

INVESTIGATING MATHEMATICS FOR TEACHING THROUGH PROBABILITY IN PRACTICE

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Abstract

This paper reports on two similar studies that investigated the mathematical work of teaching that two teachers engaged with as they taught probability. The studies involved case studies of one teacher teaching grade 8 learners in South Africa and another teaching form 3 learners in Malawi. Six categories of demands on teachers that were refined from Ball, Bass and Hill's (2004) aspects of the mathematical problem-solving that teachers face as they teach mathematics, were used to observe the mathematical work of teaching that was demanded of the two teachers. The findings show that the mathematical work of teaching that is demanded of the teacher in a multilingual context includes hearing disconnects in mathematical terms. The findings also suggest that restructuring tasks is an inevitable feature in teaching probability and that the restructuring might entail shifting the mathematical outcomes from those intended and not necessarily a scaling up or scaling down of the task as Ball et al describe.

Introduction

Probability is a relatively new topic in South Africa and Malawi curricula. In South Africa probability as a topic of study in mathematics was introduced for the first time in grades 8 in 1992 (Laridon, 1995). In Malawi it was introduced in secondary school in 1995 (Ministry of Education, 1995). Although a decade has now passed, performance of students on probability questions in the Malawi Schools Certificate Examinations continue to be poor with facility levels lower than 13% each year. This seems to indicate, among other things, that the teaching is not being effective and/or some teachers are opting not to teach probability. As a mathematics teacher educator, this is of great concern and it raises a number of questions including: how best should we prepare our teachers? And what knowledge do teachers need in order to teach probability effectively?

These are indeed appropriate questions to ask because for each topic or concept in mathematics, teachers need mathematics for teaching that topic or concept (Adler, 2005). Mathematics for teaching is more than just knowledge of the mathematical concepts, and how to solve relevant problems that entail mathematical processes and procedures. It includes

knowing what to do, mathematically, in order to make that mathematics accessible to learners (Ball and Bass, 2000; Ball, Lubeinski and Mewborn, 2001). For probability, we need to ask the question “what is it that teachers need to know and know how to do to teach probability well?” This paper discusses some results of two research studies that attempted to answer this question.

The studies were focused on probability as a specific content area in mathematics. Similar to some other topic focused studies (for example, Marks (1992) – fractions, Sanchez and Llinares (2003) – functions, Even (1990) – functions) the study attempted to identify and describe the specialised mathematical knowledge that teachers need if they are to teach probability well, and in the context of our schools. I contend that the specificity of probability as well as the specificity of the context of our schools, the study adds to the general discussion of mathematics for teaching. The general question the study was investigating is: what mathematics do teachers need to know and be able to do in practice in order to teach probability in secondary schools? To help answer this question, the study explored three inter-related questions (i) What aspects of probability are encoded in curriculum documents? (ii) What mathematical ‘problem-solving’ do teachers do as they go about teaching probability in their classroom? (iii) What knowledge resources do teachers draw on as they do what they do? This paper focuses on the second question, and reports on the problem-solving of two teachers as they introduced and taught probability, one in South Africa and in the other in Malawi.

Theoretical orientation and analytic framework

The theoretical underpinning of the study is that mathematical knowledge for teaching is situated in the practice of teaching (Adler and Davis, 2006; Ball and Bass, 2000; Ball et al, 2004). Therefore, to study it entails an analysis of curriculum in both documentation and practice. In studying teaching, the study draws on Ball et al (2004) who suggest 8 types of problem-solving that mathematics teachers do as they go about their work. I condensed these into six, as follows: Definitions, Explanations, Representations, Working with students’ ideas, Restructuring tasks, and Questioning. It is this six-part analytic framework that I used to study some teaching of probability in relation to critical question (ii) above.

