

**MATHEMATICAL KNOWLEDGE FOR TEACHING AND TEACHING PRACTICES:
A VIRTUAL-DESIGN APPROACH TO STUDY A COMPLEX RELATIONSHIP**

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ABSTRACT

Over the past three decades considerable theoretical and methodological advancements have been made in studying teacher knowledge and its association to teaching practices, and consequently to student achievement. These advancements notwithstanding, significant work still exists in disentangling the complex relationship between teacher knowledge and their teaching practices. In this paper, I first outline the affordances of using Ball and colleagues' notion of mathematical knowledge for teaching to explore the aforementioned association. I then propose a virtual-design approach based on a teaching simulation that can help address some of the challenges inherent in exploring this association.

INTRODUCTION

Attempts to explore the association between teachers' knowledge and their instruction, and consequently student learning, date to the mid 1970s. In one of the earliest works in this realm, Beagle (1979) found ambiguous results regarding the association between teacher knowledge and student learning. Concluding that "the effects of a teacher's subject matter knowledge ... on student learning seem to be far less powerful than most of us had realized" (p. 54), he urged researchers to explore other plausible contributors to teachers' instruction and student learning instead of maintaining focus on teacher knowledge. Despite his call to shift research focus, interest in teacher knowledge has persisted; in fact, over the past thirty years significant theoretical and methodological advancements have been observed in understanding teacher knowledge and its relationship to teachers' instruction, and subsequently, to student learning.

Notable among these theoretical advancements have been two significant shifts in conceptualizing teacher knowledge: the attention to the disciplinary demands of teaching a particular subject-matter – what Shulman (1986) identified as the "missing paradigm" of the then extant relevant studies – and more recently, a closer examination of the knowledge necessary for the *work* of teaching a particular subject (cf., Ball, Lubienski, & Mewborn, 2001). These theoretical advancements have been accompanied by pertinent methodological shifts: instead of using proxies to measure and explore teacher knowledge, scholars are increasingly using measures of the knowledge that teachers need to carry out the various tasks embedded in teaching; several different methods have also been utilized to

tap teachers' knowledge, including direct or videotaped observations of teachers' mathematical instruction, clinical interviews, and paper-and-pencil tasks (Hill, Sleep, Lewis, & Ball, 2007).

These advancements notwithstanding, scholars have not yet disentangled the complex relationship that exists among teacher knowledge, their instructional practices, and student learning (Mewborn, 2003). For instance, even though some recent studies (e.g., Hill, Ball, & Rowan, 2005; Sowder, Philipp, Armstrong, & Schappelle, 1998) have found that teacher knowledge relates positively to student learning gains, the particular ways in which teacher knowledge informs teachers' instructional approaches remain open to investigation. Aiming to contribute toward this direction of inquiry, the present paper has two purposes. First, it explains the affordances of focusing on a particular type of mathematical knowledge, namely the notion of *mathematical knowledge for teaching* (MKT) developed by Ball and colleagues (Ball, Hill & Bass, 2005; Ball, Phelps, & Thames, in preparation), as a means to explore the complex association under examination. Second, it outlines a virtual-design approach that can help address some of the challenges inherent in studying this complicated association.

MKT: A TYPE OF KNOWLEDGE SUITABLE FOR STUDYING THE ASSOCIATION BETWEEN TEACHER KNOWLEDGE AND THEIR TEACHING PRACTICES

During the last decade, Ball and colleagues have channeled their efforts to understand the mathematical knowledge necessary for teaching elementary-school mathematics well (Ball & Bass, 2000). A key outcome of their work has been the notion of MKT, a type of professional knowledge rooted in the practice of teaching that synthesizes the multitude of requirements entailed in the work of teaching mathematics. According to Ball and colleagues' conceptualization (Ball et al., 2005, in preparation), teachers of mathematics need to be able to explain why certain mathematical algorithms are applicable and make sense; they should be capable of analyzing, evaluating, and modifying textbook tasks so that they meet their lesson goals but also respond to their students' needs; they need to diagnose the difficulties students might encounter when assigned a task; they should be able to listen to and analyze their students' thinking and decipher the source(s) of students' errors; and they need to make judicious selections of representations and use these representations wisely to help students develop meaning. Teaching mathematics also requires being able to understand students' unconventional approaches when solving problems, as well as carefully selecting and sequencing mathematical examples and using mathematical language and symbols with care to support students' gradual construction of mathematical ideas. Rather than considering MKT as singular and unified, Ball

