

## The understanding of the integral defined as mathematical object in the university students

<b>Author (s) : <u>Aida María Torres Alfonso</u> Dámasa Martínez Martínez</b>	<b>Affiliation: Department of Mathematical Universidad Central de Las Villas, Cuba</b>
<b>E-mails:</b> <a href="mailto:aida@uclv.edu.cu">aida@uclv.edu.cu</a> , <a href="mailto:fresasjun22@yahoo.com">fresasjun22@yahoo.com</a>	<a href="mailto:damasa@uclv.edu.cu">damasa@uclv.edu.cu</a> , <a href="mailto:damasam58@yahoo.com">damasam58@yahoo.com</a>

### ABSTRACT

*The necessity to transform the didactic scenario in mathematical university student class, in the process of the students' learning and the scientific foundation of these changes with the objective of developing the understanding of the mathematical objects that put day by day at stake, has been the main motivations of this investigation. Exemplifying their results for the case of the integral defined in a course of Mathematical Analysis.*

### PURPOSES

This investigation at the moment in development in the Central University of The Villa, it sustains that all didactic design that power the mathematical understanding in an university environment, besides involving the study of the mathematical object in question, it should establish the diagnosis of the mathematical understanding of the students, also favoring as practice in the classroom the evaluations in public, as well as the own valuations of the process of the students' understanding.

We propose the design and the professors' participation, in the process of the mathematical university student in such way teaching learning that they are kept in mind the previous knowledge that the students can evidence, influencing them and modifying them through the experience characteristic of the school institution, of experiences that it can confront and practical in context. Where the professor, of transmitter of knowledge becomes a critic of his practice, confronting it and modifying it according to the demands of his practice and evaluation in the classroom. Each didactic situation that plans becomes a conjecture, a hypothesis that can only be proven or to be discarded and to be corrected if it is the case in the real teaching, the day of the class.

The authors of the paper are investigating of their practice and the classroom is a shop where they design projects for the best use in the process of teaching learning; where the students develop their capacities, habits and abilities around the social necessities under the postulates of the scientific chore, this way they trace themselves, their road toward and for the learning.

Previous results in the field of the Didactics of the Mathematics in our university center have revealed that in the basic formation of an university student of the technical sciences you had a great importance the Integral Calculation and in particular the defined integral mathematical object, is related with many of the subjects of the curriculum, for what is indispensable that the students understand it in all their development, so that they can already use the concepts, theorems and methods imparted inside those matters and in those that will receive in years superiors.

Before the current challenge that have the university professors, of becoming facilitators of the development of the understanding capacities of the students, the authors intend to organize, to design and to execute the didactic system that will contribute to the execution of those purposes, putting emphasis in that the whole mathematical activity to develop favors the development of the understanding of the integral as mathematical object instead of delighting as professors with the accumulation of the knowledge in the student.

With this article the authors contribute to the development of the thematic: Theoretical and experimental studies on the tools of analysis of the practices in the classroom and their relationship with the school processes.

## **THEORETICAL MARCO**

We present the theoretical mark of this investigative process that registers in a Qualitative Investigation, keeping in mind the relating generals of the Cultural Historical Focus, as well as the Theory of the Didactic Situations, from the perspective of the Didactics of the Mathematical.

### *Focus Historical Cultural:*

Lev Semionovich Vygotsky (1896-1934) is considered precursor of the social constructivism. Starting from him, diverse social conceptions have been developed on the learning. The fundamental of the focus of Vygotsky consists on considering the individual as a result of the historical and social process where the communication plays an essential part, also postulating that the knowledge is an interaction process among the fellow and the means, but the social understood means and culturally. In their theory the knowledge is acquired, it is built, through the interaction with the other mediated by the culture, developed historical and socially. And as for the process of development of capacities, this focus postulates that they are formed and they develop in the activity and for the activity, being acquired during the life.

### *Theory of the didactic situations:*

The Theory of Didactic Situations constitutes, from our point of view, a theory of the organized and directed learning of the Mathematical, this is, a theory of the mathematical instruction, in consonance with the budgets epistemology and cognitive expressed previously. It describes an environment of potent learning in which attention is not only lent to the on mathematical knowledge in game in the tasks, but also to the communication activities in the classroom, everything it in an organized sequence of situations and didactic moments. It constitutes, therefore, a strong theoretical mark to design a model that seeks to develop the mathematical understanding. On the other hand, the theory of the didactic situations, in its global formulation, incorporates a vision characteristic of the mathematical learning, but is distinguished of other constructivist's theories for its way of confronting the relationships between the student and the knowledge. A didactic situation is to imply a group of relationships explicitly established between a students or a group of students, some environment (including instruments or materials) and the professor with an end of allowing the students to learn - this is, to reconstruct - some knowledge. The situations are specific of the same one. So that the student builds the knowledge, it is necessary that he is interested personally in the resolution of the problem outlined in the didactic situation. In this case it is said that the refund has been gotten from the situation to the student.

