

Perspectives on Mathematical Knowledge for Teaching

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Abstract. Citizens' possibilities to develop intuition and mathematical ideas appear in modern society through instrumentation and instrumentalization: Technology does not only shape the actions of the users but also the mathematical objects to be investigated are shaped by the users. The constructivist viewpoint states that both making of mathematics and teaching of mathematics must relate to this instrumental genesis and teachers must have know-how for new kind of instrumental orchestration. Our contribution discusses the first two issues mentioned as foci of TSG27: (1) Relevant perspectives about mathematical knowledge for teaching, and (2) What teachers need to know and know how to use.

Genetic perspective

To obtain a solid view of mathematics, it is important to know how and from where mathematical knowledge and mathematical thinking appeared and came into life and action. Based on a “human laboratory within 5000 years”, Zimmermann's (2003) findings expressed by Figure 1 (2003) can be considered as a framework not only for the teaching and learning of mathematics but also for what could be characterized as ‘meta-mathematical education’ in a more general sense.

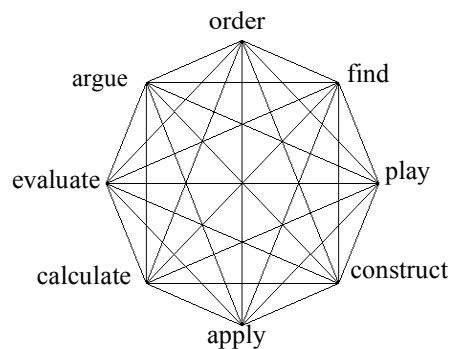


Figure 1. Sustainable networked activities and reasoning tools in producing new mathematical knowledge (Zimmermann 2003).

Eronen and Haapasalo (2007) designed a quantitative instrument based on this octagon to measure the following three so-called *Zimmermann profiles* among students and teachers:

- Math-profile: How strong each activity appears to the subject when using the term ‘mathematics’;
- Identity-profile: How good the student thinks he or she is performing each of the activities;
- Techno- profile: How suitable a computer is in performing each of the activities.

The study revealed very stereotypical profiles among all target groups. The most disturbing finding indicates that traditional mathematics teaching – also in the university - seems to be counter-productive and distorting students' views of mathematics. On the other hand, a constructivist framework using modern technology made a progressive shift towards the promotion of creative activities. The classroom experiments in their pedagogical courses for pre-service teachers showed not only changes in students' perception of mathematics but also to increase their self-confidence in both their ability to apply mathematics and to utilize the technology.

Implication: The Zimmermann components should become a vital part in every teacher's mathematics making. He or she should be able to apply them also in his/her course design, planning and assessment; speaking in more scientific terms to be explained later: to shape instrumental orchestration.

Perspective of Minimalist Instruction

The right-hand half of Figure 1 emphasizes creative mathematical activities, which very often run optimally without any external instruction or demand. Students frequently do not allocate enough time for learning how to operate technical tools and/or neglect teacher's tutoring. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be achieved without reading manuals prior to the classes. Carroll (1990) observed that learners often tend to "jump the gun". They avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulty recognizing, diagnosing, and recovering from their errors. We next pick up the following characteristics of the *minimalist instruction* (cf. Lambrecht 1999 and van der Meij & Carroll 1998):

- Learning is modelled and coached for students with unscripted teacher responses.
- Learning goals are determined from real tasks stressing doing and exploring.
- Errors cannot be avoided and should be used for instruction
- Learners construct multiple perspectives or solutions through discussion and collaboration.
- Learning focuses on reflexive awareness of the process of knowledge construction.
- Criterion for success is the transfer of learning and a change in students' action potential.
- The assessment is ongoing and based on learners' needs.

Implication: Teachers should be able to scaffold mathematics making within self-determined learning environments. The idea of Minimalist Instruction, fitting the sustainable human strategies mentioned above, should confront systems-oriented approaches in own learning or when supporting students' learning.

Educational perspective

The fact that students seem to learn effectively many skills – including mathematical ones – outside the school, forces us to ask if there is something wrong inside the school as far the question "how to learn" concerns. Namely, detailed analysis of the Finnish TIMSS and PISA results reveals that in-classroom activity does not necessarily impacts their students' performance. This opens new challenges to educational research: Which factors in our education are really important for

developing reasoning abilities? If we accept the assumption that the main task of education is to promote a skilful ‘drive’ along knowledge networks so as to scaffold pupils to utilize their rich activities outside school, it seems appropriate to re-define the term ‘education’ in more detail. For this, we adopt the following characterizations of Haapasalo & Kadijevich (2000):

- *Procedural knowledge (C)* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge (P)* denotes knowledge of particular networks and a skilful “drive” along them. The network elements can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) in various representational forms.

