

AVOIDING THE TEACHING OF MATHEMATICS

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ABSTRACT

The research results presented in this paper are only a small part of an action research performed with the main purpose of improving primary school student teachers (trainees)' understanding of mathematics. This paper reports some preliminary results of a questionnaire administered to 305 primary school teachers. The questionnaire asked them about the mathematics contents they would be prepared to omit from their teaching if schools gave them the autonomy to decide the curriculum. About two thirds of the teachers marked one or more contents they would omit. The most frequently reported reason for omitting some content was related to their perceptions about their subject matter knowledge. Their answers are contrasted with the most frequent reasons suggested by curriculum developers to exclude particular contents from the school curriculum. Some social implications derived from these teachers' answers are also discussed.

INTRODUCTION

In Amato (2004b) I describe some of the results of an action research study which included interviews with seven primary school teachers. I was interested in uncovering any difficulties they had experienced at the beginning of their careers and how an in-service course taught by me had influenced their understanding of mathematics. The intention was to use this information to help me develop a teaching programme for a similar course in pre-service teacher education. Some teachers revealed that they avoided teaching mathematics whenever possible at the beginning of their careers. My belief that mainly rote learning was going on in schools in Brasília was confirmed by what the primary school teachers mentioned in the reconnaissance interviews and by the weak conceptual understanding indicated by trainees (primary school student teachers) in the action steps of the research. What I was not expecting was the allocation by teachers of less teaching time to mathematics than to other subjects. More research was needed in order to investigate the extent of this “avoidance” problem. Therefore, the main research questions in this new reconnaissance stage were:

(1) What are the mathematics contents primary school teachers would be more prepared to omit from their teaching, if schools gave them the autonomy to decide the curriculum?

(2) What are main reasons provided by teachers to omit the contents they have selected in question 1?

(3) Which are the contents in the primary school syllabus teachers more often think they need to deepen their mathematical knowledge?

SOME RELATED LITERATURE

Students spend many years studying mathematics at school. It is easy to assume that teachers learn the mathematical knowledge they need to teach by just going to school. However, some studies that examined the outcomes of school mathematics tend to show that school experiences have not contributed much to students' conceptual understanding of elementary mathematical content. Students appear to leave school "with little more than basic whole-number computational skills" (Ball and McDiarmid, 1990, p. 442). Teachers were once children and teenagers, therefore, it is possible to make inferences about teachers' school learning based on these data. Studies of primary school teachers and trainees' knowledge of elementary mathematics provides a similar picture of school mathematical learning (e.g., Ball, 1990 and Simon, 1993). Ball (1990) concludes: "Our findings suggest that what prospective teachers have learned in their precollege mathematics classes is unlikely to be adequate for teaching mathematical concepts and procedures meaningfully" (p. 463).

There are several reasons in the literature why curriculum reformers, teachers and students avoid the teaching and/or learning of mathematics. According to Ball and Bass (2004), "teachers select and modify instructional tasks, ... and decide what to take up and what to leave" (p. 166). Yet curriculum reformers are usually the ones officially in charge of including or excluding mathematics contents from the school syllabus. According to Kline (1973), the new mathematics movement in the 60's attributed school failure to the oldness of curriculum contents. Some countries included modern contents such as set theory in the primary school curriculum and reduced the emphasis in conventional contents such as Euclidian Geometry. Later set theory was excluded from this curriculum and more emphasis was put in Euclidian Geometry.

Some years ago I started worrying about the exclusion of important contents from the primary school curriculum. It became clear to me that it is not possible to imagine a whole without some knowledge of its constituent parts and of the relationships among the parts. Chinn and Ashcroft (1993) write about the dynamic interactions of the parts and the whole in the learning of mathematics: "It is a subject where one learns the parts; the parts build on each other to make a whole; knowing the whole enables one to reflect with more understanding on the parts, which in turn strengthens the whole" (p. 3). I think that a strong conceptual understanding of mathematics (the whole) involves knowing well its content (the parts) and how the content has been put together (the connections). As a teacher I found set theory irrelevant for young students (Kline, 1973), but avoiding the teaching and learning of important pre-requisite contents, such as fractions, leaves the learner with some "missing parts" and so with less conceptual understanding.

