pupils have to perform to achieve geometrical reasoning. These tasks are: observing, identifying, describing, categorizing, building, representing, searching, transforming and finally, proving and arguing. Our previous studies established that, for all activities pertaining to spatial sense in school textbooks, 37% of all tasks performed in secondary school and 42% in elementary school were of the simplest form, observing and identifying. Furthermore, the only significant change between elementary and secondary school textbooks was the importance of the second most performed tasks: building (22%) for elementary school and representing (27%) for secondary school. Textbooks barely touch upon other tasks (Marchand 2006a). This reality has direct repercussions on the teaching and learning of spatial sense in the school setting:

Hindrances and/or divergences in interpreting the curriculum;

A lack of tools for teachers; A shift from spatial sense development towards general geometrical knowledge;

Basic spatial knowledge and curriculum objectives very similar from one grade to the next;

Several learning difficulties for pupils in all grades; and,

Tasks too easy or too hard for students.

In figure skating, coaches have access to various teaching methods that allow athletes to develop a strong knowledge of spatial sense and to learn how to use visualization, which uses geometrical transformations. This situation is inconsistent with the lack of tools available to math teachers, which results in important shortcomings in the pupils' learning process of spatial sense. The idea behind our AGA is substantiated by a multidisciplinary approach enabling connection to be made between mathematics teaching and sports. Four research fields were studied to put together the AGA:

From a didactic point of view, spatial and geometrical knowledge are both distinct and intertwined (Berthelot & Salin, 1993-1994). Spatial knowledge represents the procees of interiorizing the physical attributes of real objects so the brain can create and manipulate mental images; for example, anticipating the shape and appearance of a 3D solid based upon its spatial development. Geometric knowledge is the process used to generalize an object's real characteristics to transform it into an ideal mathematical object; for example, naming a solid refers to its established attributes and so, to geometric knowledge. Presently, geometry teaching, long reduced to logic and deductive reasoning, is recapturing its visual appeal and the underlying order of spatial structure (CREM, 2004). This change is mirrored in Quebec (and Europe as well) by the explicit intention of the education ministry to bring spatial knowledge related to visualization to the forefront.

On the neurological front, math learning has long been considered as the learning of a language structure. But recent studies, by observing the brain's actions during the resolution of mathematics activities (Whiteley, 2002), show that math learning is more a part of a visual and spatial learning process. The human brain is divided into two hemispheres: the right comprises the visual and spatial intelligence, which coordinate global vision, imagination and the ability to make analogies and associations; the left hemisphere, for its part, involves the linguistic intelligence, which relates to partial or local vision, logics, counting, linear thinking and analysis abilities (Houdé, 2004; Whiteley, 2002). During a mathematical activity, both hemispheres, and not only the left one as was originally thought, are stimulated.

Hence, the visual and spatial intelligence play an important part in math learning, and are especially helpful in developing spatial knowledge related to visualization. Therefore, it is very important to take these conclusions into consideration in future studies in order to improve the mathematics teaching and learning process.

For the psychological aspect, Piaget was the first to characterize the development of spatial knowledge, followed by both other psychologists and math professors alike. Four different models describing spatial knowledge development are the basis of all recent studies: Van Hiele (1959) explores the axiomatic and linguistic development of 2D geometry; Dion, Pallascio & Papillon (1985) target the geometrical tasks involved in the teaching and learning process for high school students only; Hoffer (1977) outlines the perceptive aspects of the teaching and learning process for elementary school pupils; and Piaget & Inhelder (1948). After careful analysis of these four models, the latter model was deemed most appropriate for our framework because it explicitly deals with the shift from physical space (concrete actions – real visual sight) to abstract space (internalized actions – mental visualization). This model deals specifically with two main ideas from our project: the development of spatial knowledge for all school grades and the important part played by actions in the shift from concrete space to abstract space.

Finally, from a sports standpoint, studies have established several mental training programs, using tools such as relaxation, positive thinking, visualization and keywords (Porter & Foster, 1990; Orlick, 1990; Partington, 1990). Of all these tools, studies have shown that visualization is the most widespread and effective (Orlick, 1992). While carrying out a pre-experiment with athletes, we observed first hand the benefits of sports programs developing visualization: figure skaters who always had difficulties in math did not experience any problems with geometric activities involving spatial knowledge related to visualization.

