Learning Mathematics for Teaching: Sources of Subject-Matter Knowledge[1]

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This research presents a two-dimensional model which analizes the knowledge relevant to Math teaching in Israel. One approach focuses on its types, while the other emphasizes its sources. This analysis is based on interviews with five mathematics teachers who have had different qualifications and seniority. Analyzing the findings according to the two approaches leads to important conclusions as to the strengths and weaknesses of teacher qualification programs and professional development programs.

One of the key issues that many researchers deal with in the mathematical education area is the educational profile of a good mathematics teacher. In order to decide how to train a good mathematics teacher, it is important to know what knowledge is required for these teachers in order to perform their duties in the best manner. In addition, it is important to find out at what points in the teachers' professional career certain types of knowledge develop and strengthen. This data will enable mathematics teacher trainers and those responsible for the teachers' in service professional development, to improve their programs. In this paper, which is part of a more comprehensive research we are doing, we focus on the teachers opinions concerning the subject matter knowledge that a teacher in elementary school needs.

1. Theoretical considerations

In the past, prospective teachers studied high mathematics at universities together with future mathematicians. In 1924, the famous German mathematician Felix Klein issued special textbooks intended for prospective teachers, "Elementary Mathematics from an advanced standpoint". Klein's idea concerning the need for special mathematics courses intended for teachers-to-be and issuing suitable textbooks for these courses functioned as a landmark in teacher education (Kilpatrick, 1992).

During the seventies of the previous century attempts were made to link the teaching quality and the number of mathematics courses in which the teachers participated during their qualification period and thereafter (e.g. Begle & Geeslin, 1972). It should be mentioned that no statistically significant links were found between the two variables. During the same period additional research was also published, in which teachers' test scores and grade point average in their university studies was examined (e.g. Andrews, Blackmon & Mackey, 1980), and this research also did not point to significant relations between the chosen variables (Fennema & Franke, 1992).

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In 1986 Lee S. Shulman proposed a new model for describing the teachers' knowledge (1986). He proposed to consider three types of teacher knowledge: a) subject matter content knowledge (SMK); b) pedagogical content knowledge; c) curricular knowledge. He further claimed that one type of knowledge cannot replace another. For example, research of Ma (1999) led her to the conclusion that pedagogical content knowledge can not replace limited subject matter content knowledge: teachers with limited mathematical knowledge are incapable of asking their students thought-provoking questions, or planning activities regarding the substantive structure of mathematics. During the last twenty years various research studies have been carried out which examined each of the aforementioned types of knowledge separately and together. These studies allowed more exact description of the types of knowledge and their effect on the quality of teaching (for example, Ball & McDiramid, 1990; Even, 1993; Leikin, 2006; Tirosh & Graeber, 1990 and many others). In comprehensive research we referred to Shulman's three types of knowledge (ibid). In this paper we will present only the findings regarding subject matter content knowledge since this is the topic of this topic study group.

Grossman, Wilson, and Shulman (1989) divided the SMK into 3 categories: *Content knowledge, Substantive knowledge* and *Syntactic knowledge*. Researchers in recent years have offered their own interpretation to these categories. We have decided to adopt those of Deborah Ball (1991, 2001):

- *Content knowledge* which includes “specific concepts, definitions, conventions, and procedures” (Ball et al., 2001).
- *Substantive knowledge* which refers to understandings of particular topics within a discipline, procedures, concepts, and their relationships to each other (Ball, 1991).
- *Syntactic knowledge* which includes "knowledge *about* mathematics, what it means to ‘know’ and ‘do’ mathematics, the relative centrality of different ideas, as well as what is necessary or logical, and a sense of philosophical debate within the discipline” (Ball, 1991).

