Middle School Mathematics Teachers' Knowledge for Teaching Percent

Kim Beswick, University of Tasmania Email: Kim.Beswick@utas.edu.au
Telephone: +61 3 6324 3167 Facsimile: +61 3 6324 3048

Abstract: Forty-two middle school mathematics teachers were presented with two mathematics problems, one requiring a percentage of an amount to be calculated and the other involving the interpretation of a pie chart showing percents. The teachers were firstly asked to provide responses, both appropriate and inappropriate, to each of the questions that they thought their students would supply, and to indicate which of these responses were appropriate. They were then asked to suggest how they would use each item in the classroom and, in particular, how they might respond to one of the inappropriate responses that they anticipated from their students. The results suggest that use of such items is an effective way to access teachers' knowledge of mathematics as well as of their students as learners.

There is broad agreement that teachers of mathematics need more than personal mathematical competence (Ball, 2000, cited in Chick, 2002). Teachers need to know the mathematics they teach in particular ways that have been described as, for example, connected (Ball, 1990) or profound (Ma, 1999). Shulman (1987) listed seven types of knowledge that he believed teachers need. Three of these were content knowledge, pedagogical content knowledge (PCK) which represented and melding of content and general pedagogical knowledge, and knowledge of students as learners (KSL). As its name suggests, PCK is closely related to content knowledge. Goulding, Rowland and Barber (2002) pointed out that the boundary between knowledge of mathematics content and PCK is not clear cut and, like Ball (1990) and Ma (1999), emphasised that it is the way in which the mathematics is known that is crucial. In relation to preservice teachers they said,

> Students who have multiple representations for mathematical ideas and whose mathematical knowledge is already richly linked will be able to draw upon these both in planning and in spontaneous teaching interactions. In such cases we would argue that the students' subject matter knowledge is ripe for exploitation and that in turn the experience of teaching will feed back into and enrich subject matter knowledge. (Goulding et al., 2002, p. 691)

PCK is also linked to teachers' knowledge of their students and, in particular, how they understand mathematics and come to understand mathematics. It includes knowing how to represent mathematical concepts in ways that are meaningful and useful to students in developing their understandings, and is closely related to such things as knowing the types of misconceptions that students may hold or develop and how these are influenced by instructional decisions. Hill, Rowan and Ball (2005, p. 373), in an extension of Shulman's work, have described "mathematical knowledge for teaching mathematics" as that used, "to carry out the work of teaching". They went on to illustrate this with a range of teaching tasks that require knowledge of mathematics, of students, and of pedagogies specific to the mathematics being taught.

Exploring teachers' knowledge for teaching mathematics is a sensitive matter which is perhaps why attempts to measure it have tended to rely on inferences from courses completed and self reported confidence levels. Studies underpinned by such views of teacher knowledge, failed to find significant links between the measures of teacher knowledge used and student achievement (Mewborn, 2001). Hill et al. (2005) adopted a more direct approach to measuring grade 1 and 3 teachers' knowledge for teaching mathematics that involved teachers responding to multiple choice items designed to tap their content knowledge and ability to diagnose student difficulties. This approach allowed them to demonstrate significant links between teachers' knowledge for teaching mathematics and their students' achievements. However, even this more sophisticated measure of teachers' knowledge provided no basis on which to claim a link between teachers' knowledge for teaching mathematics and either teachers' qualifications or experience.

The study described here went a step further than Hill et al. (2005) in terms of the approach taken to measuring the kinds of teachers' knowledge that is likely to be related to students' achievements in mathematics by asking teachers to construct their own responses to items. It used a teacher profiling instruments similar to that described by Watson, Beswick, Caney and Skalicky (2006). It was particularly concerned with teachers' PCK and KSL concerning percent, including representations of percents in pie charts. Although studies of various aspects of teachers' knowledge relevant to teaching mathematics have examined a range of mathematical topics, and difficulties have been reported in relation to rational numbers (Mewborn, 2001) there appears to be little investigation of teachers' percent understandings in particular. An exception is Chick's (2002) report that final year preservice primary and secondary teachers rated an item requiring them to provide three explanations for the equivalence of 3/8 and 37.5% the most difficult, both conceptually and computationally, of all items on an instrument designed to measure their understanding of middle school mathematics. This study illustrates an approach to measuring teachers' knowledge to teach mathematics and also provides insights into middle school teachers' knowledge to teach percent.