One of the reasons provided by the literature for avoiding the teaching of mathematics concerns a teaching focus based on affective or socialisation goals. Some primary school teachers are said to focus their teaching on the students' characteristics and socialisation goals, and do not take a strong content perspective. Bromme and Brophy (1986) advise teacher educators to discuss such issues with trainees: "for those oriented more toward affective or socialization goals than toward cognitive or content coverage goals, some consideration of responsibilities and minimally acceptable standards for content coverage seems appropriate" (p. 124). Wilson et al. (1987) argue that comprehension and representation of content is as important for the teaching of younger children as it is for secondary school students. These two goals do not seem to be incompatible, as socialisation can also happen through the learning of content. Well-adjusted citizens in society are also citizens who can critically interpret and discuss information presented in the media. It is the acquisition of cultural knowledge, such as mathematics, which makes the participation of individuals in the political life of a country possible. Research also suggests that teacher' beliefs about their students' ability to learn also influences the way they teach, the content of their lessons and students' opportunity to learn (e.g., Bromme and Brophy, 1986). Some teachers "set lower goals or are satisfied with lower performance, rationalising that nothing more can be expected from their students because they lack motivation or ability or because they come from deprived family backgrounds" (Bromme and Brophy, 1986, p. 125).

Breadth of the curriculum content or "opportunity to learn" was one of the strongest factors contributing to high achievement in international surveys (Brown, 1999). However, Ball and McDiarmid (1990) cite the results of two studies which show that curriculum content may be transformed, narrowed or avoided by negotiations made between students and teachers. In order to keep students at school, or to deal with students' resistance to complex intellectual tasks, negotiations which limited students' learning opportunities were made to make classroom life less problematic.

Bromme and Brophy (1986) point out that "teachers have been found to allocate more instruction time to subject-matter areas that they enjoy, and less to areas that they dislike" (p. 122). So the most direct influence of primary school teachers' negative attitudes to mathematics on their students' learning appears to be time allocation. In most cases those teachers are in charge of teaching all subjects, and can make their own decisions about how much time they should devote to each subject. Low time allocation may be related to avoiding the teaching of important prerequisite contents, and therefore may restrict students' opportunities to learn (e.g., Brophy, 1986 and Fisher, 1995). Mathematics is a hierarchical subject and so not learning well the contents of a certain grade may bring great problems in the learning of related contents in subsequent grades.

Some curriculum developers propose a reduction in the teaching of difficult contents for primary school students. Learning operations with rational numbers is not easy (e.g., English and Halford, 1995). For this reason, some curriculum developers in Brazil are proposing a reduction in the teaching of operations of fractions at primary school level: "It is convenient to teach the concept

of fractions in primary school grades, but not operations with them” (Imenes and Lellis, 1996, p. 46). On the other hand, some other educators think that difficult contents should be started earlier to provide the learner more time to construct and connect the related concepts, and to prevent the development of misconceptions. Orton (1987) argues that statements in which absolute levels of difficulty are assigned to particular mathematical concepts can be unhelpful. Teachers must try to find simple ways of teaching those concepts. If difficulties are inherent in particular topics, it is the teacher’s duty to provide more experiences (quantity) with the potential to improve understanding (quality). According to Nunes (in Falzetta, 2003), anticipating the teaching of difficult contents such as proportion is beneficial even to six year olds: “It is up to the school to work with a representation that the child can understand and visualize the concept of proportion” (p. 26). Another example involves the relationship between simple subtractions and negative numbers. I think that instead of saying that subtractions such as $5 - 7$ have no answer, or that a subtraction always has the largest number first, teachers could encourage the acquisition of the concept of negative numbers by challenging young students to think of ways of solving subtraction problems with very small numbers, presented and solved orally, and with the help of concrete materials: You have 4 reais (Brazilian currency) and you need to buy something that costs 7 reais. What can you do? What have you seen other people doing?