Brousseau (1981, p. 48) mentioned by Sierpinska, A. and Lerman, S. (1996) they suggested a complete investigation program for the didactics of the mathematical one that implies epistemologies studies, design of didactic situations, experimentation, comparison of the design with the processes that take place in fact, revision of the epistemologies studies and of the design, and I study of the conditions of reproducing the situations.

*Understanding a mathematical object from a perspective of the Didactics of the Mathematics:*

In correspondence with the theoretical mark of the TSD, it is assumed that to understand a mathematical object consists on being able to recognize their characteristics, properties and representations; to relate it with other mathematical objects and to use it in all the variety of problematic situations that are proposed by the professor. Under this perspective the understanding assumed by a fellow in a moment, difficultly will be total or null, but partially. It is needed therefore, to conceive on the part of the professor a sequence of activities that you/they have as objective that the emergency of personal mathematical objects as a result of a didactic organization designed a priori by the professor. In the field of the Didactics of the Mathematics it is considered to the mathematical objects as entities that arise when carrying out systems of practical corresponding to a field of problems (Godino and Batanero, 1994)

Of the other side of the didactic problem to solve, the student will have understood a mathematical object when it uses it in a competent way in diverse didactic situations, in those that will require to use different notations, as well as to transform a representation into another in a natural way, and also have the capacity to be able to express it openly, with arguments that demonstrate that his thought has evolved after a productive intellectual effort and not to equal these practical of evaluation with the active participation of the students in those which alone it is analyzed if it participated or not in classes, but not if the same one is sustained in a development of its thought and concrete mathematical activity.

## **METHODOLOGY**

The qualitative investigation is distinguished to be guided to describe and to interpret the phenomenon, it is flexible and open, active and systematic, and its design is practically emergent, that is to say, when this advances, the new situations and the obtained data generate the problem, being constantly questioned and formulating when having new data and their comparisons again. The investigation was conceived starting from the necessity of deepening in those mechanisms that help to impel the development the mathematical understanding as process, with emphasis in the qualitative thing, keeping in mind the logic of a development of interpretations that take from a process open of the reality to a theoretical conception of the necessary aspects to design a didactic model that develops the understanding of the integral defined as mathematical object in university students.

Method of Investigation Systemic: In the knowledge of the educational phenomenon it carries out an essential function the system focus that provides the general orientation for their study, like an integral reality formed by components that complete certain functions and they maintain stable forms of interaction among them.

These characteristics of the systemic method regarding the process of the education are summed up in the following way: the analysis of all the objects and fellows that participate as well as the influences and external conditions of the education process, the appreciation of the dynamic essence of the educational process that is modifying, the interaction of objects and fellows in their group, the unit of the influence that the educator exercises in the students and of the inverse influence, the entire characterization of the process of the education from the social, psychological-pedagogic, physiologic, economic point of view.

The use of this method allowed in the case of this investigation an integral analysis of the components of the system: student, professor, contents of the Mathematical Analysis; as well as of the nexus that exist between the same, it allowed the detection of the characteristics that reflect in more measure the essence of the investigated phenomenon that in our case they are not others that the indifference that the students show, the non retention of the acquired knowledge, the preference for other university specialties, the disposition lack for the systematic study. With these studied characteristics and the analysis of the typical situations: as for example that continue being the subjects of basic cycle, among those that is the Mathematics, obstacles that the students will conquer in their zeal to achieve success in their university formation. The approaches of efficiency, the levels of understanding and the conditions could settle down for the good operation of the process of teaching learning of the mathematical one in a first year of specialties of technical sciences.

Historical-logical analysis: It was used to know, with bigger depth, the antecedents and the current tendencies referred to the understanding of the integral defined as mathematical object from the perspective of the Didactics of the Mathematics. The main investigator of the paper could analyze in the sources, the following precedent investigations about the understanding of the integral one in the process of the teaching of the Mathematics. These investigations have generally been associated to two fundamental factors, on one hand; the difficulties of the students' understanding have been analyzed as for the concept, where the object mathematical limit has had great implication. And on the other hand, the almost exclusive analysis of the algebraic sense of the concept far from their interpretation as a result of processes of change.