The study reported here addressed the following research questions.

1. What knowledge of mathematical content, their students as learners, and pedagogical content knowledge related to particular problems concerning percent and its representation in a pie chart do middle school teachers display?
2. What connections are there between teachers' knowledge of their students as learners and their pedagogical content knowledge, and between these aspects of their knowledge and their years of teaching experience and self reported exposure to tertiary mathematics?

Method

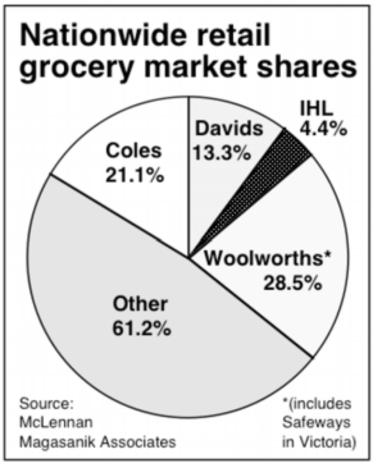
Participants. The 42 teachers in the study worked in nine rural schools in the Australian state of Tasmania. Three of the schools were primary (grades K-6), one was secondary (grades 7-10) and five catered for grades K-10. All of the teachers taught at least some mathematics to at least one middle years (grades 5-8) class.

Tasks. The tasks presented to teachers are shown in Figure 1.

What is 90% of 40?
Please explain your reasoning.

Figure 1a. Task 1, 90% of 40

Coles Myer accelerates retail purge



Explain the meaning of this pie chart.

Is there anything unusual about it?

Figure 1b. Task 2, the pie chart problem

In relation to each, teachers were asked to respond to the following two questions. The principle knowledge types these accessed were KSL (Questions 1), and PCK (Question 2).

1. What responses would you expect from your students? Write down some appropriate and inappropriate responses (use * to show appropriate responses).
2. How would/could you use this item in the classroom? For example, choose one of the inappropriate responses and explain how you would intervene.

Approximately one third of a page was provided for each response.

Coding and analysis. All responses (often several per teacher) to each question in relation to each item were firstly listed and clustered (Miles & Huberman, 1994) in order to provide an overview of the sorts of responses teachers' made. Then, the numbers of teachers who provided responses in each category were counted. Finally, the overall response of each teacher to each question in relation to each item was coded according to its degree of appropriateness and structural complexity. The rubrics used in this coding are shown in Table 1.

Teachers' numbers of years of teaching experience and their participation in tertiary level mathematics were also recorded. The former were divided into three groups: less than 5 years, 5-14 years, and more than 14 years. Tertiary mathematics experience was classified as none (9 teachers), one semester (11 teachers), one year (11 teachers), and more than one year (10 teachers).

Table 1: *Rubrics for Teacher Responses to Questions 1 and 2 for the problems in Figure 1*

Level	90% of 40, Question 1	90% of 40, Question 2
0	No response	No response
1	Response not addressing percent procedures for solution	Response not addressing the mathematical content of the problem
2	Response indicating either a correct strategy *or* an incorrect strategy	A single generic idea for the problem, e.g. use money, discuss fractions or relation to decimals
3	Response containing both appropriate and inappropriate approaches to the problem	Reference to 2 or more aspects of the solution without linking them
4	NA	Discussion including reference to part and whole concepts with specific examples
	Pie chart problem, Question 1	Pie chart problem, Question 2
0	No response	No response
1	Response not addressing the graph or its context	Response not addressing the mathematical content of the graph
2	Response indicating either the error in the graph *or* another graphical observation/ context question	A single generic idea for the problem, e.g. do percents
3	Response containing both salient and non-critical aspects of the graph	Reference to 2 or more ideas without linking them
4	NA	Discussion including reference to percents and wholes with specific examples

