

## **Implementing reform pedagogies in a mathematics classroom: Easier said than done.**

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### **Abstract:**

In this paper I consider the case of a teacher who tries to implement her interpretation of the learner-centred principles underlying the new South African reform curriculum, C2005. Data from formal and informal interviews interviews, classroom observations and field notes are used to present a vignette from one lesson. A description of a lesson is presented which describes the teachers' attempts to involve her learners in order to understand some of the connections behind the number system, instead of just telling them statements. The analysis reveals that the teacher's attempts at helping learners make sense of numbers, is actually harmful because of the inconsistencies and contradictions emanating from the teacher.

### **Introduction:**

The research field seeking to provide illumination to the question, "What knowledge do mathematics teachers need to teach mathematics?", is an ever widening one (Ball, 1991; Ball and Bass, (2000) Even & Tirosh, 1995; Adler, 2005; Brodie, 2001; Brodie, 2004; Prawat, 1989; Shulman, 1986). Shulman (1986, p.9) distinguishes between two kinds of understanding of the subject matter that teachers need to have- knowing "that" and knowing "why" :

We expect that the subject-matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major. The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so.

Prawat (1989, p. 320) expands on this to explain that pedagogical content knowledge (Shulman's term) includes:

...vital things like understanding how particular ideas will be constructed by students- what sorts of preconceptions or misconceptions might have to be dealt with when introducing new material, which forms of representation are most useful for getting certain ideas across, ....Which are the most important ideas for students to understand, and how can they be best organized and sequenced to maximize student understanding?

Ball (1991) feels strongly that mathematics teachers need to know *of* the mathematics and *about* the mathematics and that while tacit knowledge may serve one well personally, explicit understanding is necessary for teaching. Brodie (2004) comments that a situative perspective sees knowledge as an important resource in social practice, with knowledge and practice constituting and constraining each other in complex ways. She states that teachers' actions in the classroom are an important part of their reasoning and the relationship between knowledge and practice in mathematics cannot be ignored.

... teachers' mathematical knowledge and their mathematics teaching practices are mutually constitutive; that is each one shapes, creates and constrains each other, while remaining distinct analytical objects. (p. 72)

Within mathematics education, any discussion of classroom practice will be incomplete without a reference to the role of the discourse tradition in the classroom. Cobb, Yackel and Wood (1993) have identified two distinct mathematics classroom traditions- the "school mathematics tradition" (which follows an elicitation pattern where the solution is the driving force – these interactions can be described by a three part process comprised of teacher initiation, learner reply and teacher evaluation [IRE]) and the "inquiry mathematics tradition" (resembling a discussion pattern in which learner explanations are the driving force). Discourse in the inquiry mathematics tradition involves interaction which begins with the teacher asking information-seeking questions that require learners to explain how they interpreted and solved tasks(Cobb et al, 1993).

Most curriculum reform initiatives advocate the use of the inquiry tradition as being important for teachers and students to engage in, in order to foster students' understanding of mathematics (Borko, Peressini, Romagnano, Knuth, Yorker, Wooley,

Hovermill, & Masarik, 2000; Peressini, Borko, Romagnano, Knuth, and Willis (2004); Peressini & Knuth, 1998; Brodie, 2004; Cobb et al, 1993). Brodie (2006) comments that many teachers are faced with the dilemma of validating learners' responses and developing their thinking. South African studies (Mattson & Harley, 2004; Brodie, Lelliot & Davis, 2002; Taylor & Vinjevold, 1999) show that many teachers choose to focus on learners contributions at the expense of the mathematics. Many classroom have been observed where most attention is focussed on carrying out group work or increased learner talk, instead of engaging with learner ideas to take their mathematics thinking forward.

In South Africa the new curriculum – C2005 – was introduced in 1997 (DoE, 1997). The philosophy and design of C2005 reflects progressive principles which have been well received worldwide, such as child-centred learning, development of critical thinking and continuous assessment (Rogan & Grayson, 2003). The C2005 image of a teacher is one who is a facilitator of learning and not the central person in the classroom. The idealised classroom is one where learners work in groups, are responsible for their own learning and are motivated by constant feedback and affirmation. The C2005 teacher will integrate the content and make real life connections wherever possible (DoE, 1997, p. 6-7). Brodie (2006, p.15) cites studies from the US and the UK which show that teaching aligned with reform pedagogies produces “better and more equitable achievement, learning and identities in mathematics”. However she notes that there is little similar evidence in South African schools.

