APPRENTI GÉOMÈTRE, AN INSTRUMENT FOR LEARNING GEOMETRY AND MAGNITUDES MEASUREMENT

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The technological developments influence more and more the didactics of mathematics. Numerous researches are led to determine the gains and the optimal conditions of the use of these new tools of education. In this setting, the Centre de Recherche sur l'Enseignement des Mathématiques (CREM) (Nivelles, Belgium) produced a software of dynamic geometry, named Apprenti Géomètre, primarily devoted to the learning magnitudes, fractions, measurements and geometry. To deal with these various fields, it proposes two different "working environments", two different "geometrical spaces". It was also conceived as an original complement to paper and pencil context. Two years of experimentation with children from 10 to 12 allowed to assert some positive changes in the learning.

A DYNAMIC GEOMETRIC SOFTWARE

Until the end of the last century, in most cases, the teaching and learning of the first concepts of Geometry were based mainly on a descriptive approach. Objects were proposed to the learner, who had to extract their characteristics (Let us however mention as a notable exception the cartoons of J.-L. Nicolet). We think this is a narrow approach, not ambitious enough for beginners. Another approach, more dynamic and more effective in the formation of thought processes is, to our opinion, to handle geometrics objects, to cut and merge them, to vary them, to drive them towards their limits to discover their characteristics.

This approach seems more accessible today with the appearance of new technologies. In this context, the CREM elaborated in 2003, a software of dynamic geometry called *Apprenti Géomètre* (AG), for 8 to 12 year-old children. The second version (available on the website www.crem.be) will offer more activities intended for the beginning of secondary school, and also the possibility to review the way of resolving with a history file.

AG is not a crowderian or skynerian software. It possesses common points with *Cabri géomètre* (a French software conceived by C. and J.-M. Laborde dedicated to secondary school and beyond), and *Logo* (conceived by Papert). AG is a "microworld", however, contrary to Logo, all the manipulations are realized via the mouse, the pupils do not have to learn a symbolic language. Also contrary to *Cabri*, the forms are predefined in AG and can be thus manipulated directly.

An epistemological analysis of fractions, magnitudes and measurement preceded the elaboration of AG (CREM, 2003). AG offers some original functions like cutting and

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assembling shapes. Another particularity is that it doesn't offer measures with standard units.

A workshop for experimentations

AG leaves the entire initiative to the user. At a first level, the learner meets a rather intuitive field for the appropriation of knowledge and the acquisition of mathematical knowledge and competences. The geometric figures appear with preset orientation and dimensions, and have between each other simple ratios of lengths, areas and angles.

For example, some figures are in the "family" of the equilateral triangle (Figure 1). All can be built from this triangle with dividing, cutting or merging. The family of the square and that of the pentagon are built in the same way. This level is devoted firstly to the learning of magnitudes, fractions, and measurements. Some of these figures seem like usual pattern blocks to manipulate.

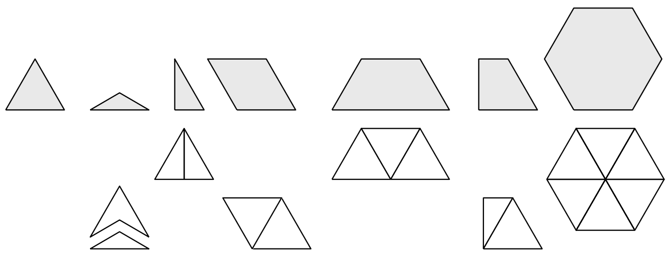


Figure 1: Figures of equilateral triangle's family.

At the second level, the figures are drawn with orientation and dimensions chosen by the user acting with the mouse. The figures are collected according to their geometrical characteristics: triangles, quadrilaterals etc. This level enables the user to modify the figures drawn and to apply to them the usual isometries.