Results and Discussion: Research Question 1

The range of suggested student responses to 90% of 40 and the pie chart problem are shown in Tables 2 and 3. Most of the responses to 90% of 40 suggest that the teachers believed that their students would find this problem difficult. Callingham and Watson's (2004) finding that the level of difficulty of this item was such that only 3% to 15% of students in grades 5-8 might be expected to successfully answer it suggests that the teachers' concerns may have been well-founded. One third of the teachers who provided suggestion included responses that indicated that they believed that at least some of their students would not engage with the problem. Only 26 (62%) of the teachers provided a clear indication that they could solve the problem. Five teachers (12%) provided only responses that were correct answers, appropriate procedures, or sensible estimates. That is, they did not provide any evidence of understanding the possible difficulties and misconceptions that their students might have. Including the six who did not respond to this question, 26% of the teachers provided no indication of these aspects of KSL in relation to this question.

Table 2: *Suggested student responses to 90% of 40 (Question 1)*

Response type (from 36 teachers)	Frequency	No. of teachers with responses in this category
Comments suggesting failure to engage, e.g., "This is too hard", Don't know", "This is stupid"	16	12
Comments suggesting students' intention to attempt the problem, e.g., "What does percent mean?"	19	13
Other comments, e.g., "Wouldn't expect many appropriate responses"	3	3
Incorrect responses without explanation, e.g., 9, 90, 360, 3600, $39.15	11	8
Incorrect/incomplete responses with explanations, e.g., 90/100 x 40, 90 x 40, 90-40 = 50, 90 + 40 = 130, 40/90 x 100, 30 because 90% of 100 is 90 so take 10 off	22	17
Correct answer, 36, without explanation	7	7
Appropriate procedures, e.g., 1/100 of 40 is 0.4 so 90/100 of 40 is 0.4 x 90, 10% of 40 is 4 so 4 x 9 = 36, 10% of 40 is 4 so remainder is 36, 90% of 10 is 9 so 4 x 9 = 36	34	19
Estimates, e.g., 50% is 20 so it's between 20 and 40, 90% is a large part of 40, more than 35, bigger than 30 because 30 is 75% of 40	11	5
Total	123	

As shown in Table 3, teachers were concerned that their students would have difficulty understanding various aspects of the pie chart problem. These related to its literacy demands as well as its mathematical content. Seventeen teachers explicitly noted the error in the pie chart and a further four provided responses that suggested they were aware of it. This leaves 21 teachers (50%) whose recognition of the error is in doubt. The teachers who were clearly aware of the error provided the 24 (26%) of the total of 93 responses that related directly to the relevant mathematics, and included just one of the five teachers who indicated that they would not use the problem with their students. Fourteen teachers (42% of those who responded to this question) provided only responses that related to matters other than the mathematics, whereas just four (19%) of the 21 teachers who clearly recognised the error provided no other type of response. Including the 10 teachers who did not provide a response this means that 14 teachers (33%) gave no indication of being aware of possible student difficulties in relation to this problem.

Table 3: *Suggested student responses to the pie chart problem (Question 1)*

Response type (from 33 teachers)	Frequency	No. of teachers with responses in this category
Observations re relative shares, e.g., Woolworths are bigger than Coles	23	12
Explicit mentions of problem with chart, e.g., Should be 100% - the whole of something	17	17
Comments pointing to problem with chart but not mentioning 100%, e.g., numbers don't add up, 61.2% is more than half but the diagram doesn't show it	7	4
Questions re specific aspects of language and comments re likely difficulties of reading demands for students, e.g., Don't understand "accelerates retail purge"	7	7
Lack of initial Coles figure with which to compare	4	4
Questions about the meaning of Other, e.g., Who are other?, need to be itemised	11	11
What is IHL?	4	4
What does the % sign mean?	1	1
Should use a different form of graph, e.g., should be a bar graph	2	2
Don't have decimal percents	1	1
Nothing unusual	1	1
Other comments not related to mathematical content, e.g., One bit is black, IHL is outside the circle, colours not different from one another	9	7
Wouldn't use, e.g., "… too complex for low ability students …", "Would not teach this."	5	5
"Been a while since I've done this so leave it"	1	1
Total	93	

