MATHEMATICS KNOWLEDGE FOR TEACHING WITHIN A FUNCTIONAL PERSPECTIVE OF PRESERVICE TEACHER TRAINING

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In this paper we look at some issues concerning the first two questions that frame Topic Study Group 27. On the one hand, we describe a functional perspective of preservice mathematics teacher training and learning. This perspective is based on the consideration of didactic notions as instruments to analyze the mathematical subject matter from different viewpoints. On the other hand, we propose some concepts and methods that can allow researchers explore preservice teachers' learning and help trainers assess the impact of methods courses' design and development.

Professional approaches to preservice teacher education are concerned with the importance of providing theoretically grounded solutions for professional activities. These approaches highlight the problem of exploring future teachers' learning processes in courses where the students still don't have teaching practice and even the moment of the real practice could be remote for them. Besides, in some cases, as the Spanish one, most of the future secondary mathematics teachers develop in-depth academic mathematical knowledge before starting to think about didactic questions. In this context, the complexity of the processes by which future teachers in university classrooms make links among didactic notions and pupil's mathematical learning at school needs attention. We tackle this problem by adopting a functional view of teacher training. We propose the use of a set of didactic notions, the curriculum organizers (Rico, 1997), structured around the process that an ideal-expert teacher would follow to promote pupils mathematical learning in a constructivist framework, the didactic analysis (Gómez, 2007). This approach attempts to capture the complexity of the mathematics curriculum and serve as theoretical support to structure the variety of meanings of a mathematical notion that have to be negotiated in a mathematics classroom. Future mathe-

matics teachers learning under this approach can be interpreted assuming an instrumented perspective (Vygotsky, 1982). We focus our work in the analysis of how a future teacher learns a curriculum organizer. For this purpose we will describe the epistemic, heuristic and pragmatic functions of a curriculum organizer and we will describe some possible relations among them.

Functional Views of Teacher Training and Didactic Analysis

We think about mathematics teacher's knowledge from a functional perspective. According to this view, teacher's knowledge can be established from the analysis and description of the activities needed to plan, manage and evaluate a mathematics lesson. Thus, the problem of the teacher's knowledge can be considered as the integration of knowledge, abilities and attitudes for action. Instead of thinking on what the teacher should know, we ask ourselves what he should be able to do in a specific context of students' learning. Therefore, we start by adopting a functional view of school mathematics, and then we reflect on the teacher's activities that can promote students' learning in that context (didactic analysis, see below).

Didactic analysis is set up around a set of notions that we call *curriculum organizers* (Rico, 1997). The way we use these notions in future teachers training is coherent with the functional view we advocate: curriculum organizers are considered as methodological and analytic tools with a didactic purpose. That is, we pinpoint our approach by postulating "a set of tasks, a set of conceptual tools and a subject that, when performing the task using the available tools [the curriculum organizers], put into play and set forth his/her competency in carrying out the processes involved" (Rico, 2007, pp. 49-50).

Didactic analysis can be used as a task planning procedure in preservice mathematics teacher training (Gómez, 2006). With it, the teacher can specify (and differentiate) the goals, content, methodology and evaluation scheme of each topic in planning. We claim that in the specific context of the planning of an hour of class or a didactic unit, the teacher can organise instruction based on four analyses (Gómez, 2007):

1. *subject matter analysis,* as a procedure by which the teacher identifies and organises the multiplicity of meanings of a concept;
2. *cognitive analysis,* in which the teacher describes his hypotheses about how the students can progress in the construction of their knowledge of the mathematical structure when they face the tasks that will make up the teaching and learning activities;
3. *instruction analysis,* in which the teacher designs, analyses, and chooses the tasks that will constitute the teaching and learning activities that are the object of the teaching; and
4. *performance analysis,* in which the teacher determines the capacities that the students have developed and the difficulties that they may have expressed up to that point.

We use *didactic analysis* to refer to a cyclical procedure that includes these four analyses, attends to the factors conditioning the context and identifies the activities that the teacher should perform to organise the teaching of a specific mathematical content.

Any cycle of the didactic analysis begins with the identification of students' knowledge for the subject matter at hand on the basis of the information provided by the last phase of the previous cycle. With this information, and taking into account the global planning of the course, we expect the future teacher to make a proposal for the goals he wants to achieve and the mathematics content he wants to work on. The next step of the cycle involves the description of the mathematical content from the viewpoint of its teaching and learning in school. The subject matter analysis is based on three aspects of any given mathematical topic: its representations, conceptual structure and phenomenology. The future teacher can use this information in the cognitive analysis for analyzing the mathematical topic taking into account the possible errors, objectives, capacities, competencies and learning paths followed by students. The information from the subject matter and cognitive analysis allows the teacher to carry out an instruction analysis to produce a planning sequence. This planning is assessed in the performance analysis in order to determine students' performance, produce better descriptions of their current knowledge and review the planning in order to start a new cycle.