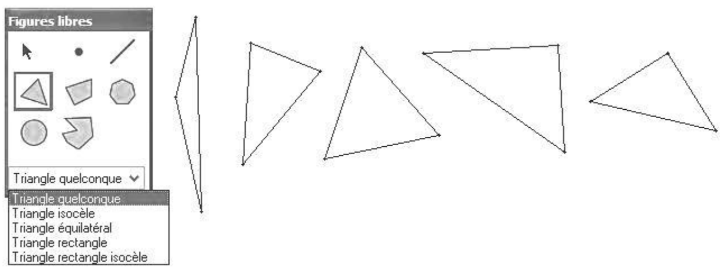


Figure 2: Figures of triangle's family.

The first version of AG offers the possibility of applying to the objects several types of simple and intuitive operations, such as *cutting*, *assembling* and *merging*, and also three types of motions, namely *sliding*, *turning* and *returning*. It offers also the three classical plane isometries: translation, rotation and reflection (Rouche & Skilbecq, 2006).

THE IMPACT OF THIS SOFTWARE ON LEARNING

During our last research period (2005-2007), our aim was to show the influences of AG on the learning of perimeter and area by 10 to 12 year-old children. 255 pupils participated in this study. Reference classes were confronted with traditional activities, whereas experimental ones used AG. The subject matter was the elaboration of formulae for polygonal perimeter and area.

First, we tried to determine some theoretical frames to build some activities. These frames are briefly explained below. At the same time, progressively, we tried to determine AG's functional area, by taking into account its potentialities (dividing, cutting, merging, moving figures) and its constraints (no measure with usual units and no tool to measure).

An epistemological frame

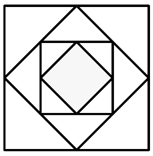
Several mathematical techniques are associated with the notions of perimeter and area. We determined three main approaches using these techniques. First, a *qualitative perception*: comparing by superposing, by division, by decomposition, by complementarity, by multiplication. Secondly, *quantification* by covering with a unit figure or a fraction of it (notion of common measure), by tiling. And third, "*numerisation"*, i.e. the replacement of magnitudes by numbers generated by usual measurement or calculation. After that, all reasonings and manipulations concerning areas for example can be replaced by reasonings and calculations on numbers. The introduction and using of formulas is included in this approach.

A didactical frame

As any technological tool, AG has its own functioning rules. Its use requires some instrumental knowledge that we wanted to be most intuitive. However, from instrumental and procedural points of view, these rules differ on the one hand from those of the user in his previous experiences of any kind and on the other from those applying, for instance, to the manipulation of cardboard forms or drawing instruments. For such reasons, AG is an "environment" (Brousseau, 1998). So, with the software, the pupils have to name every motion. They have to proceed step by step, analyzing the situation with a high degree of awareness. These new modes of operation induce new learnings.

All the tools are not equivalent in terms of potential learning (Threlfall, Pool & Homer, 2007). In connection with his instrumental theory, Rabardel (1999) insists on the system of instruments. Similarly, Assude and Gelis (2002) show the necessary complementarity "old – new" along with their presentation of *Cabri géomètre* at primary school. So, all throughout our experimentation, we tried to recognize the complementarity between "paper and pencil" or manipulations and AG.

We made the hypothesis, following the works of many researchers such as Gamlick (2002), that the transfer of tasks from one context to another would facilitate a mental reorganization of knowledge or mathematical procedures.



For example, drawing a series of squares fitted into each other using cardboard forms, AG, or drawing instruments does not confront the pupils with the same constraints nor with the same mathematical techniques and notions.

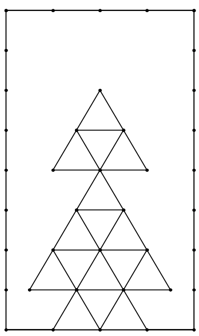
Cardboard	AG	Drawing instruments
The pupils assemble forms from a global sight of the construction. Often they measure to find the middles of the sides.	The pupils draw forms from a global and local sight of the construction. The pupils draw squares with 2 points (vertex). They need middles of two consecutive sides.	The pupils draw forms from a local sight of the construction. They have to break the figure in many lines. They need middles of sides or diagonals or medians. Often they measure, sometimes they use a pair of compasses.

Table 1: Drawing a series of squares fitted with different tools.

With these squares, some work on areas, and fraction of areas are realisable with pupils.