Fewer teachers than were able to suggest student responses, provided ideas concerning how they might use each of the problems. Their suggestions are shown in Tables 4 and 5. The most commonly suggested classroom uses of 90% of 40 (Table 4) involved explaining a procedure that would lead to the answer (26%) and/or relating the problem to a real context (also 26%). Given the concern for student engagement with this problem evidenced by one third of the teachers in relation to question 1, relating the problem to a real context was possibly intended to engage students with the task. The three responses classified as 'Other' were all expressions of discomfort with teaching percent and none of these teachers provided other responses. These three teachers with the 12 teachers who did not respond, mean that more than one third (36%) of the teachers did not make any suggestion about how they could use the problem in their classrooms.

Table 4: *Suggestions for the classroom use of 90% of 40 (Question 2)*

Response type (from 30 teachers)	Frequency	No. of teachers with responses in this category
Demonstrate or explain a procedure, e.g., 10% of 40 is 4 so 9 x 4 =36; match the words with operations; explain what percent is (per 100) and go over the steps	10	8
Encourage estimation by reference to appropriate fractions, e.g., 50% is a half so …; go back to 10%, 20%, 30% … tenths; try 10% of 40 and consider how this can help with 90% of 40	8	7
Explore students' thinking, e.g., "How did you come to this solution?"	4	4
Brainstorm strategies/techniques that work	2	2
Relate the problem to real world context, e.g., money, discounts in sales, test results	10	10
Go back to an easier example	1	1
Other, e.g., "I am teaching part-time and I take the strands of Measurement and Space", "Unsure with percentages. Need to do more reading up on the concept."	3	3
Total	38	

For the pie chart problem (Table 5) the most common suggestions were to address the concept of the whole as 100% and to discuss the context rather than clearly addressing any mathematical ideas. Nevertheless, of the 11 teachers who suggested that they would explore the context, six also provided responses in other categories that addressed some aspect of the mathematical content. The fact that only 19 (45%) of the 42 teachers provided any ideas at all suggests that the majority of the teachers found it more difficult to envision how they might use this problem than 90% of 40. None of the three teachers who responded to this question by stating that they would not use the problem were among the five who made similar statements in response to the first question about the pie chart.

Table 5: *Suggestions for the classroom use of the pie chart problem (Question 2)*

Response type (from 19 teachers)	Frequency	No. of teachers with responses in this category
Address the issue of the whole being 100%, e.g., Point out that % total is >100, get students to add up the percentages, "What % should the pie graph represent as a whole?"	11	10
Consider pie charts, e.g., provide information and practice how pie charts are done	3	3
Consider specific information shown in the chart, e.g., What % is owned by …?	5	5
Revisit percentages/fractions, e.g., talk about percentages	2	2
Graph in Excel or another form and compare	2	2
Discuss 0.1% as part of 100%	1	1
Explore the context, e.g., "What other grocery shops do you know?", "Explain retail …"	13	11
Wouldn't use, e.g., >100% would cause too much confusion, not relevant to adolescents,	3	3
Total	40	

The results of the assessment of teachers' overall responses according to the rubrics in Table 1 are shown in Table 6. Approximately one third of teachers were able to suggest both appropriate and inappropriate students responses (31% for 90% of 40 and 36% for the pie chart problem) suggesting reasonable knowledge of their students as learners. Fewer demonstrated reasonable levels of PCK by scoring 3 or 4 for Questions 2 (29% for 90% of 40 and 23% for the pie chart problem).

Table 6: *Levels of overall responses to Questions 1 and 2 both problems (n=42)*

	90% of 40					Pie chart problem				
Level	0	1	2	3	4	0	1	2	3	4
Question 1	6	10	13	13	NA	9	6	12	15	NA
Question 2	12	4	14	8	4	19	4	9	8	2

Of even greater concern is the fact that, for both problems, more than one third of teachers suggested no student response that contained any mathematics (38% for 90% of 40 and 36% for the pie chart problem). For question 2 38% of teachers similarly made no reference to mathematical ideas in relation to 90% of 40 and this figure was 55% for the pie chart problem.