Research has shown that many school students’ (e.g., Ni and Zhou, 2005) and even trainees (e.g., Domoney, 2002) see fractions as two separate natural numbers and not a single number and develop a conception of number that is restricted to natural numbers. Ni and Zhou (2005) suggest that the teaching of fractions should start earlier than is it is often recommended by curriculum developers in order to avoid the development of what they call ‘whole number bias’. More recently, research has stressed the importance of not downplaying the teaching of fractions because of the connections to the understanding of algebra concepts (Geary et al., 2008). I think that teachers should at least provide fifth graders (10 year olds) with an informal presentation to operations with fractions, with a focus on less symbolic representations and practical work and games in order to avoid the development of misconceptions, such as “multiplication always makes bigger” and “division makes smaller”.

Another influence on what students learn from school may be the “inadequacy of their teachers’ knowledge of mathematics” (Fennema & Franke, 1992, p. 147). Research tends to show that teachers and trainees present similar misconceptions to those presented by school students (e.g., Graeber and Tirosh, 1988). Simon (1993) argues that teachers with weak conceptual understanding tend to provide conceptually impoverished instruction. This was my case when I started teaching (Amato, 2004a). It was impossible for me to teach conceptually what I had learned mostly by rote. I remember well the first student’s question which made me aware of such fact. After just 3 months teaching, an 11 year old was puzzled by the result of $1/2 \times 1/3 = 1/6$ and asked “ $1/6$ is smaller than $1/2$ and $1/3$. Why do we get a smaller result number when multiplying fractions?”. Therefore, sometimes without much awareness, teachers may avoid teaching important relationships.

METHODOLOGY

I carried out the initial steps of an action research at University of Brasília through a mathematics teaching course component in pre-service teacher education (Amato, 2004b). This new part of the reconnaissance stage of the research had the aim of understanding the avoidance problem found during the first reconnaissance stage and reported in Amato (2004a). A questionnaire with three questions was designed and administered to primary school teachers:

(1) Mark with an X in the list below the contents that you would not teach if a school gives the autonomy to omit parts of the syllabus of a certain grade.

(2) Why would you omit the contents that you have selected above?

[Space was given here to answer question 2. Questions 1 and 2 occupied the first page of the questionnaire].

(3) The contents listed in the table below represent the main mathematics concepts and operations that the primary school teacher will have to help his or her students to develop. In which of those contents do you think you need to deepen your mathematical knowledge?

[Question 3 occupied the second page of the questionnaire].

Tick your answer in the appropriate column according to the following code:

N	S	M
No deepening	Some deepening	Much deepening

[To avoid unnecessary repetitions the lists in questions 1 and 3 are omitted here, but they are the same as the list in Table 1].

The questionnaire was administered anonymously so that more valid answers could be obtained from the teachers. The data were analysed following the phenomenographic method outlined by Marton (1988), who argues that it is a research method to investigate the distinctly different ways in which people conceptualise phenomena: “An effort is made to uncover all the understandings people have of specific phenomena and to sort them into conceptual categories” (p. 145). Therefore, the analysis of the open-ended question 2 was based on an extensive categorisation of all teachers’ responses. After the first reading or even during it, certain similarities in the responses were identified. Those similarities constituted the initial categories and were represented with a string of three letters which were related to their meaning. For example: [HEL] was used to represent symbolically the category “I would need help for teaching these contents”. Altogether, each response was read about six times. Normally after a third reading, the borderlines of most of the categories were defined. Each of the subsequent readings was considered a check of the categories, as the readings were done on different days. The analysis was performed with the help of a Microsoft Access data base. The queries performed in the data base also provided a final check of the categorisation. The result of each query was a table combining all responses which belonged

to a specific category. In that way the responses related to a single category could be read together to check if any of them did not follow the main pattern behind the category. It is acknowledged that it was not the ideal validation check and that an appropriate checking would involve asking a colleague to generate another set of categories and checking the convergence of the two sets. However, there was nobody available to perform such a job.

SOME RESULTS

The questionnaire was administered just before starting a four hours workshop in Brasília and five other cities near Brasília and it was answered by a total of 305 primary school teachers. I explained to the whole class that their answers would help me select the contents for the next workshops with other teachers by focussing on the contents they said they need more deepening. Each questionnaire received a number code from 1 to 305 to order them. Table 1 presents the frequency of answers to questions 1 and 3. About two thirds of the teachers ($n = 199$) who answered question 1 marked one or more contents they would omit. It can be noticed that most of the contents they would omit are related to rational concepts and operations and geometry and measurement. Question 3 was not answered by 28 teachers.

