FROM “REAL LIFE” TO MATHEMATICS: A WAY FOR IMPROVING MATHEMATICAL LEARNING

Annie Savard

McGill University

This paper presents a way to improve mathematics learning by using different contexts present in the classroom. We proposed to study the game of Heads and Tails in a 4th grade classroom. Students used their previous knowledge of the game in a socio-cultural context. The teacher created more complex conditions in the classroom in order to help the students further refine their conceptions about probability, and to develop a better mathematical knowledge about it.

CITIZENSHIP IN SCHOOL: STUDYING “REAL LIFE”

Contextualizing mathematical knowledge within a significant learning situation is a major issue in Quebec Education. The emphasis is put on competency development in order to develop citizenship. This contextualization is anchored in broad areas of learning set out in the Quebec Education Program (Ministère de l'Éducation du Québec, 2001). These broad areas of learning refer to social and political questions taken from “real life” situations. In fact, they are grounds for learning. Thus, studying how some gambling activities work could be as educational in mathematics as it is in citizenship learning. This topic is even more important today because we know that children gamble, and are being put at risk of developing problematic gambling behaviour (Ladouceur, Dube & Bujold, 1994). Studying some of these activities can improve their knowledge about their probabilities of winning and help them to be critical about it. Therefore, the prior knowledge children have about these games is redesigned based on the teaching they will receive.

A BRIDGE BETWEEN OUTSIDE AND INSIDE THE CLASSROOM

In order to create learning situations involving probability, and in order to enhance citizenship, we used an ethno-mathematic model developed by Mukhopadhyay and Greer (2001). The model suggests beginning a lesson by studying an object from the “real world” based on its socio-cultural context, and then moving it to its mathematical context. This model compels students to take into consideration the relationships between the quantitative variables. Mathematical procedures are used to find a result and this result will be evaluated and interpreted within the studied context. A political context must also be considered. This encourages critical thinking (Lipman, 2003) on the process itself and on the studied phenomenon. Thus, students have the opportunity to mobilize their knowledge in order to construct a mathematical model. We also added some elements to this model in order to improve it and thus the political context became a citizenship context as well; our goal was

Paper presented at the 11th International Conference on Mathematics Education (ICME-11) for *Topic Study Group 2: New developments and trends in mathematics education at primary level*, Monterrey, Mexico, July 2008.

that the students develop critical thinking towards gambling. In addition, we designed the context by adding these kinds of situations: action, formulation and validation (Brousseau, 1998) and by adding the conceptual field. In fact, the mathematical knowledge studied, probability, consisted of three distinctive approaches within this conceptual field (Vergnaud, 1996): theoretical probability, frequentist probability and subjective probability (Caron, 2004; Konold, 1991).

In order to study conceptions about probabilities, Fischbein and Schnarch (1997) researched among students between 10 and 18 years old. They used a questionnaire to collect data. One question asked what the outcome of the fourth toss in Heads or Tails would be if the first three tosses had been Heads. Almost all students tried to solve the issue by ignoring the independence of each toss. Seven students out of twenty showed a deterministic conception about it. The authors qualified this behaviour as misconception. On the other hand, Amir and Williams (1999) studied the cultural influence and the probabilistic reasoning with 236 students between 11 and 12 years old. They wanted to know which informal probabilistic knowledge could be an asset for learning in school. Their results showed that superstitions, religious beliefs and personalized interpretations about the manipulatives interfered with the probabilistic reasoning. This is why it is important to begin this type of lesson with informal probabilistic knowledge. Conceptions that try to explain a probabilistic outcome by manipulatives are called determinist conceptions (Savard, 2008) because people use a deterministic reasoning based on socio-cultural issues instead of a probabilistic reasoning to reach conclusions. When conceptions use a probabilistic reasoning, they are called probabilistic conceptions. When they are used in a mathematical context the conceptions based on beliefs are called affective conceptions and they are used in a socio-cultural context. But if these researches showed us some conceptions, nothing was done to complexify them.

Conceptual complexification is a way to learn by restructuring knowledge instead of destroying the old knowledge (Larochelle & Désautels, 1992). In fact, each conception is a personal knowledge that tries to explain a phenomenon and it has its own domain of validity. In order to study how personal knowledge could be of use in the classroom, in this study we asked this research question: How can affective, determinist and probabilistic conceptions be complexified?

A TEACHING EXPERIMENT IN AN ELEMENTARY CLASSROOM

We proposed a teaching experiment in a French grade 4 classroom in a Quebec City suburb. The researcher was the regular teacher of a 27 student classroom. Six learning situations were contextualized using these activities: drawing two pictures, answering two questionnaires, giving two formative evaluations and doing a bibliographic research. Each lesson used a probabilistic approach in order to develop citizenship and mathematical competencies. Each situation was started using a gambling activity. We recorded and videotaped each lesson and then we transcribed it in a verbatim. We analyzed data with the software Atlas/ti. We used DeBlois's

(2003) theoretical framework for interpreting data. This model focuses on the representation of the situation by the learner, the procedures employed, and by the effects of the didactical contract (Brousseau, 1998). Coordination between them can create awareness toward the concept.