The studies involved observing two mathematics teachers teaching probability. The two studies were done consecutively; first in Johannesburg and thereafter repeated in Malawi. The

first teacher was teaching in grade 8 at a township secondary school in Johannesburg, South Africa. The second teacher was teaching in form 3 at a township secondary school in Blantyre, Malawi. Grade 8 and Form 3 were chosen because that is when probability is introduced at secondary schools in South Africa and Malawi respectively. Township is a context of interest in that it is similar to many schools across towns in Africa, and particularly because I work with teachers in such or similar contexts. The two teachers were an opportunistic sample, both known to the author through University of the Witwatersrand and University of Malawi, and interested in exploring their own teaching of this topic. An important point to note here is that the idea was not to evaluate the teachers' teaching but to learn from it, and particularly about the mathematical demands of teaching probability, and in this context. A total of eight lessons in South Africa and five lessons in Malawi were observed and video recorded.

Problem solving demands on the teachers

Analysis of all lessons revealed that each of the six aspects of mathematical problem solving by the teacher was evident, but in uneven ways. There were many instances of 'working with students' ideas' and 'restructuring tasks'. There were also some instances of 'defining', especially in the Malawi classroom. In comparison, 'explaining', 'representing' and 'questioning' were marked more by their absence than their presence. Therefore, I will focus on 'working with students' ideas' and 'restructuring tasks' with the purpose of illustrating the kind of problem solving that was demanded of the teachers.

Extract 1 - hearing disconnects

During the first lesson in the South Africa classroom, the teacher asked learners the question "what does probability mean?" His aim was to find out if the learners had any familiarity with the idea of probability. At first there was no response from the learners, then the teacher encouraged them to try. The extract below captures the discussion that followed in the class

Extract 1

Teacher: okay, anyone who can give me a try ... just give a try ... you are allowed to guess, educational guess is good
(pause, class is quiet)
Teacher: take a guess...what do you think probability could mean?
(points to one boy raising his hand up)
Learner 1: (standing) it is about disabled people
Teacher: it is about?

Learner 1: disabled people
Teacher: disabled people he says probability is about disabled people. What are you saying (addressing the class) What do you have to say? (points to another boy raising his hand)

Learner 2: (standing) it is about all things we can do
Teacher: it is what?
Learner 2: all things we can do
Teacher: all things we can do
Learner 2: yes
Teacher: is probability
Learner 2: yes
Teacher: aha, somebody says it is all things we can do ... what are other people saying? Am going to take the last guess (pause) ... anyone to ... make an attempt? Let's give another person a second chance of ... eer... an attempt. Anyone to attempt?

Teacher: (points to Learner 3) Learner 3?
Learner 3: am still thinking
Teacher: you are still thinking
Learner 3: hhhmm
Teacher: okay, so ... other people you don't want to make an attempt ne? and it means you are hearing the word for the very first time ... probability

Learners: (inaudible)
Teacher: okay

The problem for the teacher here was that the responses from the learners were both unexpected and unintelligible in his terms. Talking with the teacher after the lesson, and as is apparent from the text, he said he did not expect the responses learners gave, and that he did not know how to make sense of the learners' ideas. The source of the learners' ideas is not the focus of this paper. However, it has particular relevance here, that is a function of learners in this class learning mathematics in English, where this is not their main language. From experience, learners who are not first language speakers of English often equate words that sound alike. In this case, the word 'probability' sounds like 'disability' or 'ability'. From this perspective, the two learners' responses of "disabled people" and "all things we can do" are a function of the sound of the word, rather than any experience of the use of the word.

On the face of it, an obvious move is to enquire into the strangeness of the learners' responses. In the messiness of classroom life, it is precisely these way out meanings that are ignored. Yet, in the context of multilingualism, attentiveness to how words sound as well as mean is important. As Adler has argued (2001), different pronunciations, and so sound alike words, can become sources of confusion in mathematics (e.g. size, sides, sights were all used by learners in a trigonometry lesson to refer to the size of an angle). The mathematical work of teaching has linguistic entailments, and the problem-solving a teacher is required to do on their feet is to pay attention to what is said, how it is said and what could be meant, if they are to enable learners in multilingual settings to work with the language resources they bring to class. This linguistic aspect of problem-solving tasks of teaching mathematics is not

highlighted in Ball et al's more general framework, and is an important aspect of working with learners' mathematics. The example here suggests that teachers need to be able to hear disconnects in mathematical terms, and reconnect these in mathematical ways – disconnects like 'disability' are indicative of what it is learners bring to the topic under discussion.