and colleagues proposed that MKT comprises four distinct domains: *common content knowledge*, which is the “mathematics knowledge and skill used in settings other than teaching” (Ball et al., in preparation, p. 34); *specialized content knowledge*, which is the mathematical knowledge and skill individuals engaged in teaching mathematics need; *knowledge of content and students*, which intertwines knowledge of mathematical notions with knowledge of how students think or come to understand these ideas; and *knowledge of content and teaching*, which refers to the type of knowledge that combines knowing about teaching and knowing about mathematics.

I argue that there are at least three significant affordances in using MKT as the basis for exploring the association between teachers’ knowledge and their teaching practices, each of which relates to respective limitations of previous research efforts to understand if and how teachers’ knowledge affects teachers’ instructional approaches. In particular, MKT is *rooted in practice*, is *informed by disciplinary demands*, and is *validly measurable*. I elaborate each of these affordances in turn.

First, instead of just theorizing about the knowledge necessary for the work of teaching, Ball and colleagues have closely scrutinized and analyzed several actual records of practice (e.g., videotapes of teaching mathematics, copies of student work, teacher plans, notes, and reflections) seeking to untangle and understand the several demands embedded in teaching mathematics as well as the configurations of knowledge necessary to efficiently respond to these demands. In doing so, these scholars have addressed one of the main limitations of the educational production function studies undertaken since the mid 1960s. In these latter studies, researchers have mainly used several proxies (e.g., courses taken, degrees earned, and performance in certification exams) to measure teachers’ knowledge and determine whether this knowledge could predict teacher effectiveness. However, these proxies could not satisfactorily determine teacher knowledge, let alone explore whether this knowledge relates to teachers’ instructional approaches, and consequently to student learning. Consider, for instance, the extent to which a teacher holds a master’s degree in applied mathematics. Earning this degree might deepen a teacher’s knowledge of differential equations and numerical analysis, to use an example, but it does not necessarily render the teacher more effective in teaching mathematics.

Second, the notion of MKT not only elaborates Shulman’s conceptualization of pedagogical knowledge for teaching (Shulman, 1986), but it closely attends to the specific *disciplinary demands* of teaching mathematics. Take, for example, the activities of making educated choices of which representations and examples to use while teaching a particular concept or the postulating and testing

of conjectures. These activities that are both constituent components of MKT lie squarely at the heart of doing mathematics, as Polya's seminal work on problem solving (1957) and Lakatos' work on *Proofs and Refutations* correspondingly suggest. By directly considering the disciplinary demands of teaching mathematics, the work associated with MKT addresses one of the main limitations of the process-product studies (e.g., Brophy & Good, 1986), which, by focusing on generic teaching skills, they largely ignored the disciplinary demands of teaching certain subjects.

Third, starting from 2001, Ball and colleagues have developed instruments to measure MKT. So far, the analysis of data collected via these instruments has provided converging evidence that validates the theoretical assumptions underpinning the construct of MKT: that it is multidimensional (Hill, Schilling, & Ball, 2004); that it relates to student learning (Hill et al., 2005); and that it integrates mathematical reasoning and pedagogical thinking (Hill & Ball, 2004). Collectively, these findings also suggest that these instruments can provide valid measures of MKT. Hence, the development of these instruments comprises an additional affordance of using MKT to study the association between teacher knowledge and their teaching practices, especially if one considers the enduring challenge to obtain valid measures of teacher knowledge (cf., Hill et al., 2007).