This paper seeks to explore the flip side of implementing reform pedagogic ideas: How does a teacher with extremely weak mathematical knowledge put some of her interpretations of reform curricula into practice? I present a case study of Nonku who is keen to put into practice the pedagogic principles underpinning the new South African curriculum, but whose personal mathematics knowledge lets her down.

### **Case study of Nonku**

It is important to note that the data reported in this paper was collected for a larger study which sought to explore the professional knowledge of grade 9 teachers (Bansilal, 2006). For the larger study, data in the form of 19 classroom observations of four teachers based at three schools was collected and analysed. The setting for the study was the mediation, by the grade 9 teachers, of the recently implemented external assessment tool-Common Tasks for Assessment (CTA) Nonku was one of the participants in the study and some of the insights gained in the larger study are reported here. In addition to the classroom observations, data from interviews with the four participant teachers and their learners was collected.

I now present the case study of Nonku which has been constructed by using an excerpt of one lesson, excerpts from two interviews, and notes from informal conversations. I observed 4 of Nonku's lessons altogether. The description is organized under 3 headings: Nonku's views about learning and teaching mathematics, Nonku's situation at school and Nonku's teaching.

*Nonku's views about learning and teaching mathematics:*

During all my interactions I was struck by Nonku's' enthusiasm for the curriculum principles. She verbalised many of the progressive principles underpinning the curriculum. In her interview, Nonku made a comment about learning:

We need to find out where we are now. Where are you now? How did you do?"...for the children it is important to construct their own knowledge...[A]nd then must give them chances to come to the front to express their views... ... We as teachers we need to present the subject in a way that is interesting to the pupils because if we got this tendency to go to the class and tell them that mathematics is difficult then what image are we portraying to the kids? We must present mathematics in such a way that the kids will be able to see that it is important, it is interesting and is meaningful in their lives...we must motivate them. They must be observant about each and everything they come across in their lives. They must look at the distance they take from home to school...they must be able to see that mathematics is alive and isn't abstract, and we as teachers, in tests and exams in whatever assessment we are doing we must involve the real life.

These comments show that Nonku believed in the importance of allowing learners to construct their own knowledge, finding out where her learners were in their understanding and finding ways that she could use lead them to better understandings. Nonku felt that teachers needed to find opportunities to teach mathematics in a way which linked to the real life relevance of mathematics. These verbalizations are in line with views espoused by the new South African curriculum (C2005) about the role of the teacher (DoE, 1997).

Her desire to move towards a learner-centred class is confirmed by the following extract from the interview, where she spoke about how important it was for teachers to give learners the opportunity to express their own views.

Sometimes you can feel perhaps you direct them in a certain way that is needed by them and I don't like that one. I think when designing the task [CTA] they must be open. They must allow the children to be free to like the subject. They must express their views freely and then must give them chances to come to the front to express their views even to the groups."

This makes it clear that she wanted to involve her learners in her lessons and that she encouraged learners' original contributions. These views seem to be aligned with the "inquiry mathematics tradition" (Cobb et al. 1993). The interview with Nonku, conveyed an image of a teacher who had positive perceptions of the South African reform curriculum (C2005).

*Nonku's situation at school:*

Nonku was sent to the school as part of a rationalisation exercise by the provincial Department of Education a few years previously. During this exercise, schools which were identified as having too many teachers (based on a specific teacher student ratio), were asked to identify teachers they considered as excess. These excess teachers were

then sent to other schools which had been identified as having too high a teacher: student ratio. Her displacement is ironic, because in that exercise it was mainly the black schools that had teacher student ratios which were too high, and the displacement of teachers was usually to these black schools. For Nonku, her displacement was from a former black, under resourced school to a well resourced ex Model C school.

Nonku was not comfortable at the school although she did whatever was asked of her. In the current year, she was not allocated fixed classes to teach. This meant that when mathematics teachers took leave or were sick, she taught their classes. Her itinerant situation meant that she had to often prepare her lessons at short notice. This added to her daily workload. During my period of observation, she often spoke about how late she worked in the evenings in order to complete her marking and to prepare her lessons for the next day. In fact the class that Nonku taught during my period of observation was one that was assigned to a young intern who was away studying for his examinations

*Nonku's teaching:*

As part of the curriculum reform process in South Africa, a new assessment programme was introduced in 2002 at the exit level of Grade 9. The Common Tasks for Assessment (CTA) is an externally mandated assessment tool which teachers are expected to manage in their Grade 9 classes and consists of a series of performance based tasks which have to be completed over a fixed period of time. During the mediation of Section A of the CTA, teachers are allowed to intervene to “help their learner succeed”, (DoE, 2002, p.12). a description of part of one of the four “CTA” lessons that Nonku mediated with her learners, is presented below,

As I watched this lesson, I experienced mixed emotions of sympathy, disappointment and dismay. Sympathy for Nonku, because the learners did not show her any respect. Disappointment with the learners because of their attitude toward this older teacher. A few learners made fun of her style of speaking e.g. she punctuated each of her phrases with the word “what”, in the way one might use the expression “er” or “umm”. The word “what” was not used as a question directed to her learners. As I watched I was also filled

with an increasing sense of dismay because of the way she approached some mathematics concepts. I found her approach and explanations incomprehensible most of the time, and often incorrect.