Restructuring tasks – shifting mathematical outcomes

Ball et al (2004) discuss making judgements about mathematical tasks and modifying them accordingly - restructuring of tasks, as an important aspect of mathematical problem solving for teachers. By restructuring, they refer to scaling down of the task if it is too difficult or scaling it up if it is not challenging enough for the learners. In this study I identified another kind of restructuring of tasks – one that requires shifting of mathematical outcomes. I illustrate this using two activities, one in Malawi and the other in South Africa.

Malawi Classroom

In the Malawi classroom, the teacher divided learners into five groups and gave them some tasks involving coins, cards and dice. The dice task was to throw the dice 16 times and record the frequencies of appearance for each number on the die. The table below was drawn on a chart and placed on the chalkboard. The teacher recorded as the groups presented their results.

	Gp 1	Gp 2	Gp 3	Gp 4	Gp 5	Total
1 dot	3	3	1	2	0	9
2 dots	1	3	2	1	5	12
3 dots	2	0	2	6	1	11
4 dots	4	2	3	1	4	14
5 dots	2	3	5	1	1	12
6 dots	4	5	3	5	5	22
Total number of trials = 80						
P(even number) = $48/80 = 0.6$						
P(odd number) = $32/80 = 0.4$						
P(1 dot) = $9/80 = 0.1125$						
P(5 dots) = $12/80 = 0.15$						

As can be seen from the table, total frequencies for each number of dots on the die was calculated and recorded. The teacher wanted the learners to compare the relative frequencies of some events (e.g. event of getting an even number) obtained from results of individual groups against relative frequency obtained from the class (total of all groups). His aim was to show that the larger the number of trials the better the approximation to the theoretical probability. Extract 2 shows the teacher's attempt to do this.

Extract 2

- Teacher: Remember that there were some errors that were made (referring to errors when picking cards) but still we see that the results are not very far from the actual results from theoretical probability isn't it? So close to the theoretical probability.
This can be testified if maybe we just choose like let's take this die eti? Say *mwinamwake chifukwa tinapanga mma* (maybe because we did it in) groups of 8 *eti?* (not so?) In each group *aliyense anapanga kawirikawiri eti?* (everybody did it twice not so?) *Mpakana tinapanga imodzi, group imodzi ija apanga ... apanga ka 16* (that suppose we take only one group, one group did it 16 times) so the total number of trials will be 16 so here it's like the total number of experiments will be 16 (writes). So let's see *kuti* (that) Probability of Even *anakakhala bwanji anakakhala kuti akupanga anthu 16 okhawo*, (what will it be if we consider only the 16) so let's see:
(checks group 4's results on chart, and adds the frequencies for even number of dots)
- Teacher: (writes $P(\text{even}) = 7/16$)
7 over 16 which is equal to?
- Learner: (calculates using a calculator) 0.4375
- Teacher: 0.437?
- Learner: 5
- Teacher: 0.4375 which is almost 0.44.
So we see *kuti* (that) that one, which is closer to 0.5 than the other? Between this one which we found (refers to 0.6 on the chart – the calculated $P(\text{even}) = 48/80 = 0.6$) and that one (refers to 0.4375) which is close ... closer to 0.5?
- Learners: (some inaudible noise)
(teacher pauses, looks at the chart and checks his addition)

South Africa Classroom

A similar incident occurred earlier in the South Africa classroom. The teacher gave his class a task to do in groups. Each group was to toss a coin 30 times and record the frequencies of heads and tails. Results from all the groups were recorded cumulatively in form of a table on the chalkboard. The completed table was as shown below.

