

Statistical knowledge for teaching: Exploring it in the classroom

Tim Burgess

Massey University, New Zealand

Address:

School of Curriculum and Pedagogy

Massey University

Private Bag 11-222

Palmerston North

New Zealand

Ph. +64 6 350 5799 ext. 8863

Fax +64 6 351 3472

Email t.a.burgess@massey.ac.nz

Abstract

There is a significant amount of research on teacher knowledge for mathematics. In comparison, research on teacher knowledge for statistics is more scarce, especially research that has been conducted in the classroom and at the primary school level. This paper reports first on the methodology of such a study. The methodology included videotaping of a sequence of lessons that involved students in investigating multivariate data sets, followed up by audiotaped interviews with each teacher. These stimulated recall interviews were based on the viewing of edited episodes from the classroom videos. The video and audio data were analysed against a teacher knowledge framework that had been formulated in relation to categories of teacher knowledge and components of statistical thinking. The framework is validated as a useful way of examining the teacher knowledge that is needed and is used in teaching statistics through investigations.

Introduction

Although statistics is part of school mathematics curricula, it is generally a more recent addition. In New Zealand, statistics has been included in the primary school curriculum since 1969, and as such, Watson (as quoted in Begg, Pfannkuch, Camden, Hughes, Noble, & Wild, 2004) considers that New Zealand can be considered a world leader with regard to statistics in the school curriculum. The latest curriculum explicitly requires that students at all levels be engaged in statistics investigations.

There is an extensive research literature that has examined the teacher knowledge needed for teaching mathematics. Statistics education research has a much shorter history and the research literature in this field pertaining to teacher knowledge is relatively scarce. The study being reported here goes some way to filling this gap.

Literature Review

More recent research in mathematics knowledge for teaching has a focus on questions such as how mathematics knowledge is used in teaching. Ball, Lubienski and Mewborn (2001) claim that significant mathematical reasoning and thinking occurs as teachers go about their everyday work. Such work includes “figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting and modifying textbooks; deciding among alternative courses of action; and steering a productive discussion” (p. 453). These aspects of a teacher’s role are indicators of different categories of teacher knowledge, such as pedagogical content knowledge or knowledge of students.

A model is needed that categorises teacher knowledge and describes the knowledge required to deliver high quality instruction to students. Such a model for teacher knowledge in mathematics is outlined by Hill, Schilling, and Ball (2004), and in this model, common knowledge of content and specialised knowledge of content are differentiated. These two categories of content knowledge are further clarified by Ball, Thames, and Phelps (2005): *common knowledge of content* includes the ability to recognise wrong answers, spot inaccurate definitions in textbooks, use mathematical notation correctly, and do the work assigned to students; in comparison, *specialised knowledge of content* needed by teachers (and likely to be beyond that of other well-educated adults) includes the ability to analyse students’ errors and evaluate their alternative ideas, and give mathematical explanations and use mathematical representations. Just as mathematical content knowledge is subdivided into two categories, so too is pedagogical content knowledge (Ball et al., 2005), into *knowledge of content and students*, and *knowledge of content and teaching*. These two types of teacher knowledge bring together aspects of content knowledge that are specifically linked to the work of the teacher, but are different from specialised knowledge of content. *Knowledge*

of content and students includes the ability to anticipate student errors and common misconceptions, interpret students' incomplete thinking, and predict what students are likely to do with specific tasks and what they will find interesting or challenging. *Knowledge of content and teaching* deals with the teacher's ability to sequence the content for instruction, recognise the instructional advantages and disadvantages of different representations, and weigh up the mathematical issues in responding to students' novel approaches.

The categorisation of teacher knowledge into four components was developed from work in the number and algebra fields. However it is recognised that statistics is, in some ways, different from mathematics, particularly because of the uncertainty surrounding the conclusions that one can draw from data (Pereira-Mendoza, 2002). There is no research literature that could be found that reported on classroom-based investigations into primary teacher knowledge and statistics. A statistical thinking model (Wild & Pfannkuch, 1999) that accounts for the uncertainty and non-deterministic processes of statistics provides a useful way forward for examining teacher knowledge in statistics. A framework that incorporates both statistical thinking and the four categories of mathematics teacher knowledge (as described above) was developed (Burgess, 2007), and this paper evaluates its usefulness for identifying teacher knowledge for statistics as used in the classroom. The major question that arises is: How effective is this framework for identifying teacher knowledge that is needed for teaching statistics at the primary school level?