TABLE 1 FREQUENCY OF ANSWERS TO THE CLOSED QUESTIONS 1 AND 3

X = frequency of teachers who said they would not teach the contents listed in question 1;

N, S and M = frequencies of teachers who said they would need No (N), Some (S) or Much (M) deepening in the contents listed in question 3;

T = frequency of teachers who responded question 3 ($T = N+S+M$); and

$D = (S+M) 100/T$ = percentage of teachers who thought they needed some or much deepening.

	Mathematics Content	X	N	S	M	S+M	T	D%
(1)	place value ideas (units, tens, hundreds, etc.)	8	98	71	48	119	217	54,8
(2)	addition of natural numbers	3	119	51	41	92	211	43,6
(3)	subtraction of natural numbers	3	108	61	44	105	213	49,3
(4)	multiplication of natural numbers	3	98	70	46	116	214	54,2
(5)	division of natural numbers	4	82	82	51	133	215	61,9
(6)	proper fractions	10	62	103	47	150	212	70,8
(7)	improper fractions	39	49	113	59	172	221	77,8
(8)	mixed numbers	32	48	111	57	168	216	77,8
(9)	equivalence of fractions	41	44	112	65	177	221	80,1
(10)	addition of fractions with same denominator	15	75	96	41	137	212	64,6
(11)	addition of fractions with different denominators	33	56	97	60	157	213	73,7
(12)	subtraction of fractions with same denominator	15	75	91	45	136	211	64,5

(13)	subtraction of fractions with different denominators	36	57	97	61	158	215	73,5
(14)	multiplication of fractions	25	64	91	57	148	212	69,8
(15)	division of fractions	30	57	97	57	154	211	73,0
(16)	decimal numbers	21	55	98	64	162	217	74,7
(17)	addition of decimals	15	59	100	56	156	215	72,6
(18)	subtraction of decimals	17	60	102	56	158	218	72,5
(19)	multiplication of decimals	21	57	104	61	165	222	74,3
(20)	division of decimals	36	41	114	75	189	230	82,2
(21)	measurement of length	29	46	110	62	172	218	78,9
(22)	measurement of area	50	35	113	88	201	236	85,2
(23)	measurement of volume	57	36	111	87	198	234	84,6
(24)	classification of some plane shapes (polygons, circles, trapeziums, parallelograms, rhombus, quadrilaterals, pentagons, hexagons, etc.)	82	32	101	108	209	241	86,7
(25)	area of some simple plane shapes (rectangles, parallelograms, triangles, etc.)	138	31	98	126	224	255	87,8
(26)	volume of prisms with rectangular faces such as a cereal box.	158	29	88	142	230	259	88,8

With the exception of addition and subtraction of natural numbers, all the other contents received more than 50% of the answers of the type [S] or [M]. This means that these teachers thought they needed some deepening in their mathematical knowledge (the sum of answers in the categories “Some deepening” and “Much deepening”, that is, S+M). This result seems to indicate that many teachers do not feel confident about their mathematical knowledge to teach most of the primary school mathematics curriculum. With the exception of addition and subtraction of fractions with the same denominator and multiplication of fractions, all the contents concerning fractions and decimals received more than 70% of answers which involved the need for some deepening (S+M). The geometry and measurement contents were the ones which the teachers seemed to feel less confident, with more than 80% of answers asking for some improvement. The only exception is measurement of length (78.9%). This result may be another consequence of new mathematics movement which avoided Euclidian Geometry (Kline, 1973). In question 1, one teacher only marked contents about geometry, area and volume (items 23, 24, 25 and 26) and explained in question 2 that school teachers tend to omit these contents:

[Teacher 208] Because these contents generally are never taught at school. The teachers always leave them for the last two months of the school year, that is, to the end of the school year. So

there is never enough time to work with them. Later, when we must know them in order to teach, we know nothing about them.

