EFFECTIVE PREPARATION FOR TEACHING OF ALGEBRA AT THE PRIMARY LEVEL

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Algebra experiences in elementary school are essential for building thinking skills and to prepare students for more formalized algebra study in middle or secondary school. Elementary teachers must understand algebraic content, understand how students learn, and use instructional strategies that foster learning to develop algebraic thinking. One online module designed to develop algebraic habits of mind with preservice teachers was implemented with one (B) mathematics methods section (n=28) and another was used as a control section. Post-test results indicated a statistically significant difference (p < .001) with section (B) improved more than the control (A) section (n=33). The shared online classroom simulations in the module allowed for collegial learning in professional learning communities.

CONTEXT OF THE STUDY

Successful teachers can have a powerful, long-lasting influence on their students. Teacher effectiveness has been found to have a greater effect on student achievement than any other teacher factor. It is important for primary K-4 students to learn mathematics from effective and knowledgeable teachers (Ball, 1990; National Council of Teachers of Mathematics, 2000; Reys & Fennell, 2003). According to Reys and Fennell, the teacher is the key and teaching time is a vital factor (Muijs & Reynolds, 2000). Simply taking more mathematics courses is not the answer (Reys & Fennell). Teachers must understand mathematics content, know how students learn mathematics, and be able to use instructional strategies that foster the learning of mathematics (Hill, Schilling, & Ball, 2004).

There is limited time to work with preservice teachers (PTs) during their mathematics methods course. Thus it is important to provide meaningful activities to help them (Gilovich, 1981), and ultimately their students (Schifter, Bastable, & Russell, 2008; Yussen, 1985) develop and maintain effective habits of mind. Mathematical knowledge can be sustained, significant, and long lasting when these habits become internalized. In order to develop these habits, PTs must use knowledge meaningfully, extending, refining, acquiring, and integrating that knowledge (Marzona, 1992). In order to develop these habits, carefully designed questioning needs to occur. How can teachers tap into students' interests? Are these habits of mind like/unlike real world knowledge (Bolin & Yarema, 2007)? How can the teacher make connections to arithmetic

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but foster more generalized solutions? How does this all relate? Will this work in all problems like this one? How do I know?

This study focuses on algebra content at the primary kindergarten through fourth grade level. “However, the word *algebra,* often associated with content covered in a traditional middle school or high school course, can evoke feelings of anxiety and raise questions of appropriateness when discussed in relation to elementary school children” (Bay-Williams, 2001, p. 196). Young children will not be required to solve problems such as $4x-6=22$ but they do need a foundation in algebraic thinking (Taylor-Cox, 2003). Early algebra establishes the necessary groundwork for ongoing and future mathematics learning. According to Bay-Williams, algebra experiences in elementary school are essential for building the habits of mind that serve as an important precursor to more formalized algebra study in the middle or secondary school.

Algebra is characterized as generalized arithmetic and the backbone of school mathematics (Christmas & Fey, 1990; Usiskin, 1995). Being able to reason algebraically cannot be overestimated. Algebra is regarded as the gatekeeper course having the capability to advance a student into higher-level courses and multiple career choices (Chappell, 1997; Kaput, 2000; Ladson-Billings, 1997). The opportunity to learn algebra should be given to **all** students. According to the National Council of Teachers of Mathematics’ *Principles and Standards for School Mathematics* [NCTM, 2000], “algebraic competence is important in adult life, both on the job and as preparation for postsecondary education” (p. 37). All students should learn algebra from effective teachers.

“We need to open the gate to algebraic thinking in elementary schools” (Chappell, 1997, p. 267). Students in elementary school should have experiences that prepare them for more formal study of algebra in the later grades (Carpenter, Franke, & Levi, 2003; NCTM 2000). Students’ development of algebraic thinking in the earlier grade is a big challenge for elementary teachers. They must focus on mathematical activity and thinking to help students learn algebra in the future (Carpenter, Levi, Franke, & Zeringue, 2005; Stump & Bishop, 2002). Researchers argue that algebraic notation should be taught in early instruction (Schliemann, Carraher, & Brizuela, 2006). Connecting algebraic concepts within and across the curriculum is important. PTs need to know how to represent information, model quantitative data, and problem solve effectively.

To develop essential algebraic habits of mind (Bolin & Yarema, 2007; Driscoll, 1999), PTs need real situations in which to practice these habits. Neiss (2005) contends that preparing teachers should include skills that develop their technological pedagogical content knowledge. She suggests purposefully developed online vignettes and virtual episodes will not only afford habits of mind opportunities but also additionally encourage PTs to integrate technology meaningfully into mathematics content. This study seeks to determine if online virtual modules integrated into the methods class help preservice teachers better understand algebra concepts.