The one issue was Nonku's weak knowledge of mathematics. Another contributing factor was Nonku's inflexibility around accepting answers different from what she wanted- this was in marked contrast to what she had espoused in her initial interview with me. Nonku tried to pose questions often enough to her learners, but was unable to use their responses meaningfully. A third factor contributing to the uncomfortable classroom environment was Nonku's explanations – she struggled to explain clearly what was in her mind. Her learners often did not make any sense out of what she was saying. The irony was that Nonku by posing questions to her learners believed that she was involving them and perhaps believed that she was trying to find out where they were in their thinking.

The CTA activity under discussion was:

*Bongani, a maths enthusiast working for Work4U, challenged the rest of the staff and you to solve the following problems.*

*3.1 Write an expression for the area of one runway if the width is  $x$  metres and the length is  $y$  metres. (1)*

*3.2 If the width and the length of the runway of an airfield in (3.1) above, is in the ratio 1:20, express your answer (3.1) in terms of  $x$  (6)*

*3.3 If the airfield has two identical and perpendicular runways, write down an expression for the total area in terms of  $x$ . Simplify your answer. (6)*

*3.4 If the total area of the airfield is  $97\,500\text{ m}^2$ , determine the width if the runway using the expression in (3.3). Motivate your answer. (6)*

*3.5 Why do you think the two runways intersect each other and are not parallel? (2)*

*[21]*

Nonku first wrote the number  $\frac{1}{2}$  on the board. She then posed the question: “I got this, who can read this for us. Hands up please.” A student answered “one over two”. She continued:

- 1 T: What kind of a, please don't talk now, I need your heads. What kind of a number is this one, please listen. What kind of a number is this?
- 2 S: Fraction
- 3 T: It is a fraction. Is this fraction falls under the natural numbers? or rational numbers? Or...?
- 4 S: real
- 5 T: I am not talking about what the real numbers are at the moment. It falls under natural numbers or rational numbers or irrational numbers. I am looking at these only. I am waiting for them. Okay I have got half which is a fraction. Do this first. Does half fall under natural numbers or rational numbers or irrational numbers? Yes?
- 6 S: I think irrational
- 7 T: Irrational number. Irrational number, remember is a number that recurs when you put it in your calculator, okay. If you could, just don't laugh, please don't laugh. [This comment was an admonishment to the learners that they should stop laughing and pay attention.]
- 8 S: Sorry, we did not know that. I did not know that.
- 9 T: Just put one divided by two in your calculator and tell me as it is you get. Put it there. Saying one divided by two, what do you get?
- 10 S: 0,5
- 11 T: 0,5. Now this number you get, does this number recurs? No. Doesn't fall under irrational number, I am sorry, what kind of number does it fall under?
- 12 S: Irrational
- 13 T: Irrational number. Quiet please. It doesn't fall under irrational number because it doesn't recur when you put it in your calculator. Okay. Does it falls under natural numbers?
- 14 S: No ma'am.
- 15 T: No why. Hand up please. Why do you say that half doesn't fall under the set of natural number?
- 16 S: Not a whole number.
- 17 T: It is not a whole number. What is the first element of natural numbers? Hands up.
- 18 S: One

- 19 T: One good, therefore we can say that half doesn't fall under the set of natural numbers. Which set are we left with?
- 20 S: Rational numbers
- 21 T: Rational numbers. Therefore we say  $\frac{1}{2}$  is a rational number.

As mentioned already, this was a very uncomfortable lesson to watch because of the disrespect shown by the learners towards the teacher. The learners' attitudes caused her to become flustered which affected the coherence of her statements or explanations. However the excerpt show that clearly Nonku was trying to put her interpretation of a learner-centred class into practice, by posing questions to them regularly.