This framework shows the relevance of this study for the advancement of this field and lays down the foundations for the genesis of our AGA on spatial sense by integrating the various tools provided by sports studies. Moreover, this AGA combines all four fields of reference by extricating the main " variables didactiques " to consider in the teaching and learning process of spatial knowledge as well as combining elements related to the development of spatial knowledge, geometric tasks available to teachers, mental images that can be manipulated through visualization, the nature of the manipulated mathematical objects and what we call the "magic key", which represents the link with sports.

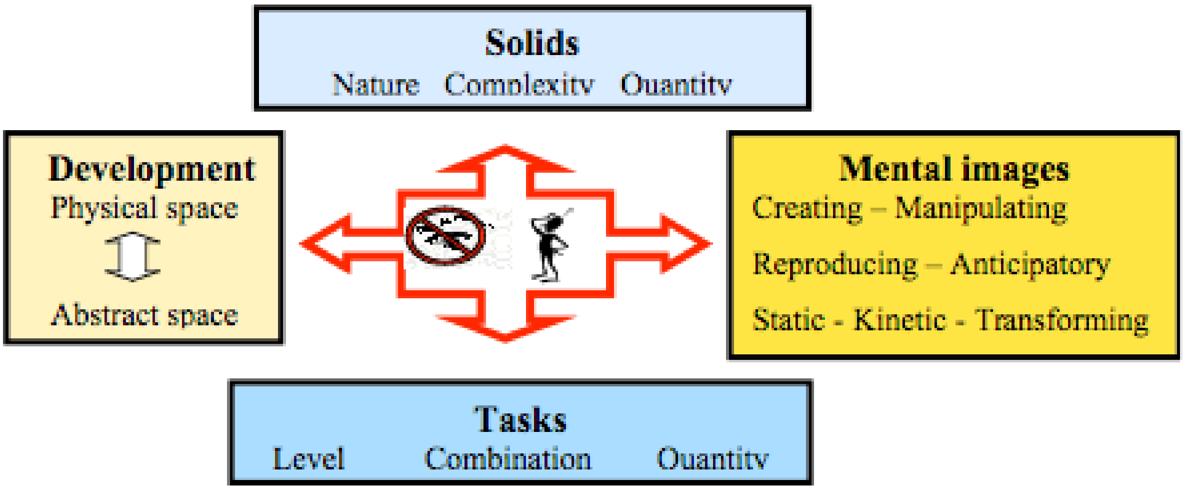

Figure 1: The AGA

The "magic key" represents the input of sports in the creation of future activities and implies an essential step that must be performed in all activities (and sometimes more than once within a single activity), during which concrete objects are hidden from sight, forcing pupils to internalize the various characteristics of the objects, thus instigating the explicit use of visualization. This stage alone is, however, not sufficient: it is imperative to question students on this phase to understand their learning curve and facilitate their progress. These two components have to be closely connected to the four " variables didactiques " surrounding the AGA to adequately construct a progressive sequence of activities (see Marchand, 2006b, for a complete description of this model). The goal of this study is to give concrete shape to the theoretically imagined AGA by acquiring an activity that is currently being used in our classrooms and using the AGA to generate several versions, enabling an increase in difficulty from one grade to the next while taking into account the multidisciplinary theoretical viewpoint and promoting the development of visualization.

GENESIS OF THE AGA

The AGA meets two precise objectives: it can be a reference tool to analyze lesson plans, teaching practices and learning processes in the classroom, or it can be used to generate occasional activities or a sequence of activities of increasing difficulty targeting the development of visualization. Both goals will be presented but the emphasis will be put on the latter.

Analysis tool

To date, we have analyzed several characteristics of the teaching and learning process of visualization: lesson planning (from teaching standpoint only) and selected activities in the classroom setting.

A priori analysis of a lesson plan

An experienced teacher had to build a spatial sense and visualization activity without any prior knowledge of our AGA for students in secondary grade 3 (13 & 14 years old). Here is the main objective and the description of the activity:

From a written description, using only straws and pipe cleaners, build a house composed of a rectangular prism and a rectangular pyramid.

The directions were to build a house with the available materials following the description: "Build, with straws and pipe cleaners, a house made out of a rectangular prism and a rectangular pyramid." The materials, available at all times, were straws, pipe cleaners, scissors and rulers. The questions were directed towards comparing the resulting houses: what are the differences between your house and your friend's house? What are the similarities? This was not a usual lesson since students rarely handle materials in secondary school. This activity allowed the teacher to observe students in action, cutting edges and comparing their mental images of both solids with the results. The teacher's intention was to find out if students would use the prism's and pyramid's attributes or trial and error to build the house.