In Orton (1983) the results of an experimental investigation are presented carried out with students of Secondary Basic, the one that shows that the learning of the integration like it adds it constitutes an obstacle for the understanding of the concept. This study is enlarged in Czarnocha and collaborators (2001) who show in a study carried out 32 university students, using the theory APOS that an appropriate coordination doesn't exist between the graphic outline of the sums of Riemann and the limit of the numeric succession, that which makes them propose curricular changes. Schneider (1988), as objective of their Doctoral Thesis, it shows the difficulties that the students present as soon as the understanding of the fundamental Theorem of the Integral Calculation. Analyzing the epistemologies obstacles that appear in the conceptions when the students using the area concepts and volume, appealing to erroneous mental representations. Turégano (1994) it carries out a study on the concepts around the measure and the learning of the infinitesimal calculation, being the purpose of their Doctoral Thesis to elaborate a didactic proposal that allows to introduce, at conceptual level, the integral one defined in secondary students that have not been begun in the study of the infinitesimal calculation. Already from another perspective, inside the community of Mathematical Education that one comes developing in Latin America, we can appreciate firstly in Cordero (1994) a study about the knowledge of the integral one and the construction of their meanings, Doctoral Thesis defended in the Department of Mathematical Educational, Cinvestav, IPN. And another important investigation in the topic is the one that we find in Farfán (1997) that points out the necessity to use, in our work educational university student, escaping from the anachronisms that can be presented, the historical reconstruction of the aspects with more rigor of the Analysis that we teach. At the moment it is developed in Spain a Project of Investigation about the conceptions and the conflicts that can be appreciated in the books of texts by those which the Mathematical Analysis the first years university student is studied in the different careers; the same one is driven by the professor and investigating Angel Contreras of the University of Jaen and one of their more recent results can already analyze it in Contreras and Ordoñez (2006) where it characterizes the complexity of a text book and the conflicts that can cause in the students when presenting the topic of the defined

integral, where the understanding of the integral one is one of the declared objectives of the process of teaching learning.

**Analysis-synthesis methods:** It was used to characterize the investigation object, that is to say, process of teaching learning of the Mathematical Analysis and the field of action of the same that it is in this case: the understanding of the integral one defined in the students that study the first year of the university. As we have pointed previously, the Mathematical Analysis has direct influences in all the branches of the Mathematics and of the Science and the Technique. Also, in the contents of this discipline he/she is the germ of mathematical and applied theories that they are imparted in subjects of the superior years and that they are the starting point to develop investigations of present time.

*Participant observation:* The experience of more than twenty years imparting the Mathematical Analysis as subject of the cycle basic university student, it has allowed to verify to the main author of the investigation that is one of the subjects that causes with more force the educational drops and emphasizes without intending it the indifference of the students For what this didactic problem to which we face constitutes a rupture among the Mathematical one that one knows, the mathematical activity that should be carried out in and outside of the classroom and the Mathematical one that we should understand to use it in a competent way in different practical, not alone when graduates, but during all its formative process.

In summary, the combination of the methods and technical previously described in function of the current Cuban university context and of having a strong theoretical mark, as well as of being able to verify the results of some of the investigations that we have indexed in this work in the classroom in a practical way, they allowed the authors to define the objective of the investigation project that at the moment develop: I model didactic that favors the mathematical understanding in university students, in which they defend arguments and they contribute suggestions about how to teach and to learn mathematics with understanding. Not being objective of the current project and like part of him, neither of this work, to analyze the understanding of the integral defined from a perspective that centers the interest in the study of aspects like their nature, operation or evolution.

## **SYNTHESIS AND DISCUSSION OF RESULTS**

The didactic strategy conceived by the authors to develop the understanding of the integral defined as mathematical object in the university students proposes three fundamental stages:

### **♦ *Necessity to carry out a diagnosis of the understanding of the defined integral, in university students***

To understand means to take the knowledge and to use it in new activities. It is the capacity to carry out a great variety of reflexive tasks, such as the generalization, explanation, application, search of evidences, or representation of a topic. The understanding is not achieved by means of the development of calculation algorithms or observing to other to follow them, but we learn carrying out activities of understanding.

This conceptualization it is come off that to influence the understanding of the students of a classroom of first university year one prevails to know in what state is its intellectual development and the intellectual possibilities, social and affective of each student.