CONTEXT ROLES IN THE CLASSROOM: SOME ILLUSTRATIONS

After the first drawing about gambling activities, we gave each student a questionnaire with 17 questions. One question was about student gambling habits, and the results showed us that all the students had already played Heads or Tails and that seven of them had even played the game with betting involved. Another of these questions was the same as Fischbein and Schnarch (1997): After getting three Heads where the only possibilities are Heads and Tails, what will be the next outcome? The student answers showed some representations about their game experiences. Six students answered Tails and five students answered Heads. It is supposed to be Tails because: “It is its turn” (Anne), or it is supposed to be Heads because: “It is Heads all the time; it is going to be Heads again” (Ralph). Only three students showed a deterministic conception. But fifteen students answered they could not predict the outcome of the next toss or that it could be either Heads or Tails. Their representation was focused on chance. They knew the outcome could be either Heads or Tails, but only one student could explain it by the equiprobability of the coins in this case.

We proposed some other probabilistic situations related to the other approaches before we introduced this situation. At the beginning of a lesson in the frequentist approach, we briefly discussed the expression “Heads or Tails”. The socio-cultural context was prevailing. But rapidly, some students brought their questions within a mathematical context by using their mathematical knowledge; they expressed their knowledge by acknowledging the fairness of the game. They concluded that they had a one out of two chance of obtaining Heads and the same probability of getting Tails. In fact, they talked about the theoretical probability of getting equiprobability, but some of the students were more preoccupied with trying to rig the coin in order to control the outcomes of the tosses. The next figure shows us the context role in a discussion:

	Mathematical Context	Sociocultural Context	Citizenship Context
1		Teacher:	
	But why it is fair if the coin isn’t cheated?	We use a coin because it is chance.	
2	Marco’s first representation:		
	Both participants have equal chances to win, because the coin has two		

	sides	and we cannot say which side the coin will fall because it is not you who throws it.	
3	Teacher: Can we predict with certainty the outcome of Heads or Tails?		
4	Marco’s answer: No.		Marco’s second representation: I can trick it by weighing it.
5			Patrice’s first representation: It is fair to play at Heads or Tails because the two participants agree. Me and my stepbrother have played Heads or Tails.
6	Teacher: Did you have the same chances?		
7		Patrice : No.	
8		Marco’s third representation: The outcome of a throw in Heads or Tails depends on the person who throws it and how they throw it.	
9		Patrice’s second representation: It depends on the strength [of the throw].	
10	Teacher: Does it have a lot of		

	chance?
11	Marco’s fourth representation: There are few chances. Chances to win are equal: 1out 2. It is like the half.
12	Teacher: She reformulates and she asks if it is what he said.
13	Marco’s fifth representation: It is equal. Both are equal means you don’t have more chance than the other.

Figure 1: Context influence in a classroom discussion.

It seems that in this example the teacher is trying to bring students into a mathematical context by asking questions within this context. At first the intervention of the teacher begins in the socio-cultural context and it finishes in the mathematical context. Marco is within this context when he talks about the equiprobability of the coin and in the socio-cultural context when he talks about the manipulation of the coin. It is possible that Marco shows critical thinking about cheating in this case, but when he does it is mostly about the handling of the coin. The third time, the teacher asks the questions within a mathematical context again. Marco seems to lean towards this context, but he gives critical explanations about the possibility of cheating the coin, which goes into the citizenship context. Patrice interrupts and gives an answer to the teacher’s question. The teacher tries again to bring them into the mathematical context, but the boys are still in the socio-cultural context since they continue to talk about the manipulation of the coin with a personalist interpretation. Marco only views the question in a mathematical context permanently after the tenth time.

After this discussion, students talked about the game and we proposed to study this game through experimentation. We wanted to record the frequency of each side of the coins (Heads and Tails). Each team of two students made 100 trials and recorded them on paper. They were asked to write every outcome out so that they could read out the sequence of the experimentation. Many of them tried to control the coin because they wanted to get a certain outcome instead of the other possibilities. The socio-cultural context was prevalent at this time because students focus on the manipulation instead of mathematics.