Results and Discussion: Research Question 2

Associations between teachers' responses to the two questions they were asked in relation to each problem are shown in Table 7. Just one teacher was able to provide an integrated discussion, focussing on relevant mathematics, of the possible classroom use of a problem (90% of 40) having provided a student response that did not address the relevant mathematics. It seems that suggesting classroom uses for a problem was more difficult than suggesting student responses. Of the seven teachers whose responses to both questions in relation to 90% of 40 were rated at least 3, four were included among the nine who provided responses of a similar level in relation to the pie chart problem. These teachers, approximately 10% of the total, appear to have a sound KSL and PCK in relation to percent. The fact that there is not greater overlap between the teachers who provided sophisticated responses to each of the problems suggests that teachers' knowledge may be quite specific to particular mathematical ideas and types of problems.

Table 7: *Associations of levels of responses to Questions 1 and 2 for each problem*

	Questions 2, 90% of 40							Questions 2, pie chart problem					
	Level	0	1	2	3	4		Level	0	1	2	3	4
Question 1, 90% of 40	0	6	0	0	0	0	Question 1, pie chart problem	0	8	1	0	0	0
	1	1	2	4	2	1		1	6	0	0	0	0
	2	5	0	6	2	0		2	4	0	7	1	0
	3	0	2	4	4	3		3	1	3	2	7	2

Table 8 reveals little association between years of teaching experience and levels of responses. Although there is some indication that the most experienced teachers are best able to make comprehensive suggestions of student responses there is no evidence they are better able to discuss possible classroom use of these problems. It seems that experience may help teachers to know their students as learners but not necessarily result in increased PCK.

In Table 9 the categories for the amounts of tertiary mathematics studied are indicated by the abbreviations, 'Sem' for one semester, 'Yr' for 1 year, and 'More' for more than 1 year. A lack of association between levels of responses and tertiary mathematics background is evident, although having one year of tertiary mathematics was the most common experience of teachers who did not respond to the question about classroom uses of 90% of 40. The data provide no evidence that requiring teachers to study more mathematics at tertiary level leads to better knowledge of students as learners or better PCK. This finding aligns with those of other researchers (e.g., Hill et al., 2005).

Table 8: *Association of years of teaching experience and levels of responses to Questions 1 and 2*

	90% of 40						The pie chart problem					
(n=40)	Question 1			Question 2			Question 1			Question 2		
Years of teaching experience	<5	5-14	≥15	*<5*	*5-14*	*≥15*	<5	5-14	≥15	*<5*	*5-14*	*≥15*
Level 0	2	1	1	*5*	*2*	*3*	4	2	1	*8*	*3*	*6*
1	5	4	1	*1*	*2*	*1*	2	2	2	*0*	*3*	*1*
2	3	5	5	*5*	*5*	*3*	4	3	3	*5*	*2*	*2*
3	4	3	6	*1*	*3*	*4*	3	6	5	*1*	*4*	*3*
4	NA	NA	NA	*0*	*2*	*2*	NA	NA	NA	*0*	*1*	*1*

Table 9: *Association of tertiary mathematics and levels of responses to Questions 1 and 2*

	90% of 40								The pie chart problem							
(n=41)	Question 1				Question 2				Question 1				Question 2			
Tertiary mathematics	None	Sem	Yr	More	*None*	*Sem*	*Yr*	*More*	None	Sem	Yr	More	*None*	*Sem*	*Yr*	*More*
Level 0	1	1	2	1	*1*	*1*	*6*	*3*	1	2	3	2	*3*	*4*	*6*	*5*
1	2	3	2	3	*1*	*1*	*1*	*1*	1	2	2	1	*1*	*1*	*1*	*1*
2	3	2	5	3	*3*	*5*	*3*	*3*	3	2	5	2	*3*	*3*	*2*	*1*
3	3	5	2	3	*2*	*3*	*1*	*2*	4	5	1	5	*2*	*3*	*1*	*2*
4	NA	NA	NA	NA	*2*	*1*	*0*	*1*	NA	NA	NA	NA	*0*	*0*	*1*	*1*