METHODOLOGY

Participants

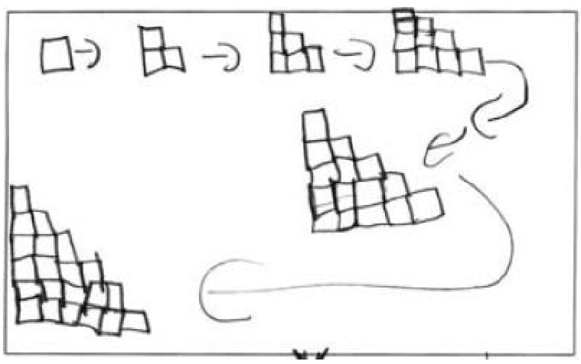
Participants (*N*=61) for this study were female undergraduate PTs enrolled in two sections of methods courses, A and B respectively (*n*=33; *n*=28), at a major university in the Southwest. The majority of the students were Early Childhood majors (*n* = 51; 83.6%), with all of the remaining students in Bilingual Education (*n* = 10; 16.4%). Most participants were White (*n* = 49; 80.3%), followed by Hispanic (*n* = 10; 16.3 %). The mean participants' age was 22.9 years (*SD* = 5.17), with the youngest participant being 20 and the oldest being 51 years of age. This sample matched closely the population of the education students enrolled at this university.

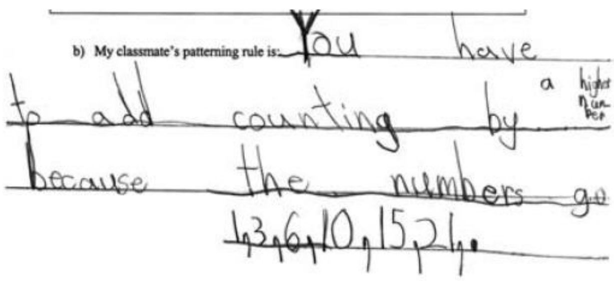
Instruments

Online modules that used simulations and interactive examples for PTs to develop algebraic thinking *habits of mind* (Driscoll, 1999) were developed. Module topics were designed to address the three algebraic habits of mind: building rules for representing functions, doing and undoing, and abstracting from computation.

Module 1: *Building Rules for Representing Functions* included recognizing patterns and making generalizations. The PTs must be alert for error patterns (Ashlock, 2006) as they examine scanned tasks of patterns completed by elementary students as opportunities to: a) learn about students' knowledge, b) identify misconceptions in student work examples, and c) observe snapshots of student work. PTs practiced analyzing student work, looking for and identifying error patterns, and planning for needed instruction. An example is shown below:

Look at the student work (growing patterns) and match it to the rubric below.





4	The student employs an appropriate problem-solving	The student shows an in-depth understanding of	The student creates and extends	The student uses, pictures and words to clearly and

	strategy to identify, extend, and create a growing pattern, ending in an accurate solution	the relationship between growing patterns and addition	patterns, making few errors or omissions.	accurately illustrate and explain their growing-pattern rule

Student Paper A_________

Figure 1: Student work and rubric for a score of 4.

Module 2: *Doing and Undoing* included understanding the process and working backwards. The PTs observed a streamed video of a primary classroom working backward from an answer to a starting point. They identified what steps came before, described what would happen if the teacher started at the beginning and not the end, and finally described the process that reversed the one the students were using.

Module 3: *Abstracting from Computation* included thinking about computation without numbers, predicting without calculation, and involvement in a simulation of the beginning of a lesson where primary students are using computation. They a) offered a strategy for what the teacher could do next to attempt to have students move toward generalizations, b) predicted what students might do next with and without calculations, c) recorded questions they could ask students in the simulation lesson to develop algebraic habits of mind, d) suggested different representations that could have been used, and e) linked representations back to the original problem context.

A nine-item pre/post assessment was designed to measure the PTs pedagogical content knowledge of patterning and their ability to generalize those patterns. Tasks included finding the pattern core, naming appropriate primary patterning activities, determining repeating patterns, and finding the 10^{th} item in a growing pattern. An example is shown.

Sketch the fourth train in the pattern below, find the perimeter of the 4 trains in the patterns, and then determine the perimeter of the 10^{th} train without building the train (3 pts).

Perimeters_________________ Draw the 4^{th} train___________________

10^{th} train perimeter_____

Figure 2: Growing pattern generalization task example

Methodology

A nine-item algebra assessment was administered on a pre/post-test basis in both methods classes. This pre/post assessment was evaluated on a 15-point scale because certain items had multiple parts as demonstrated above. Both sections received classroom methods instruction in algebra based on the Van de Walle (2006) text. Only one section (B) of PTs completed Module 1. The module was evaluated using a performance rubric to determine accuracy and completion. For purposes of this study only completion was analyzed. An experienced instructor, who was not part of the research team, taught the sections. This instructor had more then 15 years of K-12 mathematics classroom experience and 5 years of college level teaching. Mathematics methods courses are designed to help PTs bridge their mathematics content with appropriate mathematical pedagogy. Both sections received the exact same instruction from the same textbook; the only exception was that one section used the online module.