A different approach to knowledge relevant for teaching can be found in Kennedy's (2002) article which proposes three separate sources of knowledge:

"First, there is ‘*craft knowledge’*, which is largely acquired through experience and which tends to be a-theoretical and idiosyncratic. Then there is ‘*systematic knowledge’*, which is acquired mainly through colleges and universities, research articles, journals and professional associations, and which tends to be more theoretical, codified and abstract. Finally, there is ‘*prescriptive knowledge’*, which is generally acquired through institutional policies and which tends to consist of ‘should’ and ‘ought’ statements. Prescriptive knowledge is more codified than craft knowledge but is less theoretical than systematic knowledge" (Kennedy, 2002).

Leikin (2006), in her research which deals with learning by teaching, proposed to examine mathematical knowledge relevant for teaching using a 3D model linking types of teachers’ knowledge, sources of teachers’ knowledge and forms and conditions of knowledge. In this paper we present only findings which are the result of interviews with Math teachers, so we will use only two of Leikin's dimensions:

- *Types of teacher knowledge***:** content knowledge, substantive knowledge, and syntactic knowledge;
- *Sources of teacher knowledge*: craft knowledge, systematic knowledge and prescriptive knowledge.

2. Method

In this paper we will present an analysis of the interviews that we carried out with the teachers teaching mathematics in elementary school. Among the interviewees were: an elementary school teacher with more than a 5-year experience who became a mathematics teacher only taking professional development courses (T1); an elementary school teacher who is highly experienced and who has never had any formal training in mathematics teaching (T2); a highly experienced teacher who has both a B.A. and M.A. degrees in mathematics teaching (T3); a teacher who has very little experience in Math teaching and who has never had any formal education in mathematics teaching (T4); Last but not least, a teacher who has very little experience but who has a degree in Math teaching (T5).

The interview included 4 questions:

a) Tell us about yourself in regard to the teaching of mathematics in elementary school.
b) How do you see the role of the mathematics teacher in elementary school?
c) What should a mathematics teacher know in order to fulfil his/her role in the best possible way?
d) What, in your opinion, are the best ways to construct this knowledge?

At the end of the interview the interviewee was asked to write a summary of the basic proficiencies that an elementary school mathematics teacher must have, and those that he / she should have. Interview analysis was performed using the *cross-case analysis method*.
In our analysis of the interviews, we first wrote the transcript of each interviewee. Then we filled in a chart based on the categories mentioned above (*subject matter content knowledge* and *sources of teacher knowledge)*, with the information supplied to us by the interviewees. Each researcher filled in the chart separately, to increase its validity and then we compared the results. While writing the analysis, we used a conceptual theoretical approach: we used the informants' language and their culture focusing on their theoretical conceptualization of the phenomena while connecting these with theoretical constructs common amongst the academic community (Shkedi, 2003).

3. Findings

The following table displays the results from the interview analysis according to the two-dimensional model we presented above:

	Craft knowledge	**Systematic sources**				**Prescriptive sources**		
		Systematic Studies in Colleges and Universities	Professional Development Programs	Reading Research Articles, Journals and so forth	Colleagues	Tests and Accountability Systems	Curriculum Standards and Guides	Texts of Diverse Nature
content knowledge	V	V	V	V	V	V	V	V
substantive knowledge	V	V	V	V	V	V	V	V
syntactic knowledge	V		V				V	

Table 1. Types of teachers' SMK and sources of teachers' knowledge.

By analysing the interviewee themes, it seems that a certain amount of disciplinary knowledge is necessary, but this knowledge should focus on profound understanding of subjects related to the elementary school curriculum. The teachers claim that it is preferable that there be wide knowledge of mathematics, but that it is not imperative. The teachers spoke about the disciplinary knowledge that they are supposed to bestow upon their students and tended to speak less of the disciplinary knowledge they themselves are required to have.

An analysis of the table shows connections between the two-dimensions of mathematical knowledge that the teacher requires for elementary school. By filling in the chart based on the information supplied to us by the interviewees, we found out that many of their statements could fit into certain slots (for example, between content knowledge and craft knowledge), other slots could contain only a few of their statements (for example, between syntactic knowledge and knowledge gained by interacting with colleagues), and the rest of the slots remained vacant (for example, between syntactic knowledge and collegues).