	Heads	Tails
After 30 flips	15	15
After 60 flips	34	26
After 90 flips	48	42
After 120 flips	62	58
After 150 flips	72	78
After 180 flips	89	91
After 210 flips	105	105
After 240 flips	119	121
After 270 flips	131	139

Extract 3 below is of the discussion that followed the activity. The discussion was in response to the question “what happens as you add more and more flips?”

Extract 3

- Learner 1: If we add more flips the numbers become more and more
- Teacher: What is she saying?
- Learners: If we add the number become more and more

Teacher: what becomes more?
Learners: the number that we have
Teacher: another group?
(some noise from learners)
Teacher: please people you are making noise... I want one at a time .. should come here.. I want you to explain what is happening to the frequencies
Learner 2: when we add more flips the numbers increase
Teacher: is there any group which is saying different other than the numbers become more and more?
Learner 3: in our group we say the ... as we add more and more flips the number is improving
Teacher: what do you mean the number is improving?
Learner 3: it is becoming more
Teacher: who has something different? I want you to look at the frequencies, what is happening as more flips are added
Learner 4: the number of tails is more different
Teacher: what about the relative frequency? I want you to look at the relative frequencies, what can you say?
(Silence)

I should mention that both these tasks were taken from textbooks. The teachers' aim (and also the textbooks') was to use this to illustrate that as the number of trials increase the better the experimental probability approximates the theoretical probability. In other words, the more the trials the closer the experimental gets to the theoretical probability. For the Malawi experiment, it was expected that whole class result of many trials would be close to 0.5 (the theoretical $P(\text{even})$) than the individual groups' results of much fewer trials. While for the South Africa experiment, it was expected that as more and more flips were added the closer the number of heads and tails would get to being equal.

I will not go into what happened next in each of these lessons, instead I want to focus on the problem the teachers were faced with, that is, the fact that the task didn't throw up the concept of 'law of large numbers' as expected. The tables generated by the tasks did not visually nor conceptually support the concept as intended. Restructuring of some kind was needed, in order to continue the lessons. The restructuring required here is not a scaling up or scaling down. The learners successfully completed the task, the problem for the two teachers was that they were presented with results they did not expect and could not use as planned.

So what would be a good next step when in such a situation? What do the teachers need to know about and for probability, and know how to do when the activity in which learners are engaged is not able to function as planned i.e. to provide the means to reflect on the law of large numbers. One possibility is to extend the task (as opposed to rescale) and generate more data so as to provide more possibilities for leading to the intended outcome. A different possibility is to use the table as is with the same data, but shift the outcome from 'law of large

numbers' to other possibilities for developing probability concepts that the table presents. For this to happen, teachers need the knowledge to realise that although the task did not give the expected results, it offered opportunities for other concepts such as 'uncertainty', 'randomness', 'possible outcomes' and 'likelihood', which the teachers could explore with the learners. There is a lot to gain from these results but the teachers require the knowledge to see that and restructure the task accordingly. The restructuring requires shifting the mathematical outcome from those intended. This kind of problem solving for the teacher arises in the context of probability because of the uncertainty of outcomes from probability activities done in class. I thus propose the shifting of appropriate mathematical outcomes as an additional component of restructuring tasks.

Conclusion

I have presented three instances in the teaching of probability in township (and multilingual) schools in South Africa and Malawi, and have used these to discuss some of the kind of mathematical problem-solving teachers face. The first extract brought issues of language to the fore. This is not elaborated in Ball et al's framework and suggest a specificity important for teachers to know and be able to act on. On the spot problem-solving is needed in multilingual settings when learners are working to understand both new concepts and the language in which these are being presented. A mathematical ear is needed to hear and then engage learner utterances that reflect sound-alike and not only mean-alike ideas. The second and third extracts bring out the issue of restructuring of tasks. I argue that restructuring tasks is an inevitable feature in teaching probability and that the restructuring is not necessarily a scaling up or scaling down of the task as Ball et al describe. Rather restructuring might entail shifting the mathematical outcomes from those intended. These add to the description and understanding of mathematical knowledge for teaching.

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