Rational numbers, geometry and measurement were also the topics with highest avoidance responses (category [X]). These results are similar to the ones obtained in the first stage of the reconnaissance stage (Amato, 2004a). So I continued to place a great emphasis in the teaching of these topics in the action steps of this research, with trainees and in workshops and in-service courses for practising teachers. The activities for rational numbers, geometry and measurement were started at the first week of the semester, and they continued until the last day of each semester. The idea was to provide trainees with several opportunities for revising past content through activities involving extensions of the content and relationships with other contents. The number of activities for place value and operations with natural numbers alone was greatly reduced, but there were still many activities about operations with rational numbers which included a natural number part. Through operations with mixed numbers and decimals (e.g., $35\frac{3}{4}+26\frac{1}{4}$ or $24.75-12.53$) with the use of versatile representations, trainees experienced further activities related to operations with natural numbers and had the opportunity to make important relationships between operations with natural numbers and operations with fractions and decimals. These changes proved to be quite effective in helping other classes of trainees overcome their difficulties in relearning mathematics conceptually within the time available (Amato, 2006).

Table 2 presents the categories and frequency of answers to question 2. This open question was not answered by 114 teachers (categories [NOA] + [NOB]). Some teachers marked the contents they would omit in question 1, but did not write any reasons for omitting them in question 2 (category [NOA]). The number of teachers who said they would not omit any content was 106 (categories [NOT] + [NOB] + [NOC]). The most frequently reported reason for omitting some content was related to their perceptions concerning their subject matter knowledge (category [SMK]) (Shulman, 1986), with 46 negative remarks. There was only one positive remark about this category [SMK+]: [Teacher 180] “No content would be omitted as I love mathematics and feel confident about it. I am successful in helping my students to learn by basing my teaching on the concrete”.

TABLE 2 CATEGORIES (C) AND FREQUENCY OF ANSWERS (F) TO QUESTION 2

C	EXAMPLE OF RESPONSES	F
SMK	I do (do not) master them, or I do (do not) feel confident about them. (Positive remarks = 1, negative remarks = 46)	47
PRA	They are not relevant to the practical everyday life of the students.	42
PCK	I do not know how to teach them.	31

	I do not feel confident about teaching them.	
HEL	I would need help for teaching these contents.	23
BAS	They are not fundamental (basic, important) contents for grades 1 to 4 (or for younger students).	16
DIF	I consider these contents difficult for my students.	14
LAT	They should be taught in later grades.	14
ATT	I do (do not) like (enjoy) them, or I do (do not) find them interesting. (positive attitude = 2, negative attitude = 5)	7
TIM	There is no time to teach them.	5
PRE	The students do not have the pre-requisites to learn them.	4
MAT	The students are not mature enough to learn them.	3
SUP	I would work only superficially with them.	2
INT	The students are the ones who decide when to teach them.	1
NOT	I would not omit any of these contents. 17 teachers added that “All these contents are important” (subcategory [IMP]).	37
NOA	The teacher did not answer question 2, but marked some contents in question 1.	47
NOB	The teacher did not answer question 2 and did not mark any content in question 1.	67
NOC	The teacher answered question 2, but did not mark any content in question 1.	2

The second most frequent category was practical relevance to the everyday life of the students [PRA]. Their perceptions concerning their pedagogical content knowledge [PCK] (Shulman, 1986) was the third more frequent category. In the category [HEL] the teachers mentioned that they would need help or to improve their knowledge for teaching the omitted contents, but they did not specify the type of help or knowledge needed. That is, they did not specify whether it was related to categories [SMK] or [PCK]. When the frequency of answers more directly related to teachers' professional knowledge (professional categories [SMK] [PCK] and [HEL]) were combined, 101 remarks were totalised, which represents about 50% of all the remarks related to avoiding the teaching of some contents (206 remarks). Brasília and its surrounding cities (Federal District) are atypical as the majority of their primary school teachers have completed a college course. Some teachers in less developed regions of Brazil have not done any teacher education course. Therefore, the situation could be even worse in these regions.

These “professional” reasons provided by teachers’ are very different from the reasons often provided by curriculum developers to exclude particular contents from the school curriculum. With the exception of the category “relevance to the practical everyday life of the students” [PRA], which

had a high frequency (42 remarks), the least frequent categories were the ones more compatible with the reasons proposed by curriculum developers ([BAS] [DIF] [LAT] [TIM] [PRE] [MAT] [SUP] [INT]). The affective category [ATT] also had a low frequency, and this may indicate that the avoidance problem is more related to professional knowledge, and less related to attitudes. Yet in the case of primary school teachers there is some evidence about a relationship between the affective and cognitive domains (Amato, 2004c). Research also tends to show that primary school trainees and teachers tend to blame procedural teaching for their negative attitudes to mathematics (e.g., Haylock, 1995 and Brown et al., 1997).