Methodology

Because of widespread advocacy for conducting research on teacher knowledge in the context in which it is needed, namely the classroom, methods are needed that might provide such insight. Although students are also participants in the classroom, videotaping of the teacher during classroom lessons is considered sufficient for examining teacher knowledge (Schoenfeld, 1998) because a focus on the teacher explains a significant proportion of what takes place in the classroom, particularly as it relates to teacher knowledge. In the current study, videotapes of a sequence of four or five lessons from each teacher provided the first data source. It is recognised that a researcher's

interpretations of what is going on can be problematic (Phillips & Burbules, 2000). One solution to this problem is to use stimulated recall interviews with the teacher, as a way of reducing the limitations of one researcher's interpretations, and as a type of data triangulation. An edited video of classroom 'episodes of interest' provided a stimulus for discussion between the researcher and the teacher. Such episodes of interest are considered 'critical incidents' on which to base stimulated recall interviews (Lyle, 2003). Gass (2001) points to the importance of minimising any delay between the original classroom event and the stimulated recall interview. The stimulated recall discussion between the teacher and the researcher, which was audiotaped, was generally scheduled the same day as the lesson itself. This audiotaped discussion provided a second source of data for analysis.

The two sources of data were imported into Annotape software (Rosehill Software Limited 1999-2002) for subsequent coding. The coding categories were developed from the framework that had been proposed, with each cell of the framework having a unique code. As well as codes and timecodes being 'attached' to each segment (video or audio), notes could also be attached. These notes took the form of either transcriptions or field-type notes. The software included a search facility that enabled searching by codes or by terms/phrases within notes, and the search returned all video or audio instances that contained the code or term/phrase being searched for.

Results

Descriptions of teacher knowledge in relation to both statistical thinking components and the four categories of teacher knowledge were developed. These descriptions are not however within the scope of this paper. As part of the investigation into the suitability of the framework for identifying teacher knowledge as used in the classroom, or as needed but not used, a profile for each teacher was developed. Each teacher's profile showed which components from the framework were used by that teacher, which components were needed but not used, and which components were identified as not needed. One example of such a profile is shown below (for Linda).

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				M
	Reasoning with models			M	
	Integration of statistical and contextual				M
Investigative cycle					M
Interrogative cycle					
Dispositions					

Key: = direct evidence of that knowledge used; = indirect evidence of that knowledge; M = missed opportunity related to that knowledge.

Figure 1: Summary of Linda's teacher knowledge

The key indicates that four types of situations occurred with regard to particular teacher knowledge. First, the grey-shaded cells of the framework indicate that direct evidence was obtained from the video or interview for that particular aspect of teacher knowledge. For example, Linda used *specialised knowledge of content: transnumeration* when she had to evaluate whether a student's sorting of the data cards into two piles would be suitable to allow the student to compare these two piles and make a sensible statement from them. Second, for some cells on the framework, indirect evidence was found for that aspect of knowledge. These cells are shown with the 'hatched' shading. For example, *common knowledge of content: transnumeration* was not directly observed for Linda, as she did not sort data cards or otherwise transnumerate the data for herself. However, when Linda had to evaluate a student's sorting of the data, it was clear that if she had had to sort it appropriately for herself, she would have been able to do so. Consequently the conclusion can be drawn that she had *common knowledge of content: transnumeration*. Third, for some cells, that component of knowledge was not identified in any episode, nor was it seen to be needed. Such cells remain unshaded. Fourth, some evidence was found when teacher knowledge could have been used in a particular interaction with a student or group of students, but for some reason that knowledge was not used. Such instances were referred to as 'missed opportunities', and are shown in the profile with an M in that cell. Missed opportunities are further discussed below.

The type of investigation that was undertaken by students involved multivariate data that was provided, or was 'replicated' with data from the class. As such, the students' statistical thinking did not require an understanding of a need to gather data to answer a question; they already had the data. So, the framework row that corresponds to 'need for data', as one aspect of statistical thinking, was not in evidence as part of teacher knowledge in this particular teaching unit. Dispositions, as another component of statistical thinking, although apparent during the teaching, was not able to be subdivided in relation to the four categories of teacher knowledge as all the other components of statistical thinking were. Consequently, in the profiles, that row of the framework is blank.

All remaining 24 cells of the framework were identified in at least one teacher's practices, with 21 of those cells represented in either three or four teachers' profiles (see diagram of profiles for all four teachers). Common knowledge of content was the only type of knowledge that could be inferred as being present due to the presence of other types of knowledge, most commonly due to specialised knowledge of content. The other three types of teacher knowledge (specialised knowledge of content, knowledge of content and students, and knowledge of content and teaching) were considered to be more focused and with specific 'roles', and therefore it is unlikely that evidence of these can be inferred from other types. The diagram below shows the profiles for all four teachers.