A cognitive frame

Duval (2005) showed that geometric activities need four ways of seeing. Often, in the classroom, only two of them are used: “the botanist one” (recognizing a figure and associating its name) and “the surveyor one” (measuring the figure). Two other views are also necessary: “the builder one” (building figures with drawing instruments) and “the inventor/handyman one” (building figures with others). AG allows training of these two ways of seeing thanks to the number forms at disposal, and to operations such as dividing, cutting and merging.



For the pre-test and post-test, we asked 146 pupils of 5th grade and 109 pupils of 6th grade to copy this figure. After analysis, we determined that three ways of drawing may signify three ways of seeing for those pupils: “fragmented seeing” (drawing line after line), “global seeing” (noticing a structure in the drawing) and, between these, “local seeing” (noticing some structures only).

	Fragmented seeing		Global seeing		Local seeing	
	Pretest	Post-test	Pretest	Post-test	Pretest	Post-test
5th	32%		4%		55%	
6th	40%	30%	15%	15%	40%	55%

Table 2: Ways of drawing and seeing.

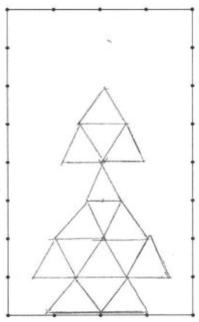


Figure 3: Fragmented seeing.

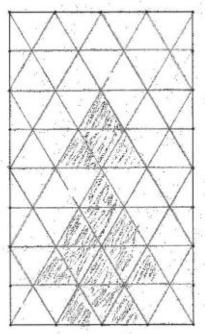


Figure 4: Global seeing.

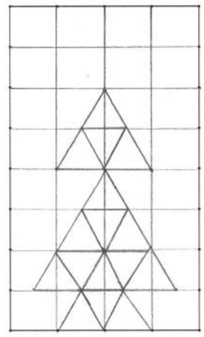


Figure 5: Local seeing.

A functional area for AG

With these theoretical frames and during the experimentations, we defined a functional area for AG to teach and learn perimeter and area. We try to do this also for other tools (see Figure 6). Our target was to build a system of instruments to meet the notions of perimeter and area. For AG, the functional area is composed by “the inventor/handyman” and sometimes “the builder” ways of seeing, and *qualitative perception* and *quantification.*

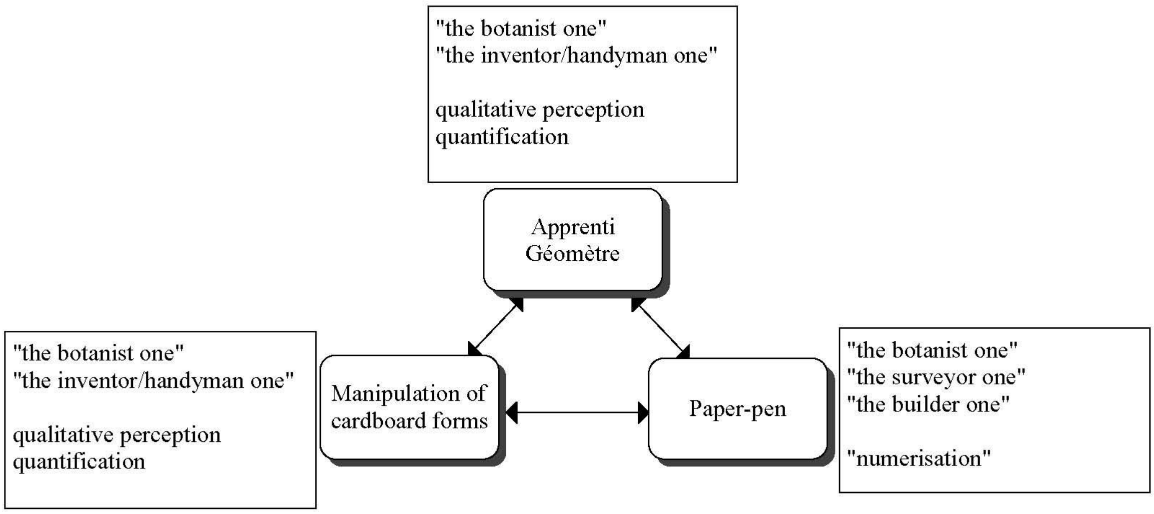
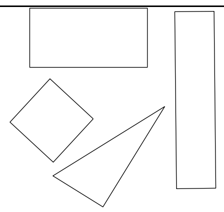
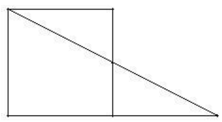
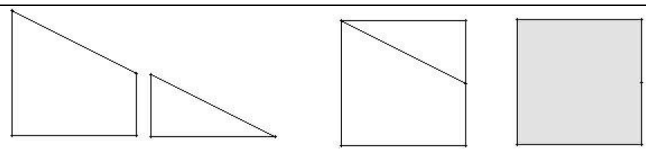
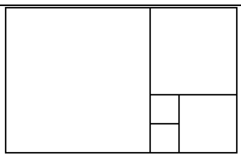