INSTRUMENTAL LEARNING IN TEACHER TRAINING

From our functional perspective of teacher training, a future teacher learns by putting into practice a set of notions, the curriculum organizers, for analyzing a mathematical concept with didactic purposes. Therefore, the future teacher's activity is centred in the use of conceptual and methodological tools (the curriculum organizers) for performing two types of tasks: (a) analyzing the mathematical concept and (b) using the information resulting from such analysis either in other analysis or in planning a lesson. Understanding the tool is a process that takes place while using it. The future teacher's actions while performing the task enhance his understanding of the tool. And this improved understanding enhances his performance of the task.

This view of learning is rooted in Vygotsky's perspectives and his consideration of instruments as mediators of the individual activity (Vygotsky, 1982, p. 67). We will consider the curriculum organizers as mediating instruments between the future teachers' action and the activity. Future teachers' design and selection of pupils learning tasks —in brief, task planning activity— can be seen as an instrumented practice when the future teacher uses curriculum organizers to produce and use information to propose solutions to this activity. As Trouche (2005, p. 155) has claimed, "the study of instrumented action schemes requires studying, beyond the techniques themselves, their epistemic, heuristic and pragmatic functions". In this paper we will focus on the epistemic, heuristic and pragmatic functions of the curriculum organizers. These three functions characterize the three aspects of the use of a curriculum organizer by a subject: the subject (a) needs some understanding of the curriculum organizer in order (b) to use it for analyzing a mathematical concept, producing useful information that, in turn, (c) can be used possibly in conjunction with others organizer's information, with a concrete didactic purpose. We will now briefly describe how we conceptualize these three functions of a curriculum organizer in the context of preservice teachers' training and we will show how these ideas can be used to explore and describe future teachers' learning.

Meaning and Uses of a Curriculum Organizer

We raise then the question of how a future teacher learns a curriculum organizer. Our interest will focus on the epistemic, heuristic and pragmatic functions of the curriculum organizers. Corresponding to these functions, we will talk about their meaning, technical use and practical use.

From the disciplinary perspective, the *meaning* (M) of a curriculum organizer is the option that the trainers have taken for the formal meaning of the mathematics education notion to which it refers, from the multiple meanings that are proposed in the mathematics education literature. For instance, trainers have to decide which definition to adopt for the notion of system of representation from the several definitions available in the literature (Goldin & Janvier, 1998). It is the meaning that we, as trainers, expect the future teachers develop along their training. But, as a tool of the didactic analysis cycle, each curriculum organizer has a pragmatic function. This pragmatic function, that we call the *technical use* (TU) of a curriculum organizer, refers to the set of strategies and techniques that, as trainers, we consider necessary for analyzing a secondary school mathematics topic and producing relevant didactic information about the topic. For instance, the technical use of the notion of learning path gathers the strategies and techniques necessary for identifying the learning paths of a given learning goal. The information that emerges from the technical use of a curriculum organizer can be used for didactic purposes. This is the pragmatic function of the curriculum organizer, that we call *practical use* (PU). It refers to the set of strategies and techniques that, as trainers, we consider necessary for using the information produced with the technical use in other analysis of the didactic analysis procedure (i.e., in the technical use of other curriculum organizers) or in the design of a didactic unit on the topic at hand. Figure 1 shows a schematic representation of these ideas.