Capitalizing on the affordances of MKT as outlined above, scholars working on MKT have recently started exploring the association between teacher knowledge and teachers' instructional practices into more depth by dissecting videotaped lessons of teachers who differ in their level of MKT (Hill et al., accepted). Analyzing a series of lessons taught by five teachers with different levels of MKT, these researchers found that, compared to their low-MKT counterparts, high-MKT teachers committed fewer mathematical errors, responded more appropriately to their students, and chose examples that more effectively helped their students construct meaning of the targeted concepts and processes. The high-MKT teachers were also more successful in sequencing mathematical examples and tasks, restating textbook definitions, and using representations efficiently.

Despite its significant insights into the association at hand, the abovementioned study was limited in three ways, all related to exploring teachers' instructional approaches in an in-vivo situation. First, the participating teachers used different curriculum materials. Even though one could argue that teachers' knowledge might shape their use of curriculum materials, one cannot ignore that curriculum materials themselves might also inform teachers' instructional decisions and actions (cf., Ball & Cohen, 1996), thus rendering curriculum materials a potential confounder of the association between MKT and teaching practices. Second, these teachers were not teaching the same topic, a limitation that

should not be underestimated given the results of previous studies documenting that even the same teacher might pursue different instructional approaches when teaching different topics (e.g., the case of Ms. Jackson in Fennema and Franke, 1992, or Ms. Lehava, in Kahan, et al., 2003). And third, given that the participating teachers were teaching different students in different classrooms and schools, these teachers' instructional approaches might have been shaped – at least to some extent – by several classroom, school, and outside-of-school contextual factors identified in previous studies (e.g., the existing classroom norms, students' prior knowledge, beliefs, values, and dispositions, the availability of certain instructional resources, parental or administrative expectations, and the organizational and policy context in which these teachers worked).

A VIRTUAL-DESIGN APPROACH TO EXPLORE THE ASSOCIATION BETWEEN MKT AND TEACHING PRACTICES

Taking the limitations associated with exploring how MKT might play out in informing teachers' instructional practices in an in-vivo situation, over the past year I have developed a *virtual-design approach* that allows for addressing some of these limitations. In what follows, I briefly present the foundations of this approach, elaborate its design, explicate its affordances, and explain how it can be used to explore the association between teachers' MKT and their teaching practices. At this juncture it is important to clarify that the proposed design should not be pitted against current in-vivo approaches to explore the association at hand, because, due to its artificiality, this design bears the limitations germane to studying teaching in virtual environments. Hence, the proposed design should be considered *complementary* to in-vivo approaches to explore the association between teachers' MKT and their instructional practices.

The core of the proposed design is a *teaching simulation* which builds on some of the ideas and work by Herbst and Chazan (Herbst & Chazan, 2003, 2006) on representations of teaching. These scholars have used animations and slide shows of cartoon characters as a means to evoke, elicit, and explore teachers' *practical rationality*, namely the set of resources that practitioners draw upon to make decisions and the actions in which they engage in different instructional situations. The teaching simulation consists of three parts which correspond to the three phases of a teaching cycle: planning, instruction, and reflection upon practice (cf., Wilson, Shulman, & Richert, 1987).

In the first part of the teaching simulation (i.e., planning), teachers are presented with two textbook pages that both address the same mathematical topic (i.e., division of fractions, a rich topic for exploring teachers' MKT and the ways in which this knowledge plays out in teaching) but approach

this topic rather differently. By presenting the pertinent algorithm and a series of related exercises, the first textbook page mainly seeks to help students develop procedural fluency in the topic at hand. In contrast, the second page mainly aims to foster students' conceptual understanding of the aforementioned topic: instead of presenting the algorithm at hand, this textbook page presents students with several situations in which division of fractions should be employed and expects them to deduce this algorithm based on their answers to the given mathematical situations. The participating teachers are asked to comment on the affordances and limitations of each textbook page and explain how they would use these pages (either only one or both of them) to plan an introductory lesson on division of fractions.