Figure 6: Functional area for three contexts.

Some activities

	Classify these figures, from the smallest area to the largest one.	With AG, pupils can superpose the figures. By transparency, figures appear which can be cut. It’s not so easy with cardboard.  Pupils can also merge the figures, after turning, and superposing.

		
	The smallest square has an area of 2. What is the area and what is the perimeter of the large rectangle? With paper/pencil, often pupils measure… With AG, sometimes they try to measure on the screen! But they can also use the little squares to tile the rectangle. And sometimes, because duplicating and tiling is slow, they “see” that the greater squares can be built from copies of the smallest squares. They calculate the measure of their sides from the measurement of the smallest square. They use the geometric characteristics of the square and, progressively, they “see”!	

Impact of the software

At the end of two years research and from the implicative analysis of the results in the initial and final tests, we conclude carefully that firstly, statistically the pupils having used AG possess a better “global vision” (CREM, 2003) of geometric figures and secondly, they do better at completing certain figures (triangles for example) and computing areas. These results can be interpreted according to Duval's theory about the ways to “see” in geometry (2005).

Another source of knowledge is the way with which operations and motions are identified and collected. In paper/pencil work, these are more or less unconscious. The hand acts in connection with the live visual analysis of the situation. The motions (sliding, turning) are used in an undiscriminating way and leave no tracks. With AG, pupils have to choose the require motion to apply via its name. They use only one motion at a time, one after the other… And before doing that, they have to analyse the situation. It is not a decision on the spot, it is a decision based on a visual and conceptual *a priori* analysis.

Throughout the research, the teachers observed that the students using AG anticipated, better than the others, the requires sequences of operations.

CONCLUSIONS

Apprenti Géomètre is a dynamic geometry software. It allows to divide, to cut, to merge, to duplicate a lot of figures. Some of them are predefine, others no. The user can also move those figures and apply some deformations to the free figures.

These fields allow the user to approach measurement, fraction and magnitude without standard units. This way, pupils think about measurement, and not only about number with a standard unit.

In using AG, pupils have to think about what they want to do. And they have to translate that in different mathematical actions that pupils have to name. Those mathematical techniques, also the *a priori* analysis, help pupils to understand and learn mathematical notions.

We are conscious that it is necessary for pupils to manipulate in class. AG is an additional tool which has to be used with the others already present. And we are also conscious that is not easy for teachers.

We would like to close this paper with a French quotation from a primary teacher which we worked with for six months:

“Ce qui est pratique avec l'informatique, c'est que les enfants devaient mieux se représenter au départ ce qu'ils voulaient réaliser parce que sinon ils étaient piégés. À la limite, quand ils peuvent disposer de matériel qu'ils peuvent manipuler, ils vont chipoter. Tandis qu'avec l'ordinateur, ils ont tout intérêt s'ils veulent être efficaces rapidement à imaginer là où ils veulent aller!... bien se représenter et réfléchir avant d'agir. Ils pourraient chipoter aussi avec l'informatique mais en règle générale ils ne le font pas, car ils se rendent compte qu'ils sont vite piégés!”

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