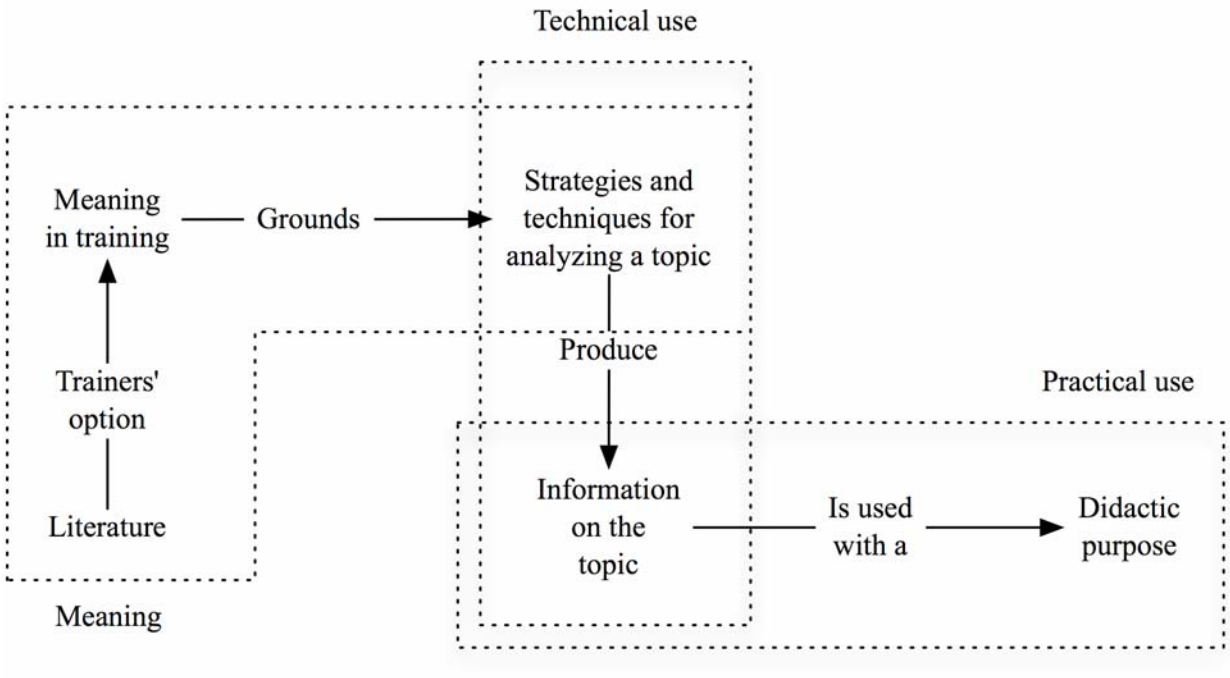

Figure 1. Meaning and uses in teacher training

Our purpose is to show that it is possible and relevant to explore and characterize future teachers' learning of curriculum organizers in terms of the ideas of meaning, technical use and practical use. In particular, these ideas can be used for the design of instruments for coding audio transcripts of future teachers working on a given topic with the didactic analysis procedure. In that sense, we will say that, for a given episode in a transcript, a future teacher advances in the *meaning construction* of a curriculum organizer, if he proposes examples of it, or declares, discusses or reflect on its properties, definition or relationships with other notions. Similarly, we will say that a future teacher *develops the technical use* of a curriculum organizer, when in a given episode there is evidence that he puts it into play in order to analyze a mathematical topic. A future teacher's technical use of a curriculum organizer is usually based on his interpretation of the meaning of the notion and can involve specific methods or other notions of the didactic analysis procedure. Finally, we will say that a future teacher *develops the practical use* of a curriculum organizer when there is evidence that he uses the information emerging from its technical use for didactic purposes.

SOME RESULTS AND DISCUSSION

Given the disciplinary definitions of the notions of meaning, technical use and practical use of a curriculum organizer given above, one could expect that the process of learning a curriculum organizer should follow a sequence: first to construct the meaning of the notion, then interpret this notion in practice in order to develop strategies for analyzing a topic with it (technical use), and finally use the information emerging from the technical use for other analysis or the design of a didactic unit. The diagram M → TU → PU can represent this sequence.

But in practice, when preservice teachers have solved the tasks proposed in a methods course in order to plan a lesson, we have found in some preliminary explorations that preservice teachers do in fact enact different sequences for different curriculum organisers[1] (González & Gómez, Forthcoming). The appearance of different sequences depends on several factors. For instance, based on their experience, preservice teachers might interpret the curriculum organizer with its everyday meaning, and start working directly on its technical use (sequence 1 in Figure 2). In some cases, developing the technical use of the curriculum organizer can promote the development of its meaning, which in turn informs the further development of its technical use (sequence 2 in Figure 2). Similarly, the development of the practical use can inform the development of its technical use (sequence 3 in Figure 2). In some cases, a step in a sequence might not inform other steps, as is the case of sequence 1 in Figure 3. We depict this situation in the diagram by a dotted line.

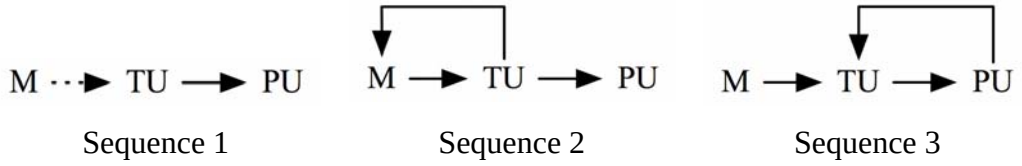

Figure 2. Examples of other learning sequences

We have not yet explored why and under which circumstances some sequences are more prone to be enacted than others. It seems that preservice teachers' previous knowledge and some characteristics of particular curriculum organizers promote some specific sequences.

[1] We do not have space here to describe these research results. This can be done, perhaps, in a extenden version of this paper.

Answering in detail the questions of what sequences appear and why, can help us, as researchers, understand how learning takes place in a methods course based on a functional perspective of teacher training and learning. It can also help us, as trainers, in assessing the design and development of our training programs.

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