Results

Results for the nine-item assessment revealed section A PTs had mean pre/post-test total scores of 10.2 (SD=2.3) and 11.09 (SD=3.1) respectively. Section B students had mean pre/post-test total scores of 10.39 (SD=2.6) and 12.61 (SD=2.2), respectively. Thus section B, which completed Module 1, improved 2.22 points on average from pre-to post-test while group A, which was the control group, improved 1.4 points.

Section	Test	Mean	Std. Deviation	95% Confidence Interval	
				Lower	Upper
A	Pre-total	10.42	2.305	9.68	11.46
	Post-total	11.09	3.076	9.82	12.25
B	Pre-total	10.39	2.587	9.39	11.40
	Post-total	12.61	2.200	11.75	13.46

Note. section A, n = 33; section B, n = 28; 15 point scale.

Table 1: Descriptive Statistics of Total Scores with Different Class Sections.

The data were analyzed using 95% confidence intervals (CIs) for each group's mean (sections A & B) by the pre-and post-total scores. Confidence intervals were employed and interpreted (Capraro, 2005) to "make clear what the data has to say" (Cumming & Finch, 2005, p. 170). This analytic method provides for comparison that limits inflation of TYPE I error, which occurs when multiple univariate tests are used (Thompson, 2002). An examination of the CIs in Figure 3 showed that while there were no statistically significant differences between the pre-total and post-total scores of section A at the $p < .05$ level, there were statistically significant differences at $p < .001$ between the pre-total and post-total scores of section B. This significance can be observed by using the inference by eye (Cumming & Finch) - looking at section B in Figure 3 one can see that there was no overlap in the error bars indicating a statistically significant difference at $p < .001$ from pre-to post-total scores. The scores from section A have almost a 75% overlap from pre- to post-total resulting in no statistically significant difference. This analysis supports section B improved more in their algebraic thinking from their completion of Module 1 than section A. Variances were similar for the both groups.

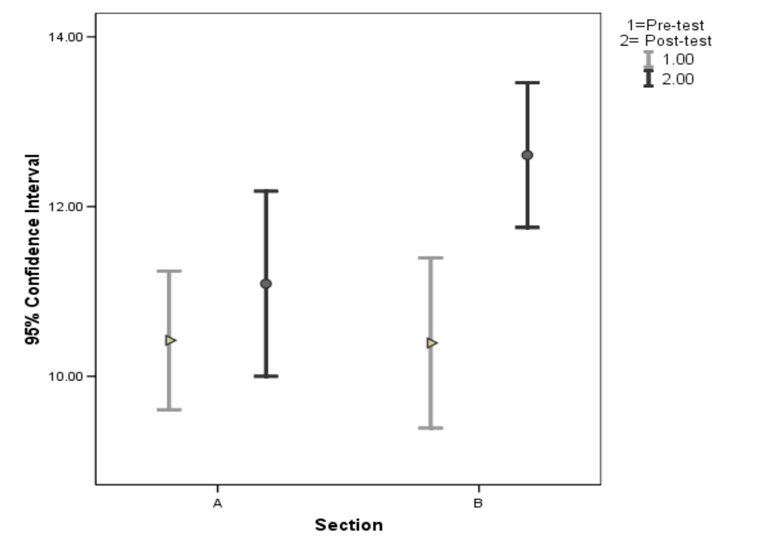


Figure 3: Confidence Intervals for the pre-total and post-total test for section A & B.

DISCUSSION

From the results above, it can be inferred that PTs need more practice in developing algebraic habits of mind than can be covered in a face-to-face methods class. Algebra at times is not covered as completely as it should be during an elementary methods class (Stump & Bishop, 2002); therefore, online modules to assist preservice teachers in developing algebraic habits of mind were created. These modules can provide opportunities for PTs to first develop their own algebraic habits of mind before they are required to help their primary students develop habits of mind. With preservice education there is at times a dissonance between what goes on in methods classes and what students observe in their sometimes less than ideal field-based classrooms. These modules provide PTs with multiple varieties of activities in a controlled-simulated environment. All students in the group view the same scanned student work, lessons, vignettes, and streamed videos creating a perfect opportunity for shared online discussions, chats and reflections on the same topic. These reinforcements and shared online classroom simulations allow for collegial learning in professional learning communities.

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