Because of space limitations we will present the analysis of three slots, each showing a relationship between one of the parameters of Shulman's approach and one of the parameters of Kennedy's approach.

3.1 "Content knowledge" and "craft knowledge"

The teachers report that there is a very strong connection between content knowledge and craft knowledge. This connection is expressed in a number of ways. All the teachers claimed that they reached new mathematical insights as a result of teaching a certain subject. The meaning of this is that experience strengthens content knowledge by widening the "knowledge circle". Interviewee T2: "Two years ago I taught the subject of the "circle" for the first time. When I tried to prepare for the lessons, I realized that my knowledge about this subject is insufficient. I felt "grey" and could not be as creative as in other subjects that I feel that I know better."
In addition, actual teaching strengthens teachers' understanding that they need to know mathematics. Interviewee T4 says that during her studies in college she did not really tried

too hard to learn the subject matter. Only now does she understand how important it is and she said that she would be happy to take those same courses again.
At this connection it is important that the teacher know the algorithms and definitions of concepts ("instrumental understanding") and also the "relational understanding" by Mellin-Olsen (in: Skemp, 1976) as well. For example, interviewee T1 spoke about adding of half to a quarter: "one should not perform this calculation according to the algorithm, unless you have mathematical insight and you can do it based on numerical sense. Insights such as these also come from experience and from dealing with similar questions in class."
Elementary school teacher needs to develop his/her own number sense if he/she is to promote number sense in his/her students. Interviewee T1 claimed: "I assume that it is important that the teachers themselves have a sense of what half plus a quarter is" She added that it is also important that the teachers have the ability to oversee the children's work and ask the kids after each exercise what the solution should be, and, at the end of the exercise, to check if they came up with a similar solution.
Subject matter knowledge, in the teachers' opinion also develops during the error analysis process that the teachers perform when checking tests and assignments. One of the interviewees commented: "Throughout the years I have learnt that one needs to understand mathematics in order to deal with the students' mistakes and to understand what their source is." (T1).
The teachers stated that the emphasis must be placed on basic mathematics. Interviewee T5 said that it is advisable that a mathematics teacher know higher mathematics, however it's not essential.
The citations show that all interviewees talked about the connection between subject-matter knowledge and insights that come from experience

3.2 "Professional development programs" and "substantive knowledge"

While examining the statements given by the more experienced teachers, we could see that they made the connection between professional development programs and substantive knowledge. However none of the less experienced teachers even spoke about it.
It is evident that this is a connection that is made over the years.
The senior teacher, interviewee T2 said: "I feel that I have gone through a process. I am more mature now." She talks about a change in her teaching as a result of educational development programs that she took part in. We can learn from these words that she now sees mathematics teaching in a different, more profound way, and this leads to a more meaningful learning process by the students. This can be achieved by making the connection between the mathematical concepts and the different ways of applying this knowledge.
Professional development programs enable teachers to create new links between the concepts and ideas of basic mathematics while viewing familiar subjects from a different perspective or from a different direction. According to the teachers, these are the characteristics of high quality in-service courses that affect them and contribute to more creativity in their work. These courses focus on the relationships among the different mathematic concepts. Interviewee T1 said: "I, for example, have undergone loads of in-service courses. There is always something one does not know or something new that improves knowledge. This makes me want to join these courses, hear new things, and learn how to use things that you already know in an entirely different way." During the courses the teachers learn different approaches to the same concept. Lecturers in these programs emphasize the links between various areas in mathematics: "When one of the instructors gave various activities about fraction: for instance circle cutting, thread cutting, drawing a

fraction in a circle, drawing a fraction in a square, namely different models of a whole that are...already by the first activity they can see what a fraction is, the students could see right away what a fraction is, and they also understand that a third is larger than a quarter, namely they see very interesting things already in the beginning. This is an activity I took from a seminar and it is very interesting." (Interviewee T1).

