DEVELOPING MATHEMATICAL KNOWLEDGE FOR TEACHING IN A PRE-SERVICE SECONDARY TEACHER EDUCATION PROGRAMME IN SOUTH AFRICA

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Introduction

The education landscape in South Africa has undergone substantial change in recent years with the introduction of new school curricula for Grades 0-12, new teacher education qualifications and related new curricula, and the shift of teacher education into the higher education sector. All this follows the demise of apartheid, and is framed by national priorities of redress, repair and equity. Concerns about student performance in mathematics abound, and many attempts continue to be made to address the situation through government- and privately-funded initiatives, yet for the most part these have had very little impact on the problem at large. It is not surprising that teachers' mathematical knowledge has been identified as a key factor in improving student performance. Coupled with this is a concern about shortages of mathematics teachers due to the small numbers entering the profession, and attrition within the profession due to low morale in teaching as a whole and as a consequence of HIV/AIDS.

A new Bachelor of Education (B.Ed) degree was recently introduced across the country and given the current climate of transformation and transition, it brings with it new possibilities for addressing some of the challenges in mathematics teacher education in the country. In this paper I reflect on the conceptualisation, design and implementation of the secondary mathematics programme at our University. I discuss some important lessons that have been learnt so far, as well as gaps that have been identified in the current design. I then suggest a potentially more ambitious framework that addresses some of the gaps and concerns.

The secondary mathematics components of the B.Ed programme

The B.Ed degree was introduced at our university in 2003. It is a four-year professional degree offered entirely within the School of Education. All mathematics courses are specifically designed for prospective teachers and are taught by staff whose primary interest is mathematics education. At a time when the traditional model of secondary teacher preparation (undergraduate degree followed by teaching certificate/diploma) is attracting very few prospective mathematics teachers, the B.Ed opens up an alternate route into secondary mathematics teaching. And with the recent re-introduction of government-funded bursaries, the number of students studying to become secondary

mathematics teachers via the B.Ed route at our University is growing substantially. Thus the B.Ed programme holds the potential to produce increasing numbers of mathematics teachers, although this is complex given the backgrounds of the students entering the programme. The majority of students who are accepted into the B.Ed programme would not be accepted into a science degree programme because their maths results in their final year of schooling are too low. Many have had a poor school education, attending poorly-resourced schools where teachers may be un(der)-qualified in mathematics. Thus, from a university point of view they would not be considered as mathematically able nor has having mathematical potential. In addition, the majority of students do not speak English as their main language. Thus the needs of the students were a key factor in the design of the mathematics programme.

A second key factor was the new secondary school curriculum which reflects substantial changes in terms of content and pedagogy, and now has many similarities to so-called "reform" curricula in many other parts of the world. For example, the curriculum has been broadened to include applied topics such as statistics, mathematical modelling and financial mathematics. In addition, greater emphasis has been placed on communicating mathematical ideas and on mathematical processes such as reasoning, modelling and proving. Our task was thus to design the mathematics components of the programme to engage the dual objects of mathematics and teaching, and to provide an opportunity for students who are under-prepared for tertiary mathematics to learn and re-learn mathematics in ways that deepen their understanding and give them confidence in their mathematical abilities and in their abilities to teach mathematics.

In the early stages of conceptualisation, the intuitive appeal of *mathematics for teaching* gave us confidence to design and argue for a programme that was a fairly radical departure from what was typically being offered across pre-service mathematics teacher education programmes in the country. The literature base available at the time (e.g. Ball, Lubienski, & Mewborn, 2001; Even, 1990; Kennedy, 1997; Shulman, 1986, 1987) provided some theoretical base for our project. While the overall mathematics programme was conceptualised holistically within the broader framework of the B.Ed curriculum, the timeframes for conceptualising, designing and implementing specific courses were such that this had to be done incrementally – on a year-by-year basis. It was thus typical for staff to be developing the detailed aspects of courses while they were in the process of teaching the course. We chose to use mathematical domains as the structuring principle for most aspects of the programme. Thus most courses were organised around mathematical content domains (e.g. functions, calculus, statistics) with mathematical practices embedded in the courses. This choice was largely a pragmatic one since mathematics courses in higher education are typically