Recapturing questions valued previously in this work, we portion of the position assumed by the author that the Integral one is a key topic in the professional's curriculum that we are analyzing, for what would be strategic to design the diagnosis of their understanding in a systematic way and interdisciplinary during the first year of the career.

- ◆ *To design of didactic situations they are kept in mind the results that the continuous diagnosis offers to the professor, so much of the group as of each student*

To demonstrate abilities to reason and to analyze mathematical information's:

To understand the concepts, theorems, advance and relative mathematical procedures to numeric successions, limit, infinite, continuity, functions and their derived.

To demonstrate to be able to apply the knowledge to the resolution of problems that are modeled using the integral defined as mathematical object.

To use the mathematical language in the communication of ideas. .

Disposition toward the Mathematical one and their habits of work singular or in cooperation.

**System of didactic situations developed in a course of Mathematical Analysis, conceived with the objective of propitiating in the university students, the understanding of the defined integral.**

The professor should guide his process of teaching learning toward didactic activities that reinforce the development of an effective mathematical thought in the students. To establish individual and collective registrations of the group as for the evolution of their actings in each mathematical activity that is developed.

The diagnosis of the understanding of the integral defined as mathematical object settles down from the beginning of the course, by means of several test it has more than enough knowledge, disposition toward the mathematics, its study, its scientific aspirations, the relationship that they know of each one of the mathematical objects that go learning with its future profession during the school course. The techniques of this diagnosis are enlarged to individual interviews and in the group; constituting part of the process of each student's evaluation during the whole course.

The educational activities don't subscribe to the exchange moment in the classroom. Being guided in a previous way on the part of the professor of sessions that link the theory with the practice where the students will model problems of the daily life, where the mathematical object will be interpreted: defined integral and that they will be resolved with the use of the technologies.

The preparation work for these activities will have two modalities: a singular and another in teams, the first one will allow assisting the individual differences, the interests, their motivations and the second will prepare them for the work in team as professional futures.

The didactic resources are used that they appear in Internet with the objective that the students carry out a critical valuation of the products assisting to the different representations that are used and to the advantages of the visualization for the understanding of the on mathematical object in game: <http://archives.math.utk.edu/visual.calculus/>

As conclusion of this created didactic situation, to each student they are proposed two problems of the text book that will solve with the aid of the new technologies and whose result was presented as a didactic product that links the different representations and the visualization in the important link of the understanding of the mathematical objects.

Several of these works they were exposed by the students in events of student scientific character for the quality that they evidenced, what not demonstrates alone that the student evolution in the process of understanding of the defined integral, but rather it was capable in designing a didactic resource so that the other ones understand the mathematical one in the first year university student.

On the other hand and as a result of this investigation, the students expressed in interviews singular and collective that these activities have motivated them for the learning of the topics of the subjects and for the career that study, also that, it has changed them the perception of the mathematics, showing the social and human of the same.

## CONCLUSIONS

The continuous reflection in the strategies that are used in the position and solution of mathematical problems should be conceived as an essential process so that the student acquires durable capacities that can be transferred to the solution of new activities in different contexts.

The development of reflection activities in the learning process as well as the use of the knowledge that has been mobilized and the evolution that has carried out the learning process, strengthens the responsibility before the study, as a value to reach in the university students

Should carry out theoretical activities and of application of each one of the mathematical objects that pay to the understanding of the defined integral, in those that the interaction is fomented with the professor and the rest of the students and that they foment the self-esteem and the spirit of collaboration.

Another significant aspect of this investigation work reveals the importance of proposing to the students, scientific projects that are solved using the mathematics that learned during the school course, with an appropriate use of the technology in function of the solution and presentation of results.

Therefore, the necessity to transform the didactic scenario in mathematical university student class, leaving of the diagnosis of the previous mathematical knowledge of the students, the possibility to use in a deliberate way the technologies of the information and the communications in the process of the students' learning; as well as to foment the evaluation of the own students like via for each student's individual development they have been the main addresses of the didactic system that we propose.