To carry out the analysis, we will cover the AGA's various components: knowledge development, chosen solids, mental images involved, tasks performed and the "magic key". The emphasis is put on physical space. Apart from reading the description, when students mentally portray the shape of the house, they never directly use mental images. The description is very simple for this grade (the prism and pyramid concepts are first tackled in 3rd grade- 8 and 9 years old). The building of the house calls for two solids: a rectangular prism and a rectangular pyramid. Without any particular features, these regularly shaped solids are very easy for these students. From the description, students will create reproductive images, because they know these solids. Also, these images will be static. Students have to read the description, anticipate (look for) the house's shape, build the house and describe their results. Despite the lack of visual depiction of the house, the required visualization is very simple. Furthermore, no feedback questions are asked as the teacher's objective is to question students on the results.

The a priori choices the teacher made to attain the main objective may be insufficient to generate visualization for secondary grade 3 students. Several important components of the AGA are missing but to fully comprehend this lesson plan, we have to analyze the activity in a classroom setting (a posteriori analysis).

A posteriori lesson analysis

To analyze the activity in a classroom setting, we filmed and transcribed the lesson.

We used two standardized analysis grids, one for the teacher's actions and the other for the students' actions, to carry out a systematic analysis. We limited our observations to explicit actions (Marchand, 2006a). As an example, regarding the proposed tasks, we identified and characterized each occasion when the teacher asked students to perform a task. We followed the same pattern for the AGA's other components. What follows are the main observations from both the teacher's and the students' standpoints: From the teacher's standpoint, as foreseen, 90% of all actions he asked the students to perform redfeered to physical, as opposed to abstract, space. The displayed solids were those expected. Only 10% of the actions mentioned by the teacher referred to visualization. These actions involve reproductive and static images. The tasks demanded of the students were: describing (39%), arguing (24%), observing (16%), building (15%). A shift towards geometrical knowledge was observed. Finally, as expected, the house was only hidden while its description was being read. No questions dealt explicitly with visualization; 53% of all questions valued geometric knowledge and the final result. Students, for their part, stayed within physical space, since no visualization was required as the required tasks were easy and concrete. They all successfully completed the activity using regular-shaped solids. Only one student, a highly trained athlete, clearly expressed using visualization while comparing his mental image of the house with the completed house. The tools used to solve the activity were: building (32%), comparing houses (24%) and geometric identifying (14%). No specific actions or information inferred any transitions from physical space to abstract space and vice versa, apart from the one student. The results observed here confirm the conclusions of the a priori analysis: the proposed activity is too simple for secondary grade 3 students. This activity, based on geometric knowledge, does not value the development of spatial knowledge related to visualization because it puts the emphasis on physical space and uses solids too simple for secondary grade 3 students thus failing to stimulate visualization (Marchand, 2006a). The "magic key" stage, requiring both the physical absence of an object and the ensuing questions on the visualization process, is essential in the development of visualization, as was clearly shown in various sports training programs (Skate Canada, 2003; Orlick, 1992).

The proposed activity, achieved in a traditional setting, shows a possible shift from spatial knowledge related to visualization towards general geometric knowledge and the important lack of consideration, in the teaching process, of the findings of sports studies on visualization development. Our AGA takes into account the results of sports studies results in the a priori and a posteriori analysis of activities, but makes the most effective use of these results in the actual creation of activities. We will now consider the tool enabling the creation of activities promoting the development of visualization in a math classroom setting.

Activity creation tool

The AGA is here considered as a skeleton structure used to generate specific activities or achieve increasing levels of complexity starting from a basic activity. The first run of this AGA allowed us to create a specific activity covering the same objectives as the above-mentioned activity but emphasizing visualization development by taking into account all components of the AGA. Here is a description of the activity.

The directions were to read the description of a house to build, to get the materials (exact quantity and size of straws needed) from the teacher before starting construction, to build the house, and to answer questions on the process. The description was: “build a house with a body composed of a rectangular prism with larger side rectangles than base rectangles and a roof made of a pyramid with an apex that is an extension of one of the prism’s edges.” Materials were available to students only once, before the start of construction. Short, medium and long straws were available. The questions were aimed at visualization: What did you see in your head? Was the house in 3D in your head? Can you make the house turn (rotate) in your head? The teacher’s intentions were to bring to light what students were visualizing in order to interest them in the visualization process, to identify their operating methods and to help the students develop mental images.