Two of the three teachers who answered question 2 for the pie chart problem by saying they would not use it had clearly identified the error in the pie chart in their responses to question 1. Their reasons for not using the problem were quite different. One said, "I wouldn't – the >100% would cause too much confusion. It would set my students back", whereas the other said, " I obviously have limited understanding myself as I can't understand why it adds up to more than 100% – so I would probably avoid talking about this, so as not to confuse students". Both of these teachers were experienced, having taught for 14 and 19 years respectively. The first had a science degree and taught grades 8-10 maths and English, whereas the second had an arts degree and had qualified as primary teacher. Neither claimed to have studied any mathematics at tertiary level. The difference appears to be one of confidence, with the first doubting his students' abilities and the second her own. In both cases the result was that a problem with the potential to stimulate students' thinking about key mathematical issues to do with percent, presented in a realistic context providing opportunity for critical thinking would not be used.

Conclusion

Research Question 1: What knowledge of mathematical content, their students as learners, and pedagogical content knowledge related to particular problems concerning percent and its representation in a pie chart do middle school teachers display?

Of the 42 middle school teachers who participated in this study, approximately one third demonstrated KSL in relation to percent that could be described as reasonable. Fewer than 10% displayed sophisticated KSL and PCK in relation to both of the problems. Teachers generally found it more difficult to suggest student responses to the pie chart problem than to 90% of 40. The percentages of teachers offering no suggestions were 14% for 90% of 40 and 21% for the pie chart problem. It is likely that at least some of these teachers did not have the requisite mathematics content knowledge to answer the questions themselves as well as having little KSL in the contexts of these problems. In providing student responses to the pie chart problem many teachers did not mention any relevant mathematics. This could also be interpreted as a lack of content knowledge on the part of the teachers or, alternatively, as indicative that they did not expect their students to respond to the mathematics inherent in the context. The higher non-response rates for question 2 (29% for 90% of 40 and 55% for the pie chart problem) compared to question 1, suggest that many teachers struggled to think of ways that the problems could be used in their classrooms. In spite of the their concerns for student engagement with the de-contextualised 90% of 40 teachers found the pie chart problem more difficult to respond to, perhaps indicating less familiarity with contextualised problems and/or less experience of using these sorts of problems in their teaching. Differences between the sophistication of individual teachers to the two problems point to the specificity of teachers' knowledge in terms of types of problems and mathematical ideas.

Research Question 2: What connections are there between teachers' knowledge of their students as learners and their pedagogical content knowledge, and between these aspects of their knowledge and their years of teaching experience and self reported exposure to tertiary mathematics?

The results suggest that PCK is more difficult to acquire than either content knowledge or KSL, in the sense that knowing the mathematics and knowing how students might respond does not imply PCK. The lack of associations between important types of teacher knowledge, principally KSL and PCK, and the teachers' years of experience and tertiary mathematics background suggest that direct approaches to assessing teachers' knowledge are warranted. The results confirm those of earlier studies (e.g., Ball , 1990; Ma, 1999) that crucial aspects teachers' knowledge for teaching mathematics can not be inferred from experience or tertiary study of mathematics and is not likely to be improved by increasing either.

Implications: Although it is not clear why some teachers did not respond to various questions it as possible that this was at least partially indicative of a lack of understanding on the part of the teachers – a few admitted this. That several teachers considered these problems inappropriate for their middle school students is also of concern. As the examples of two teachers who would not use the pie chart in their teaching illustrate, there are factors such as confidence and expectations of students that are relevant here but which were not foci of this study. Studies that incorporate such factors, usually considered to be separate from teacher knowledge, are needed. Tasks such as these also have potential value in professional learning aimed at improving teachers' knowledge for teaching mathematics and in teacher education.

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