I will try to disentangle some of Nonku's understandings about the real numbers as conveyed by her explanations to the learners. Firstly I will discuss elements of her concept image (Tall & Vinner, 1981) of the properties of the real number system. The term concept image refers to the "total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (Tall & Vinner, 1981, p.152). Nonku's understanding of the properties of the sets of rational, irrational and natural numbers are revealed in the excerpt. Note that in line 3, Nonku offers a choice of  $\frac{1}{2}$  belonging to the set of natural numbers or the set of rational numbers, as if she saw the sets as being exclusive. When the learner proffered the answer "real" Nonku did not accept it because it was not in her scheme that she was trying to present. In line 5 again, her comments reveal that she saw the set of natural, rational and irrational numbers as being separate and exclusive, because she expected the  $\frac{1}{2}$  to belong to only one of the three sets.

Secondly, line 7 reveals her view that an irrational number is one which "recurs when you put it in the calculator". She is clearly confused about rational and irrational numbers. A recurring decimal number is a rational number because it can be expressed as a fraction of two integers-e.g.  $\frac{1}{3}$ . However not all rational numbers form recurring decimals. Nonku then applies her classification rule to describe  $\frac{1}{2}$  as a rational number because it does not recur when expressed as a decimal fraction (line 13). It is interesting

to note that Nonku involved her learners in the classification exercise by asking them to find 1 divided by 2, by using the calculator and by posing questions to them.

The excerpt shows that Nonku used her concept image of the three sets of numbers (natural, rational and irrational) as being separate and disconnected, to shape the direction of the discussion. She noted that  $\frac{1}{2}$  was not a natural number (because it was not a whole number), and then  $\frac{1}{2}$  was not an irrational number (because it did not recur when expressed in decimal form). So, by eliminating the two possibilities of natural and irrational, she was left with  $\frac{1}{2}$  being a rational number.

However her concept image of the real number system also contained contradictory and irreconcilable understandings. The next excerpt from her lesson reveals these contradictions as she got to the point that she wanted to make, which was that any common fraction can be represented as a ratio.

22 T: What it means is that these numbers can be written in a ratio form. If I don't want to say 1 divided by 2, I can also say 1 is to 2. Okay, that is a ratio and fraction can be written in a ratio form because if I don't want to say half that is or 1 divided by 2, I can also say 1 is to 2. Understood? Therefore all fractions that what? recur, they fall under what? rational numbers.

In line 22 reveals that Nonku understands recurring decimals as being rational numbers, which they are. However, this understanding is contrary to her previous statement in Line 7 where she said that an irrational number is one which recurs. Nonku continued with her explanations of rational numbers:

23 T: Also the natural numbers it can be represented in a ratio form because I can say 5 over 1 which also can be 5 is to 1. Please listen, because in activity 3 you are going to make use of that. Now that you know what a ratio is, a number that can be expressed in this format now in 3.4 you can see that 1:20 can be the same as 1 divided by 2. 1:20 is the same as 1 divided by 20.

The other point of contradiction emerges from line 23. When Nonku says "... natural numbers it can be represented in a ratio form because I can say 5 over 1", this demonstrates that she understood the set of natural numbers as lying within the set of those numbers which can be expressed as fractions (rational numbers). Interestingly, this was in contrast to the impression emerging from the previous excerpt which was that they were mutually exclusive. So there are two anomalies between what she said at different times during the lesson. In the one instance she implied that a common fraction which was a recurring decimal was irrational. In the second instance she said that such a number was rational. In the one instance her explanations showed that she understood the sets of natural, rational and irrational numbers as being separate and in another instance she showed that she saw the natural numbers as being contained in the set of numbers which could be expressed as common fractions, which is true.

A second point to note is the circular argument used. Nonku started with the common fraction  $\frac{1}{2}$  and used a false notion to establish that the number was rational (which ironically it was, by virtue of being able to be expressed as a common fraction). However for the problem at hand all that was needed to relate the fraction to a ratio was the common fraction representation ( $\frac{1}{2}$ ).

Nonku continued with the lesson by going on to provide another circular argument concerning the ratio 1:20 and its fraction representation of  $\frac{1}{20}$  (Bansilal, 2006).

### **Discussion:**

The case study has provided a picture of a teacher who is keen to put C2005 principles into practice. The lesson excerpt shows that she tried to involve her learners in the discussion by posing questions to them instead of just giving them the answer. She has also involved them by getting them to verify her assertions by using a calculator. It is also clear that she tried to avoid making mathematical inferences without justifying them to her learners - she tried to lead them (using an incorrect justification) to understand that  $\frac{1}{2}$  was a common fraction. However Nonku did not display a robust knowledge "of" mathematics and neither did she display a sound knowledge "about" mathematics.

(Ball,1991). She was unsure of which direction to proceed and did not convey a clear sense of what she was trying to do. Consequently the circular arguments that she presented resulted in incoherent explanations.