Influenced by the professional development programs, the substantive knowledge of the teachers widens regarding the need to search for different ways to solve the task using various mathematical tools. For example, interviewee T2: "now I think that searching for ways such as these also contributes a lot to the construction of knowledge of both the student and the teacher."

The teachers indicate that during in-service courses they deal with tasks that develop numerical insight more than with regular tasks. Once the teachers have acquired the skill of making the connection between the characteristics of numbers and the arithmetic operations on them; to quote in interviewee T1 they are liable to pass it more easily.

The citations point to the fact that it is only the senior teachers who speak about this connection (interviewee T1, T2). This fact makes sense since during their long years of teaching the teachers have taken part in many in-service courses.

3.3 "Prescriptive knowledge from tests and accountability systems" and "syntactic knowledge"

One of the goals of teaching mathematics in elemntary schools according to the curiculum is developing students' mathematical capabilities such as numerical insight. All the teachers said that mathematical thinking and numerical insight should be developed. They further said that dealing with goals such as these makes the teachers understand what it means "to know mathematics". For example, the teachers said that constructing a concept in mathematics is a result of an ongoing and accumulative process therefore repeating the studied concepts while widening and deapening the meaning of those concepts and linking them to other subjects is essential. In other words, the teachers felt that it was their responsibility to make the student understand the rationale of the curriculum, namely, that mathematics is not a collection of facts but a connected holistic domain.

One of the requirements of the Mathematics curriculum is that the students deal with tasks that enable them to perform mathematical inquiry, and tasks that require mathematical numeracy; and indeed the teachers felt accountable for this type of knowledge. It was not stated explicitly but it was quite clear from what they said that they look for tasks that require searching for different solution approaches, and looking at familiar things from a different angle. Interviewee T3 stated that "we also teach students how to find the subject of a sentence. In fact the kids' problem is finding the connection, they see no connection, they don't manage to make the connections between the verbal instructions in the problem and the numerical data. They don’t seem to be able to ask the proper questions which will lead them to the answer." Interviewee T2 focused on her personal development in the field of syntactic knowledge and how it affected her teaching. In the past what interested her was that her students give the correct answer. She didn't care how they did it or if they gave all the different options. As a result of her acquired awareness (to syntactic knowledge and her sense of accountability to the student knowledge) she now believes that working on different ways to solve the problem and finding all the possible answers contributes both to the knowledge of the student and that of the teacher.

Interviewee T4 expressed the need for support from senior teachers who will help her achieve this kind of knowledge.

4. Concluding remarks

This research focuses on the subject-matter only. The subject matter is one part of a more comprehensive research whose goal is to find out what the required knowledge for Math teachers in elementary school is, from their point of view.

Teachers' knowledge of mathematical subject matter is one of the most important components affecting teaching quality and student achievement. There are different ways to examine these knowledge components. In the current research we examined mathematical subject matter knowledge using Shulman's and Kennedy's approaches. As a result of the interview analysis, we discovered that a two dimensional approach to the teachers' knowledge of the subject-matter helped the teachers become more aware of what is expected of them in order to improve the quality of their teaching. We in turn, learned how the sources of teachers' knowledge contribute to the different types of teachers' SMK. Furthermore, it was interesting to find out which slots contained more statements and which did not contain any statements at all. It interesting to note that it was not only the less qualified teachers who didn't speak about the link between syntactic knowledge and systematic knowledge acquired during qualification, but the university and college graduates never mentioned it either.

It is safe to assume that the use of additional research tools such as lesson observations, observations of individual work with students, focused interviews or questionnaires, would explain these points. One of the central conclusions of the research is that observing elementary school mathematics teachers by means of the dual approach enables identification of the strengths and weaknesses of the qualification programs and the professional development programs.

We are currently working on a questionnaire which is based on those same interviews and on researches concerning the Math knowledge of elementary school teachers. We intend to present our future findings in the conference topic group study.

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