organised around mathematical domains and this provided face validity for a new programme. Also, such a design is easier to implement when working incrementally on a year-by-year basis. The coherence of the programme was achieved through strong central management with continual attention being given to gaps, overlaps and opportunities for links within and across courses. An overview of the mathematics courses is given in Table 1.

Year of study	Mathematics and mathematics education courses for prospective secondary maths teachers
1	College algebra and functions Geometry and trigonometry
2	Introductory calculus Introductory vector and matrix algebra (with emphasis on geometric links) 1st methodology course
3	Mathematical modelling Statistics
4	Mathematical connections - links between different domains of mathematics Financial mathematics 2nd methodology course

Table 1

As can be seen from the table, students take eight mathematics courses (two per year), and two mathematics methodology courses. All courses are the equivalent of one semester (approx 12 weeks) but contact hours per week range from 3-8 depending on the course. In addition, students spend six weeks (two three-week blocks) in schools each year where they gain practical teaching experience. Of this total of 24 weeks, approximately 15 are focused on mathematics teaching, with the remaining nine weeks focusing on the students' other teaching subject.

Following (Boaler, 1997) we believe that what students take up from a mathematics course is a function both of the mathematical content and how it is learnt. We thus work hard to establish and maintain a mathematical discourse community characterised by a culture of participation and inquiry. Such a culture takes time to become well-established, partly due to students' difficulties in expressing themselves in English but mainly because it marks a substantial shift from their previous experiences of learning mathematics. In addition we make extensive use of spreadsheets, dynamic geometry software and graphing calculators to support investigative work.

What we have learnt

In developing a total of 14 mathematics and mathematics education courses over four years across the B.Ed programme, what came to be an "innovation treadmill" gave us continual opportunity to engage the notion of mathematics for teaching, at theoretical and practical levels. At the same time, the research literature in the field began to grow (e.g. Adler & Davis, 2006; Adler & Pillay, in press; Ball, Bass, & Hill, 2004; Davis & Simmt, 2006; Doerr, 2006; Even, 1990; Kazima & Adler, 2006);

and this continued to influence our thinking. So over the past six years we have gathered much anecdotal evidence about our programme but have not yet had opportunity to engage in systematic research to investigate these findings more thoroughly. In this section I discuss briefly seven issues that have emerged and which pertain in various ways to mathematical knowledge for teaching.

- *Students take mathematics courses more seriously than methodology courses*. This claim is based on observations of the effort expended in the different kinds of courses, student attendance, and on discussions with various students in different cohorts. There are many reasons for prioritising mathematics courses – a realisation of the gaps in their own mathematical knowledge and a desire to improve their knowledge, a desire to learn more mathematics and to know well the content they will teach, a naïve belief that teaching mathematics is easy and that the central purpose of initial teacher education is to acquire the necessary content knowledge coupled with a conglomerate of teaching tips. Although many struggle at first to both recognise and realise a new pedagogic practice (Bernstein, 1996) in the mathematics courses, by second year the majority of students are willing and able to participate actively. Students appear to take methodology issues more seriously as they progress in their degree. It appears that students pay a great deal of attention to how their teacher educators teach mathematics courses, and this has important implications for what comes to be construed as modelling the practice of teaching but this is beyond the scope of this paper.
- *Integrated courses of mathematical content and methodology are difficult to implement and are not effective in the current climate of curriculum change in South Africa*. When the new programme was first introduced, we included methodology issues in the mathematics courses. We soon observed that many students were not in a position to take up these issues, partly because they did not see the importance of such work, but also because they were struggling too much with the mathematics of the course and even the school mathematics in which the methodological issues were embedded. A typical example is the topic of integers which was included in the first year course to engage with content they were likely to face in their upcoming school-based block, to engage with learners' thinking, to interrogate some text book activities and to identify a selection of appropriate teaching strategies. Many students struggled to focus on the topic from a teaching perspective. For example, having proposed to teach the subtraction of a negative number, such as $5 - (-7)$, through reference to multiplication of negative quantities, they struggled to grasp the contradiction in sequencing subtraction before multiplication. This led to their grappling with subtraction of negative quantities as well as multiplication of two negative numbers. While this was productive to some extent, there was insufficient time to focus on integers to develop the necessary depth of understanding and