We compiled each frequency so that students could see the variability between their answers. They compared their frequencies with the theoretical probability in the mathematical context. After a discussion about the outcomes, students attempted to answer this question: After obtaining three Tails in three trials, what are you likely to get on the next trial? Students used their mathematical knowledge to answer. They referred to the ratio seen in the theoretical approach: it was supposed to be one out of two, but Melissa answered that it would be Heads because she said that the outcomes were usually not more than three in a row. She was staying in the socio-cultural context. Her answer didn't show mathematics, it showed the knowledge of the game. In response to her, Marina said the outcome would be Heads because since we had three Tails already we then had three chances out of six to get it (we have already three Tails, it is time to get three Heads!). She tried to go in the mathematical context, but her reasoning was based on a determinist conception toward the probabilities. In fact, some students remembered their experimentations and they said it is possible to get the same outcomes many times in a row. Then Sarah concluded: "We got seven Tails in a row. And we still have one chance out of two on the next trial to get Tails because there are two sides on the coin" (PF.1087). She left the socio-cultural context to get into the mathematical context with a comprehension of the independence of the events. It seems this consideration about independence between the outcomes leads to the construction of variability. Thus, she constructed new mathematical knowledge in order to develop her mathematical competencies by restructuring her knowledge in a mathematical context.

At the end of the teaching experiment, we gave the students the same blank questionnaire they had already answered in order to compare their answers. At the question: After getting three Heads at Heads or Tails, what will be the next outcome? Most of their answers stayed similar, and three students showed a deterministic conception, but their explanations were different: twenty students explained it could be both of them because they had one out two chances to get it and seven students answered Heads because they got this result in their experimentation. Nobody used their previous knowledge to explain. In fact, they used what they had learned in the lesson (the independence between the outcomes and the variability) to answer their questions.

IMPLICATION FOR TEACHING

This research tried to study how "real life" knowledge could be used in the classroom to improve student learning. According to Fischbein and Schnarch (1997) and Amir and Williams (1999), this "real life" knowledge, also called conceptions, is often wrong: students think about manipulative reasoning in order to explain the outcomes instead of using a probabilistic reasoning. Like in these studies, our research showed us these conceptions in the classroom. Comparatively, our study tried to complexify these conceptions in order to restructure student knowledge. In fact, we highlighted the role of the context in classroom. It was when students moved from a socio-cultural context to a mathematical context that conceptions complexified. It is the shift from one context to another that conducts students to complexify their conceptions. Situated knowledge in different contexts can provide the limits of the domain of validity and contribute to complexify the conceptions of learners. Thus, when the teacher provides information from the mathematical context, students can use it to confront their knowledge and then move into a different context in order to use this information. Of course, in this lesson, it was additionally possible to exploit the determinist conception about manipulation by asking students to model outcomes of a game that was cheated. Comparing the cheating model and the regular model could have had the effect of moving the students from the socio-cultural context to the mathematical context. In fact, if the teacher was aware of the shifting between contexts, s/he could have guided students towards the context that would fit better, creating learning conditions that would enable the teacher to better intervene.

References

Amir, G. S. & Williams, J. S. (1999). Cultural influences on children's probabilistic thinking. *Journal of Mathematical Behavior, 18* (1), 85-107.

Brousseau, G. (1998). *Théories des situations didactiques*. Grenoble: La pensée sauvage éditions.

Caron, F., (2004). *Splendeurs et misères de l'enseignement des probabilités au primaire*. Actes du colloque GDM 2002: Université du Québec à Trois-Rivières

Deblois, L. (2003). *Interpréter explicitement les productions des élèves: une piste...* In Éducation et francophonie, Volume XXXI (2), http://www.acelf.ca/c/revue/revuehtml/31-2/08-deblois.html

Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal of Research in Mathematics Education, 28* (1), 96-105.

Konold, C. (1991). Understanding student's beliefs about probability. Dans E. V. Glasersfeld (dir.), *Radical Constructivism in Mathematics Education* (pp. 139-156). Dordrecht: Kluwer academic publishers.

Ladouceur, R., Dube, D. & Bujold, A. (1994). Gambling among primary school students. *Journal of Gambling Studies, 10* (4), 363-370.

Larochelle, M. & Désautels, J. (1992). *Autour de l'idée de science : itinéraires cognitifs d'étudiants et d'étudiantes*. Sainte-Foy: Presses de l'Université Laval ;.

Lipman, M. (2003). *Thinking in education (Second Edition)*. New York: Cambridge University Press.

Ministère De L'éducation Du Québec. (2001). *Programme de formation de l'école québécoise au primaire*: Gouvernement du Québec.

Mukhopadhyay, S. & Greer, B. (2001). Modeling with purpose: Mathematics as a critical tool. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 295-311). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Savard, A. (2008). *Le développement d'une pensée critique envers les jeux de hasard et d'argent par l'enseignement des probabilités à l'école primaire: Vers une prise de décision.* Thèse inédite, Université Laval, Québec.

Vergnaud, G. (1996). The theory of conceptual fields. Dans L. Steffe et P. Nesher (Eds.), *Theories of mathematical learning* (pp. 219-239). Mahwah, NJ: Lawrence Erlbaum Associates.