Because the dominance of *P* over *C* seems quite natural both in the development of scientific and individual knowledge, an appropriate pedagogical idea also in mathematics could be to go for spontaneous *P*. The logical relation between the two knowledge types in this so-called *developmental approach* is based upon genetic view (i.e. *P* is necessary for *C*) or simultaneous activation view (i.e. *P* is necessary and sufficient for *C*)¹. On the other hand, it seems appropriate to claim that the goal of any education should be to invest on *C* from the first beginning. If so, the logical basis of this so-called *educational approach* is dynamic interaction view (i.e. *C* is necessary for *P*), or again the simultaneous activation view. The latter means that the learner has opportunities to activate conceptual and procedural features of the current topic simultaneously. By “activating” we mean certain mental or concrete manipulations of the representatives of each type of knowledge.

Implication: From the above-mentioned definition of educational approach follows a strict demand for teachers: They should have a thorough understanding of the links between conceptual and procedural knowledge of the topic under consideration. For being able to speak about mathematics education, the teaching process should produce dynamic conceptual knowledge, which promotes more or less useful procedural knowledge. Teachers should be able to apply empirically tested pedagogical models (as the *MODEM*-framework²) so as to promote links between different forms of mathematical representations. On the other hand, teachers must be able to combine systematic planning of learning environments within minimalist instruction.

¹ Concerning the logical relation between conceptual and procedural knowledge, four views can be found in literature (cf. Haapasalo & Kadijevich 2000). The two approaches here are based on these views.

² A large Finnish project *Model Construction for Didactic and Empirical Problems of Mathematics Education* (MODEM; see Haapasalo 2003; 2007; Internet: <http://www.joensuu.fi/lenni/modemeng.html>)

Technological perspective

When designing any learning environment, we meet the conflict between conceptual and procedural knowledge: Do we have to understand for being able to do, or vice versa (Haapasalo 2003). Implementing of technology makes this more complicated but at the same time opens new, even progressive approaches, especially when learning through design is utilized. Technology-based (or ICT-based) mathematics education has expanded to include the following solutions, many of those being used via networks or in local computers, including modern calculators and communicators (Haapasalo & Silfverberg 2007):

- computer algebra systems (CAS), dynamical geometry (DGS), and dynamical statistics (DSS);
- spreadsheets, drawing programs, and other versatile tools for mathematical modelling;
- online databases of available software, instruction, research, statistics, history, etc.;
- online communication in all of its synchronous and asynchronous forms;
- new kinds of environments to read, write and publish, including tools for support;
- tools for utilizing of the world-wide web: search engines, etc.;
- online experiments and simulations in diverse forms of digital educational content
- online libraries containing books, learning objects, other teaching materials, etc.
- learning management systems (LMS), which are used to manage students and course materials;
- digital portfolios;
- virtual worlds in the form of three-dimensional immersive environments offering, for example, shared exhibitions or other forms of collaborative functionality.

When using a tool within more or less spontaneous procedural actions, the tool, especially at the beginning, puts certain limitations on what can be investigated and how. Adapting the term of Trouche (2004), we mean by *instrumentation* the process when the tool shapes the actions of the users. On the other hand, users often find their own schemas and schemes to use the tool. In this process of *instrumentalization* not only the use of the tool, but also the objects to be investigated are shaped by the users. Not long ago, students had to make paper-and-pencil manipulation of $1/\sqrt{2}$ to $\sqrt{2}/2$, because the average-man's instrumentation did not allow making the calculation easily. Today, any sophisticated user proceeds faster by using the calculator keys SQR and 1/X. This *instrumental genesis* has an impact on how mathematical situations appear for a modern citizen. On the other hand, progressive technology in all above-mentioned forms can be interpreted as an orchestra. Therefore it seems appropriate to use Trouche's term *instrumental orchestration* when scaffolding³ student's instrumental genesis.

³ Trouche (2004, p. 296) speaks about "external steering of students' instrumental genesis", which we, however, do not find a solid expression within constructivist views on teaching and learning.

The above-mentioned opportunities together with a changed conception of knowledge and learning are leading to a paradigm shift: learning of mathematics is more distributive (independent of time, place and formal modes), constructivist (learning community centred) and technologically enhanced. Even though students use technological applications in more informal way, as on their free time, for too many curriculum designers and teachers, technology-supported learning environments appear as “interactive e-textbooks”, based on objectivist-behaviorist tradition to learn basic facts and skills. Haapasalo & Silfverberg (2007) describe in detail, how the Finnish school curriculum neglects ICT opportunities even though students’ top scores in PISA studies are not due to classroom activities. They also describe quite similar situation in other countries. In UK at Key Stages 3-4, for example, more than $\frac{3}{4}$ of the students never or hardly ever used ICT during mathematics lessons.

Implication: Teachers should become aware, what happens outside the classrooms and how making of mathematics and applying it in modern world is changing through instrumentation, instrumentalization, and Learning by Design (cf. Eskelinen & Haapasalo 2007). This should have immediate impact of instrumental orchestration inside and outside the classroom.

Problem-solving perspective

We skip describing the features of problem solving in detail, this topic being widely analysed in our community. We just invite the reader to take a look on Haapasalo’s (2007) examples, which show very poor metacognitive abilities among teachers, whereas this was not the case when students worked voluntarily in their free-time by using technology. It seems that school contaminates thinking both among teachers and students.