The second phase of the teaching simulation (i.e., instruction) corresponds to a *virtual lesson*, a slideshow that features the character set *ThExpians P*. This character set has been designed by Herbst, Steffen, and Skindzier to model a wide range of individual differences that are present in a classroom. These characters support particular Thought Experiments in Mathematics Teaching (being conducted at the *GRIP* Lab) that use representations of students to elicit how teachers perceive students as they participate in instructional situations. (For more information see <http://grip.umich.edu/> and http://sitemaker.soe.umich.edu/soe/faculty_research&mode=single&recordID=50811.) This virtual lesson presents an imaginary teacher, Mrs. Rebecca, who builds an introductory lesson on division of fractions using the second textbook page described above. Although Mrs. Rebecca uses cognitively demanding tasks to build her lesson, most of her instructional decisions and actions diminish the cognitive level at which the content is presented and experienced in her lesson. For instance, even though Mrs. Rebecca uses a representation that could support student learning, she mainly outlines a sequence of steps for students to follow when solving the assigned division-of-fractions tasks.

This slideshow is parsed into five lesson segments. The points at which the virtual lesson is parsed are strategic, in the sense that each of these segments concludes with an instance during which Mrs. Rebecca has to make specific decisions related to several teaching practices. Hence, this virtual lesson creates three distinct types of opportunities to explore teachers' decisions and actions. In particular, the researcher can explore the extent to which the participating teachers are able, without any prompting, to identify Mrs. Rebecca's decisions and actions that distort the richness and the intellectual complexity of the mathematics considered in the virtual lesson (i.e., "noticing" opportunities). The participating teachers are also asked to evaluate Mrs. Rebecca's decisions and actions at various points of the lesson (i.e., "evaluating" opportunities). Finally, teachers are asked to

outline their own decisions and actions during each of the five aforementioned end points of each lesson segment, were they asked to teach this introductory lesson on division of fractions and were they to encounter situations similar to those faced by Mrs. Rebecca (i.e., “performing” opportunities).

In the final phase of the teaching simulation participating teachers are asked to reflect upon the virtual lesson as a whole, evaluate the virtual teacher’s instructional approaches, and explain how they would capitalize on this virtual lesson to structure a subsequent lesson on division of fractions.

The aforementioned virtual-design approach is promising for exploring teachers’ instructional practices for several reasons. First, it helps simplify the teaching complexity, since it excludes some of the classroom and school contextual factors that could impinge on teachers’ decisions in an in-vivo situation. In so doing, the teaching simulation allows for a more in-depth exploration of the association between teachers’ MKT and practices. Second, performing in an in-vitro environment ensures that all teachers consider exactly the same lesson, which facilitates comparability. Third, this virtual design ensures that all participating teachers are exposed to the same student actions and reactions that require making decisions conducive to exploring teachers’ performance in several teaching practices. And finally, recent research findings (e.g., Stecher, Vi-Nhuan, Hamilton, Ryan, Robyn, & Lockwood, 2006) suggest that the decisions that teachers make in in-vitro situations are not unrelated to those that they make in real-classroom settings.

To explore the association between MKT and teachers’ practices, this virtual design can be employed in conjunction with the paper-and-pencil MKT instruments developed by Ball and colleagues. In particular, the researcher can first explore preservice or in-service teachers’ MKT using items calibrated to the topic explored in the teaching simulation (e.g., by using the *Rational Numbers* MKT-form which focuses on rational numbers and includes items on division of fractions). This close correspondence between the mathematical topics tapped by the MKT instrument and the teaching simulation can facilitate a better exploration of the association at hand. Then, the researcher can explore participants’ performance in the teaching practices the virtual lesson was designed to investigate (i.e., selecting and using tasks, using representations, providing explanations, capitalizing on students’ work, and responding to students’ requests for help). In the extended paper, I will present findings of a study in which the aforesaid design has been utilized to explore the association between preservice teachers’ MKT and their performance in the aforementioned teaching practices. I will also explain how the data collected via the teaching simulation were analyzed from both a quantitative and a qualitative perspective to better explore and understand the association under investigation.

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