DISCUSSION

This study shows that low time allocation to the teaching of mathematics may be related to teachers' professional knowledge which I consider to be a combination of SMK and PCK. Some educational researchers may consider these results uninteresting. However, research is not supposed to report only interesting results. School students will be those most affected by teachers' low professional knowledge and avoidance to teach mathematics. I think that teacher trainees need 'a map' combining basic aspects of SMK and PCK. This map should be an integral part of their initial professional knowledge, and it should be acquired in pre-service teacher education and before they have their first teaching experiences at school. Yet they should have the autonomy to use the map according to their needs and constraints imposed by schools. If the curriculum is large and there are too many students in their classes, they can teach in a more direct way by using exposition, something such as using the map to take the underground and getting straight to the main aim. If there is enough time, they can use investigations, similarly to taking a nice walk without worrying about the time, enjoying the new places and scenery, and even enjoying getting lost and seeing unpredicted places. In this case, there is no need to look at the map very often.

Having to learn SMK and PCK from teaching mathematics does not allow teachers to concentrate on other professional aspects for successful learning of their students. One of these aspects involves identifying the level of understanding of individual students, targeting them with progressive activities and questions which lead to reflection and to a deeper conceptual understanding. Another important aspect is responding to students' curiosity and unanticipated questions about contents they will be learning in later grades. Answering these unanticipated questions has the potential to expose students to important connections between what they are learning and what they will be learning in the future. Teachers should also be able to plan their teaching in such a way as to anticipate the acquisition of difficult mathematics concepts.

Taking into consideration that the avoidance problem has already been mentioned in the literature about teachers' attitudes to mathematics (e.g., Bromme and Brophy, 1986), it is suggested that teacher educators and researchers from developed countries should also investigate its existence. Negative attitudes to mathematics are still a serious educational issue, even in developed countries. Di Martino and Morselli (2006) report that some European countries have faced a decline

in the enrolment of students in science university degrees. They suggest that such decline is related to negative attitudes to the subject. This is an even greater problem for developing countries such as Brazil, which do not have the resources to employ scientists and engineers from abroad. Todeschini (2007) reports that the number of Brazilian students getting engineering degrees every year is very low compared to the needs of the country. The first action mentioned to ameliorate the problem is to improve the quality of school teaching in mathematics and science. Therefore, it is urgent to improve the mathematical and pedagogical knowledge of primary school teachers, since they teach the foundations of several important contents for students' learning of further mathematics.

In the past I had opted for teaching less content pre-service teachers education, to be able to do it in a more detailed way. After the first reconnaissance stage reported in Amato (2004a) and this new reconnaissance stage of the research, it became even more clear to me that trainees need to: (i) have some initial knowledge about how to teach the whole curriculum and not just a sample of it (i.e., the map mentioned before), (ii) learn how to teach each content from very informal pre-school stages to final stages in formalisation and (iii) acquire enough conceptual understanding to be able to teach the curriculum as a coherent and organic whole, by emphasising the relationships among concepts and operations. As trainees will be teaching different grades in a very near future, I continue to make an effort in improving their conceptual understanding of a good range of mathematical content in the primary school curriculum.

In the particular task of deciding about what contents to focus and what to avoid, I think that teachers' decisions should be based on (a) a cultural perspective, that is, how necessary the content is to their students' everyday life, and (b) a sequential perspective, that is, how important the content is to the learning of future related contents in mathematics and in other subjects such as geography, physics and biology (prerequisite learning). In the case of more difficult concepts and operations such as fractions, it should also be based on teachers' knowledge about the time students need to learn them as a coherent and organic whole.