In addition to the presence in the profiles of aspects of teacher knowledge, there were numerous instances of what was termed missed opportunities. Some missed opportunities corresponded to incorrect knowledge being used, while others related to non-use of teacher knowledge. As one example, a group of students talked about some questions that might be investigated in a given multivariate data set. One student suggested: *Put them into year and age and gender*. The teacher asked what they were trying to find out. Student: *We will figure that out once we have sorted the cards. Like, if there are more girls who are year 6 than boys who are year 6*. Another student then said: *We can add them together and do averages*. At this point the teacher made no response as to what the student meant or intended, or whether an average would have helped make sense of the data and helped to find something interesting. This incident indicates a missed opportunity in relation to *specialised knowledge of content: transnumeration*. Missed

opportunities such as this show non-use of knowledge, which could be attributed to one of three reasons: a lack of statistics knowledge on the teacher's part, a lack of recognition that their knowledge could have been used in that situation to question the students, or a conscious decision (for a range of possible pedagogical reasons) not to use their knowledge. Some missed opportunities were further explored in the stimulated recall discussions resulting in a reason for that missed opportunity becoming apparent; in other situations, the reason for the missed opportunity is unknown.

Table 5-2: Summary of Linda's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				M
	Reasoning with models			M	
	Integration of statistical and contextual				M
Investigative cycle					M
Interrogative cycle					
Dispositions					

Table 5-3: Summary of John's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				
	Reasoning with models	M	M	M	M
	Integration of statistical and contextual				
Investigative cycle		M	M		M
Interrogative cycle			M		M
Dispositions					

Table 5-5: Summary of Rob's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M	M	M
	Variation				M
	Reasoning with models		M	M	M
	Integration of statistical and contextual			M	M
Investigative cycle				M	M
Interrogative cycle		M	M	M	
Dispositions					

Table 5-6: Summary of Louise's teacher knowledge

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration	M	M		M
	Variation	M	M		
	Reasoning with models		M	M	M
	Integration of statistical and contextual		M		
Investigative cycle			M		
Interrogative cycle			M		
Dispositions					

Figure 2: Profiles of the four teachers

Conclusions

The diagram above of the four teachers' profiles indicates, because of the extent of shaded cells within the framework, that most components of statistical thinking (other than the ones discussed earlier) across the four categories of teacher knowledge are needed by primary school teachers as they teach statistics through investigations. It is, as noted earlier, interesting that 21 of the cells are present for at least 3 of the 4 teachers. There were few aspects of teacher knowledge that were not needed in the teaching. The main reason for these differences was due to minor variation between teachers in the way

that the lessons ‘unfolded’. The framework also provides a useful way of identifying for each teacher, through patterns of missed opportunities, aspects of teacher knowledge that are in need of development. Together, the aspects of knowledge identified across the four teachers and the missed opportunities for each teacher indicate that the methodology adopted and the framework are successful and useful for identifying teacher knowledge needed for teaching statistics through investigations at the primary school level.

References

- Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.
- Ball, D. L., Thames, M. H., & Phelps, G. (2005). Articulating domains of mathematical knowledge for teaching. Retrieved May 13, 2005, from http://www-personal.umich.edu/~dball/Presentations/RecentPresentations/041405_MKT_AE_RA.pdf
- Begg, A., Pfannkuch, M., Camden, M., Hughes, P., Noble, A., & Wild, C. J. (2004). *The school statistics curriculum: Statistics and probability education literature review* (Report for the Ministry of Education). Auckland, New Zealand: University of Auckland.
- Burgess, T. A. (2007). *Investigating the nature of teacher knowledge needed and used in teaching statistics*. Unpublished doctoral thesis, Massey University, Palmerston North, New Zealand [Available from <http://www.stat.auckland.ac.nz/~iase/publications/dissertations/dissertations.php>].
- Gass, S. M. (2001). Innovations in second language research methods. *Annual Review of Applied Linguistics*, 21, 221-232.
- Hill, H. C., Schilling, S., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105(1), 11-30.
- Lyle, J. (2003). Stimulated recall: A report on its use in naturalistic research. *British Educational Research Journal*, 29(6), 861-878.
- Pereira-Mendoza, L. (2002). Would you allow your accountant to perform surgery? Implications for education of primary teachers. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics (ICOTS 6) - Cape Town* (pp. CD Rom). Voorburg, The Netherlands: International Association for Statistical Education.
- Phillips, D. C., & Burbules, N. C. (2000). *Postpositivism and educational research*. Lanham, MA: Rowman & Littlefield.
- Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, 4(1), 1-94.
- Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, 67(3), 223-265.