I argue that she would have done less harm to her learners understanding of rational numbers, had she just started with line 22 where she told them that  $\frac{1}{2}$  can also be represented as the ratio 1:2. In trying to facilitate what she thought was a learner-centred discussion, she got entangled in her own incorrect views of the number system. The damage to the learners' understanding of the number system would have been compounded by the contradictory understandings Nonku presented.

However Nonku's interview comments revealed her desire to align her practices to the ideas espoused by C2005. Being positive about the ideas and philosophy behind a curriculum is the easiest part of embracing curriculum reform. Prawat (1989) states that the teacher's task is to create conditions that allow students to develop knowledge structures that are both powerful and robust. He further states that the teacher's role is to foster discourse processes which promote conceptual understanding and higher order thinking. To facilitate this type of discourse, Cobb et al (1993) state that a teacher must be skilful at posing questions, listening carefully to students' ideas, rephrasing students' explanations in terms that are mathematically more sophisticated, deciding when to provide information, and orchestrating class discussions to ensure participation by all students. However Brodie (2004) comments that teachers who engage in forms of learner-centred practices such as group work, may not necessarily promote genuine mathematical engagement and thinking among their learners. Brodie points out that it may be possible for a teacher fronted lesson to engage learners in meaningful mathematical thinking, thus creating a learner-centred lesson. She argues that good, clear explanations by teachers, which link important concepts and which engage learners in reflecting about the mathematical concepts they are being taught should be seen as good mathematical teaching. Nonku failed on both counts-although she tried to foster meaningful discourse, she was constrained by her poor knowledge base. Neither were her explanations good nor clear. However she voiced all the positive intentions of the new

curriculum, which shows that embracing curriculum reform ideas is easier said than done.

One of the reasons for the contradictory understandings that she displayed is surely because she was stressed and uncomfortable, but it is not a sufficient reason. It seems like she displayed different elements of her understanding of the number system at different points, which may demonstrate that the different understandings co-existed as disconnected schemes of knowledge, being brought to bear at different times to explain different bits of mathematics.

Adler (2005) comments that the problem facing mathematics teachers currently in practice is that they don't know enough mathematics. In her analysis of assessment tasks which were prescribed by a selection of ACE programmes offered at different sites, Adler noted that there was a dominance of compressed rather than decompressed or unpacked mathematics. She uses the terms compressed mathematics to refer to the ability to demonstrate mastery of procedures and underlying concepts (which may not guarantee an underlying conceptual understanding). Adler uses Ball, Bass & Hills' (2004) work to define "unpacking" as one of the essential and distinctive features of "knowing mathematics for teaching".

As mathematics education researchers grapple with the question of what knowledge mathematics teachers need, Nonku difficulties in the classroom may help us consider certain issues. One of the challenges of designing curricula which are based on "unpacked mathematics" is to decide which aspects in the curricula do we consider most important so that the mathematics teacher education programme can foreground experiences in unpacking these aspects. Nonku's difficulties lay in a disconnected understanding of different representations of fractions, and an incomplete understanding of the number system, two concepts which most mathematics teacher educators take for granted that high school teachers will know. Is it possible to design a mathematics teacher education programme whose curriculum foregrounds the mathematical practices of "unpacking" as opposed to "compressing" that could train high school teachers like

Nonku, to gain an understanding of crucial and fundamental concepts. Furthermore can we design a programme which can support and guide Nonku to implement the practices associated with reform curricula, that Nonku is so keen to take on.

### **Concluding Remarks:**

There are many socio-political aspects that arise in this lesson which I did not unpack. My intention was to demonstrate, by using Nonku's laboured explanations, that curriculum reform practices are meaningless without knowledgeable mathematics teachers.

One of the key determinants of success of reform pedagogies is the teachers' facility with, and deep understanding of mathematics that will enable her to engage with learners' ideas to take their mathematical thinking forward. I used the case to illustrate the futility of aspiring towards a progressive and reform based pedagogy if the fundamentals of a sound knowledge base for the teacher is not in place. Teaching which is in line with curriculum reform makes greater demands on teachers' knowledge than the old content driven curricula. I have argued that a teacher without the necessary mathematics for teaching - knowledge, does more harm by trying to apply progressive pedagogic principles such as discourse of the inquiry tradition.

Furthermore, the paper was also intended to show that the concept of mathematics teachers' knowledge for teaching is confounded by many issues. It is pertinent to reflect on Adler's (2005) concluding questions: "...can [the unpacked mathematics ] be taught, and then will it lever up the benefits we would want: more effective mathematical preparation ....".

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