Implication: Fostering students’ problem-solving abilities means emphasizing the genesis of heuristic processes and students’ possibilities to develop intuition and mathematical ideas. This can be realized when making constructivist views of teaching and learning alive: Concepts and procedures can be at least partially constructed by students themselves - and also the other way around - well-known concepts can be applied in one form or the other one. Solving of authentic problems by utilizing modern technology promotes both of these goals allowing to reflect the relationships of mathematical components under consideration. Opposite to conventional teaching, which often seems to rather contaminate than promote student’s metacognitive abilities, progressive pedagogical solutions utilizing modern technology allow us to gain profit from student’s natural activities outside the classroom.

Socio-constructivist perspective

The slogan of constructivism in our scientific community started about twenty years ago. Still, there are quite few empirically-tested models how teachers can plan and realize learning environments within that paradigm, fulfilling the demands described in the former chapters and taking into account group-dynamical and collaborative aspects. Concerning group development, the famous classification of Tuckman (1965) gives a nice framework: *forming, storming, norming, and performing*. We skip illustrating these phases, because by typing-in 'Tuckman group processes' Google produces more than 72000 appropriate hits with interesting links. There are numerous studies of group dynamics within problem-solving. However, very important aspects, such as the impacts of knowledge structure, pedagogical philosophy and support for reflective communication on the learning process are to be clarified as has been done in the *MODEM* -project (see footnote 2). We skip introducing this framework and instead of that, we place emphasis on the concepts 'collaborative' and 'socio-constructivist'.

By *collaboration* we mean that student teams define their own goals for their work and maintain active processes towards this goal, profiting from a diversity of perspectives and opinions, and a free flow of information. In an ideal collaborative learning environment many features of minimalist instruction can also be identified. With respect to our socio-constructivist framework, we apply the famous *pragmatic theory of truth* emphasized by the famous philosopher Peirce (see Figure 2). When an open problem is given, the teams work in causal interaction with this problem under collaboration. After testing the viability of radical ideas among the teams and between the teams, only those ideas finally remain which are viable for the whole social group consisting of those teams.

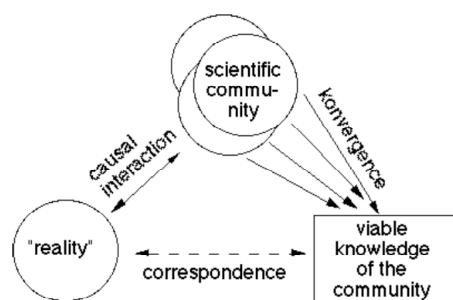


Figure 2. Pragmatic theory of truth: viable knowledge as a result of social constructions (cf. Eskelinen & Haapasalo 2007).

Implication: Teachers should be able to plan and realize collaborative socio-constructivist learning environments, based on research-based frameworks. They should be able to scaffold the constructions process of viable knowledge through radical constructions among the learners.

Managerial perspective

Teachers should be ready to encounter e.g. the following serious questions: What is wrong with mathematics teaching because its reputation is so negative? What business principles can propose ways out? To find methods, which could improve the image of mathematics teaching among general audience, teachers could ask what has to be done if it would be an enterprise with a similarly bad reputation. Hvorecky (2007) proposes long-term solutions, most of them requiring substantial changes in mathematicians' minds and in their approach to their teaching methodologies.

Implication: Instead of talking about internal problems of mathematics, teachers should be able to discuss the issue as a managerial problem. They should be ready for at least the following measures: (1) Setting up priorities, (2) Making mathematics more user-friendly, edible and digestible.

Closing words

Mathematics education is a very wide and complicated discipline as a science. In addition to the perspectives described above we could add many other perspectives more. To develop research-based theories for teaching praxis we need to combine knowledge of mathematics and its history, philosophy, psychology, sociology, physiology and ICT, for example. Therefore, to develop a praxis theory we usually need a lot of work and persistence. On the other hand the situation is analogous with industrial production: viable and sustainable products are very often beautiful, simple and easy to use -otherwise nobody would buy them. This has been the starting point in first author's MODEM -project, having been a basis for teacher education about two decades. The idea to use a quasi-systematic model for the planning of learning environments but on the other hand to allow free architecture of learning in the authentic learning situations sounds contradictory. However, this is the way our system works in everyday life. We pick our food quite randomly from breakfast buffet, even though the kitchen personnel has used all its expertise to plan and construct things for us very carefully. Unfortunately we must express our worries about the very bad reputation of mathematics and mathematics teaching. We are afraid that mathematics teaching in school can have the same destiny as Latin language, if teachers are not ready to accept the shift of their profession. Instead of trying to teach mathematics in the classroom they should accept the reality (what is already happening) that making of mathematics in the sense of Zimmermann activities runs in most natural way on students leisure time. Scaffolding this important resource in appropriate way puts even more demands on teachers' expertise, but at the same time increases the social status of teachers.

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