Starting with this house-building activity, we activate the AGA to create a progressive sequence of activities spanning from 1st grade (5-6 years old) to the 5th grade of secondary school (16-17 years old). We put forward one activity per grade to display a possible evolution, 11 in all (6 for elementary school, 5 for secondary school), to display a possible sequence of activities. The following chart shows description of all activities and the increases in difficulty from one grade to the next, bringing to the forefront the components of the AGA that were taken into account to create the progression from one activity to the next:

Grade	Elementary school activities
1	Here is a house from a far away land (square pyramid). Can you describe it to me? Remember its appearance because I will now hide it and you will have to find it among other houses. Can you find it? How did you remember it? How did you recognize it? (Wooden solids: prism, cylinder, half-sphere, pyramid and cone) **AGA elements:** 1 simple **solid**; 3 easy **tasks** (observing, describing, identifying); static reproductive mental **images**; transitions between physical and abstract spaces; hiding the solid and questioning the students on this stage.
2	Here is a house from a far away land (a **house composed of a blue rectangular prism and a red rectangular pyramid**). Can you describe it to me? Remember its appearance because I will now hide it and you will have to find it among other houses. Can you find it? How did you remember it? How did you recognize it? (Wooden solids formed out of 2 solids of different colors: one house composed of a blue prism and a red pyramid, another with a yellow prism and a red pyramid, another with a blue prism and a red cone, etc…) **AGA modifications:** adding one solid.
3	Here is a house from a far away land (composed of a yellow cylinder and a green triangular pyramid). Can you describe it? Remember its appearance because I will now hide it and **you will have to build it with the solids I will give you**. How did you remember it? How did you rebuild it? (colored wooden solids, not assembled) **AGA modifications:** replace identifying the house with building the house
4	Here is a house from a far away land (composed of a triangular prism and a square pyramid). Can you describe it to me? Remember its appearance because I will now hide it and you will have to build it with the materials I will give you. How did you remember it? How did you rebuild it? (**individual straws and pipe cleaners allowing pupils to rebuild the house from scratch)** **AGA modifications:** replacing solids with straws and pipe cleaners.
5	Here is a house from a far away land (composed of a square prism and a square pyramid). Can you describe it to me? Remember its appearance because I will now hide it and you will have to build it with the materials I will give you. **Once the house is hidden, you will have to come get your straws and pipe cleaners from me. You may come only once**. How did you remember it? **How did you anticipate the numbers of straws you needed?** (straws and pipe cleaners in the teacher's possession) **AGA modifications:** pupils do not have access to the straws and pipe cleaners at all times.
6	**Here is a description of a house** from a far away land (composed of a cube and a hexagonal pyramid). Can you build it with the materials I will give you? You may come get your materials only once, so anticipate the numbers of straws you will need. **What did you see in your head after the description was read?** How did you anticipate the numbers of straws you needed? How did you rebuild it? **AGA modifications:** replacing observation with a written description of the house; starting in abstract space instead of physical space.

Table 1: Elementary school activities description

We can summarize the choices for elementary school as follows: we concentrated on using one or two solids at a time; pupils created reproductive and static mental images; we combined at most three different tasks of which included observing or researching, describing and identifying or rebuilding; we chose to start in physical space for the first five grades and to introduce abstract space in 6th grade; we used wooden solids in the first two years and straws and pipe cleaners in subsequent years; and, for the “magic key”, questions dealt with the development of visualization and the object was always hidden at one point and was less and less visible from grade to grade. Let us now examine the chart for the secondary grades:

Grade	Secondary school activities
1	Here is a description of a building (**composed of a rectangular prism and a rectangular pyramid that is truncated by a plane parallel to its base**). Can you build it with the materials I will give you? You have one chance to come and get your materials so anticipate the number and **size** of the straws you need. What did you see in your head while the description was being read? How did you proceed to anticipate the number of straws and to rebuild it? (**straws of 3 different sizes** and pipe cleaners in the teacher's hands.) **AGA modifications:** two solids are mentioned, but one is more complex; students now have to anticipate the size of the straws as well as the quantity; students who have not seen this complex solid may form anticipatory images.
2	Here is a description of a building (**composed of a rectangular prism with side rectangles larger than the base rectangle and a rectangular pyramid with one edge forming an extension of one of the prism's edges**). Can you build it with the materials I will give you? You have one chance to come and get your materials so anticipate the number and size of the straws you need. What did you see in your head while the description was being read? How did you proceed to anticipate and to rebuild it? **AGA modifications:** complex solids; reproductive and anticipatory images.
3	Here is a description of a building (**composed of a rectangular prism with an 8 cm by 12 cm base rectangle and a rectangular pyramid with a base of the same size and a height of 15 cm**). **Can you build it using the right sized straws?** You have one chance to come and get your materials so anticipate the number of straws you need **and give me a plan of the house with accurate measurements.** What did you see in your head while the description was being read? How did you proceed to anticipate the number and size of straws you needed? How did you proceed to rebuild it? (straws and pipe cleaners in the teacher's hands, rulers and scissors for students.) **AGA modifications:** less complex solids than before but students have to anticipate and use accurate measurements; addition of a plan with accurate measurements (representational task).
4	Here is a description of a building (**with a base derived from the rotation of a rectangle around one of its longest edges and a roof derived from the rotation of a right triangle around its longest side**). **Can you describe the building? What will it look like?** Can you draw it? What did you see in your head while the description was being read? How did you proceed to visualize it? (no material) **AGA modifications:** kinetic instead of static images (rotation of solids); emphasis on appearance instead of exact measurements.
5	Here is a description of a building (**with a base derived from the rotation of an isosceles trapezoid around the axis formed by the bisector of the two bases and a roof derived from the rotation of a right triangle around the axis formed by the bisector of its longest side**). Can you describe the building? What will it look like? Can you draw it? What did you see in your head while the description was being read? How did you proceed to visualize it? **AGA modifications:** more complex solids of revolution; anticipatory kinetic images.

Table 2: Secondary school activities description

We can summarize the choices made for the secondary school as follows: we kept the same number of solids (two) but made them more complex; we added anticipatory static images and anticipatory or reproductive kinetic images; we chose to always start in abstract space to increase the use of visualization; we asked students to create more precise models by planning ahead; we used the same materials for the first three grades but gave more specific information about sizes and measurements; and, for the "magic key", the emphasis of the questions was on visualization, the object was always partly hidden and,for the last two grades, the object was completely hidden .

This example shows a possible progressive sequence of activities to promote the development of visualization from the beginning of elementary school through the end of secondary school and provides us with a glimpse of the architect's usefulness in creating these activities. We tested and analyzed the secondary grade 3 activity generated by the architect in a classroom setting. By comparing the results of this activity to the results of the traditional activity analyzed earlier, several differences relating to the development of visualization were observed. The activity generated by the arhitect promotes the explicit use, activation and improvement of visualization (Marchand, 2006a): the teacher asked more questions and more of these questions dealt directly with the students' ability to use visualization (52% of the questions targeted the visualization process), the required tasks were appropriate for the students' level (and even a bit too complicated as regards answering questions on the visualization process), the teacher put the emphasis on abstract space (53% of all required actions were internalized actions) and the objective focused directly on the development of visualization.

This first production of an activity sequence by the AGA is only the beginning. By analyzing the various activities according to levels of complexity, we see that we did not take advantage of several of the AGA's attributes (more than 2 solids; transforming images; various tasks related to 2D representations, mathematical reasoning, the development of solids...) which means that we could use the AGA to produce complete activity sequences for each grade, which will be our next challenge.

CONCLUSION

We have presented, in a theoretical manner, the AGA's possible applications in the genesis of an activity sequence. With the house-building example, we realize that the main consideration in the development of visualization does not lie with the chosen activities but in how we ask questions about these activities in the classroom. Therefore, we can adapt activities in existing textbooks to favor the development of visualization. The same rule applies to sports, as coaches who do not use the "magic key" and explicitly question their athletes about visualization may not trigger the development of visualization.

To achieve better design and testing of these visualization-developing activities in the math classroom, it is imperative to collaborate with teachers from the start of the process. Our main goal will be to use our expertise to construct a concrete tool to aid teachers while benefiting from their experiences to investigate, experiment and validate our activity sequence. This cooperative study will lead to an exchange of ideas and experiences, the genesis of a tool to help teachers teach spatial sense and visualization that responds to the need of the educational community and the validation of our AGA after the sequence has been tested in classrooms. Various documents will then share this tool with the educational community at large. This project will have direct repercussions on the way teachers approach spatial sense in school, on the training of future teachers since university professors are currenly no better equipped to handle this subject than elementary and secondary school teachers and on research bringing new information to an already effervescent field of study.

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