## REFERENCES AND BIBLIOGRAPHY

- Brousseau, G. (1981). Problèmes de didactique des décimaux, *Recherches en Didactique des Mathématiques*, Vol.2, pp. 37-127.
- (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en Didactique des Mathématiques*, Vol. 7, n. 2, pp. 33-115.
- (1997): “*Theory of didactical situations in mathematic*”. Dordrecht, Netherlands: Kluwer Academic.
- Czarnocha, B. and cols. (2001). *The concept of definite integral: coordination of two schemas*. *College Mathematics Journal*, MAA (to appear in May).
- Castorina J, A. Piaget y Vigotsky en la perspectiva de las relaciones entre comprensión y explicación, Universidad de Buenos Aires, CONICET, Argentina, artículo extraído en mayo del 2007: [www.fahce.unlp.edu.ar/departamentos/psicologia/catedras/genetica/castorinacampinas.htm](http://www.fahce.unlp.edu.ar/departamentos/psicologia/catedras/genetica/castorinacampinas.htm)
- Contreras, A. y. O., L. (2006). *Complejidad ontosemiótica de un texto sobre la introducción a la integral definida.*, *Revista Relime Vol. 9 (Número 1)*, 65 – 84.
- Cordero, F. (1994). *Cognición de la integral y la construcción de sus significados. Un estudio del discurso matemático escolar*. Cinvestav-IPN., México D.F.
- Chevallard, Y. Bosh, M. y Gascón, J. (1997). *Estudiar matemáticas: El eslabón perdido entre enseñanza y aprendizaje*. Barcelona, España: Editorial Horsori.

- Contreras, A. y cols. (2001). *La enseñanza-aprendizaje de los conceptos elementales del Análisis Matemático*, Proyecto de Investigación, Ministerio de Educación y Cultura (CIDE).
- Contreras A y Ordoñez L. (2006) *Complejidad ontosemiótica de un texto sobre la introducción a la integral definida*, Revista Relime Vol. 9, Num. 1, 65 – 84.
- Farfán, R. M. (1997). *Ingeniería didáctica: Un estudio de la variación y el cambio*. México: Grupo Editorial Iberoamérica.
- Font V. (2003) *Processos mentals versus competencia*. Spain, 3-6 December 2003, 66-68.
- Godino, J. D. y Batanero, C. (1994). *Significado institucional y personal de los objetos matemáticos. Recherches en Didactique des Mathématiques*, Vol. 14, nº 3: 325- 355.
- Orton, A. (1983). *Student`s understanding of integrations. Educational Studies in Mathematics*, 14 (1) , 1 -18.
- Perkins, D (1999): *¿Qué es la comprensión?* In: WISKE. M.S. (Ed.) *La Enseñanza para la Comprensión. Vinculación entre la investigación y la práctica*, 215-256. Barcelona, España: Editorial Paidós.
- Pirie, S.E.B. y Kieren, T. E. (1994). *Growth in mathematical understanding: How can we characterize it and how can we represent it?*. *Educational Studies in Mathematics*, 26 (3): 165-190.
- Ruiz I., M (2004). *Arcadia: La competencia pedagógica didáctica para aprender con sencillez y significatividad*. México: Editions Norma.
- Schneider – Gilot, M (1988). *Des objets mentaux «aire» et «volume» au calcul des primitives. Tesis doctoral. Université Catholique de Louvain. Faculté des Sciences. Louvain La Neuve*.
- Sierpinska y Lerman (1996). *Epistemologies of mathematics and of mathematics education*. En: A. J. Bishop et al. (eds.), *International Handbook of Mathematics Education*, 827-876. Dordrecht, HL: Kluwer
- Torres, A (2003). *Internet y la Investigación en Educación Matemática*. En *IV Jornadas de Innovación Docente*. Valencia: Universidad Politécnica de Valencia.
- Turégano P.(1994), *Los conceptos en torno a la medida y el aprendizaje del cálculo infinitesimal, Tesis Doctoral, Universidad de Valencia*.
- UNESCO (1998). *Declaración mundial sobre la Educación Superior en el siglo XXI: Visión y acción. Conferencia Mundial de Educación Superior*. Material extraído de [http://www.unesco.org/education/educprog/wche/declaration\\_spa.htm#declaracion](http://www.unesco.org/education/educprog/wche/declaration_spa.htm#declaracion)
- — — — — (2001) *Conferencia Internacional de Expertos sobre la Enseñanza de las Ciencias, la Tecnología y las Matemáticas en pro del Desarrollo Humano*, Goa, India, 20-23 de febrero de 2001, Material recuperado en [www.unesdoc.unesco.org/](http://www.unesdoc.unesco.org/)
- Vigotski, L. S. (1982). *Pensamiento y Lenguaje*. Ciudad de La Habana, Cuba: Editorial Pueblo y Educación.