According to Shulman (1986), curricular knowledge involves familiarity with "the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10). This ability to detect the "contraindications" mentioned by Shulman is extremely important to teachers' professionalism and autonomy. It involves teachers' ability to analyse innovations proposed by curriculum reformers with respect to: (a) their viability within the time and physical constraints presented in their classrooms and (b) their long term effects to students' learning and achievement. I acknowledge that surveys measuring students' achievement appear to have many drawbacks, and in many cases use tests that mainly measure procedural understanding. Yet it is quite difficult for me to accept the fact that many Brazilian students cannot cope with the very naive word problems presented in those tests, when most teacher educators world-wide are now advising the teaching of more sophisticated mathematics problems.

Not all curriculum innovations which were believed to be related to good teaching were

thought to have improved mathematical achievement. The application of behaviourist theories to the primary schools curriculum in the 1920s is considered to be an example of an unsuccessful innovation (e.g., Romberg and Carpenter, 1986). The same can be said about some of the educational implications drawn from Piaget's theory (e.g., Romberg and Carpenter, 1986), and the new mathematics movement in the 60's (Kline, 1973). On the other hand, "innovations found successful in practice were opposed by arm-chair theorists on a priori grounds" (Smith, 1965, p. 8). In the literature the ability to analyse innovations can be noticed in the teachers mentioned by Delpit (1987), Leinhardt (1988) and Pehkonen (2007). This is a deeper form of curricular knowledge not well explored in research and in teacher education. Very recently it has been recommended that the:

Training of teachers should include sufficient coverage of the scientific method so that teachers are able to critically evaluate the evidence for proposed pedagogical approaches and to be informed consumers of the scientific literature (who can keep up with advances in scientific knowledge after graduation from training programs). (Geary et al., 2008, p. 45)

Learning about the scientific method may help, but I doubt teachers will have the time to be consumers of the literature in educational research. Besides most curriculum developers and mathematicians probably know well the scientific method, but this knowledge does not seem to have prevented them from proposing reforms such as the new mathematics movement in the 60's. I prefer the idea of developing teachers' conceptual understanding of mathematics and knowledge of representations. I attribute my present ability to evaluate the long term effects of reforms to my conceptual understanding of mathematics and to my capacity to resist pressure from others. Knowing well the importance of fractions to students' learning of future topics, such as algebra (Geary et al., 2008) and to science (Penrose, 2005), prevents me from downplaying the teaching of fractions in teacher education, and from proposing any school reforms that does not have a strong focus on fractions. Similarly to Chinese teachers (Ma, 1999), most of my conceptual understanding of mathematics was acquired through "studying teaching materials intensively" (p. 130), especially by reading American and English textbooks and teachers' journals of education such as *The Arithmetic Teacher* and *The Mathematics Teacher*.

Teachers' attention to the opinions of their main stakeholders may also help them critically evaluate curricular innovations. The primary school teachers interviewed by Pehkonen (2007) are considered reflective, highly educated and dedicated professionals who "emphasized that they were autonomous. They are able to make they own decisions and do not follow 'official' schedules." (p. 65). However, they usually decide to follow a good mathematics textbook without omitting any tasks, as such omissions could harm their students' learning and affect their already heavy workload. They also attempt to satisfy their main stakeholders (students, parents, administrators and other teachers) by avoiding reforms that tend to restrict students' opportunity to learn. In particular,

parents are the ones who will later have to deal with the consequences of their sons and daughters low achievement at school.

Teachers are sometimes accused of being the greatest factor inhibiting school reform or of implementing reforms in their own restricted ways. Their beliefs are challenged and said to be traditional. I think that resisting certain reforms is a very healthy form of teachers' professionalism and autonomy, rather than the result of traditional beliefs. When reforms fail to provide the results predicted, teachers (and not reformers) have to face the accusations of incompetence on the newspapers. The concept of teachers' forced autonomy (Skott, 2004) is useful in the context of school reform, but I also would like to suggest the idea of false autonomy. Teachers are not totally autonomous. Carr and Kemmis (1986) point out that understanding of social action requires "understanding the perspective of others involved in and affected by the action" (p. 199). Teachers are social workers and are, therefore, accountable to those affected by their actions. When society notices certain reform failures, usually extreme actions are taken against teachers and they are denied even the most basic forms of autonomy. Instead of spending more money on teacher preparation, money is spent on teachers' evaluation and on testing students and ranking schools according to their students' achievement.

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