- *There are ongoing tensions between covering mathematical content and developing a mathematical discourse community.* Given the expectations of tertiary mathematics and the breadth of the South African school curriculum, tensions between covering mathematical content, attending to mathematical practices and developing a mathematical discourse community are inevitable. A compromise has to be reached but the situation is exacerbated when students enter the programme with poor mathematical backgrounds, when they are struggling to communicate in the language of teaching and learning, and when class sizes increase dramatically. A typical situation where this occurs is when students are given a problem to investigate and they use a variety of different approaches. Some approaches are more productive than others in moving the mathematics forward but if these approaches are the only ones that are ever chosen to be shared, it becomes extremely difficult to build a community that values diversity, complexity and broad-based participation.
- *There are continual tensions between breadth and depth of content coverage.* In looking at Table 1, many may be surprised at the lack of advanced mathematical topics such as abstract algebra, and the lack of additional courses in topics such as a calculus. We feel this unease daily but a second course in any topic comes at the cost of introductory courses that cover the breadth of topics included in the school curriculum, and again this situation is exacerbated when students enter the programme with a poor mathematical knowledge base. Our current thinking is that the inclusion of additional courses in advanced mathematics should not come at the cost of an in-depth study of school mathematics, which is not getting sufficient attention in the current programme anyway.
- *A space must be forged for engaging deeply with school mathematics.* We concur with (Cooney & Wiegel, 2003) that mathematics programmes for secondary teachers must revisit school mathematics in substantial ways. When students first did school maths, they were teenagers, and so their understanding of the work was inevitably incomplete and lacking in insight. This is particularly true in the South African context. Within initial teacher education they need to revisit school mathematics as adults to engage the content, drawing on their new knowledge of

aspects of advanced mathematics and from the perspective of prospective teachers rather than learners. I shall elaborate this point more in the next section.

- *There is a tension between developing mathematics courses for maths teachers and mathematics courses as service courses for other disciplines.* In a typical science degree initial mathematics courses are often prerequisites for students studying towards a variety of major subjects, such as physics, chemistry, statistics, computer science and actuarial science. So there is an expectation that the mathematics courses will include the necessary content for application in other disciplines. We have experienced a similar situation with respect to pre-service science teachers. Our courses thus need to fulfil two functions in this respect – to prepare future mathematics teachers in the ways described above but also to cover content necessary for physics and chemistry. There are instances where the necessary science-related mathematics is not deal with early enough in the four-year programme to be useful to science students when they need it.
- *Dealing with large classes demands a reconsideration of many of the key principles in our design.* When the programme began in 2003 our classes consisted of 20-30 students in second year with even smaller groups in fourth year. In the past two year our numbers have grown substantially, and in 2008 there are 150 students in the first year class and 75 in the second year class. Such growth in numbers is desirable but brings with it many new challenges such as difficulties in building a discourse community where students have opportunity to participate actively in class discussions, access to computer resources for investigate work, and providing academic support for students who are struggling. It is clear that the success our programme is now dependent on our ability to scale up, to identify and retain the key aspects of the programme, and to forego the others in the interest of producing more mathematics teachers.

Gaps in our programme

Our programme is still young and needs refinement and adaptation, particularly with respect to scaling up. We are particularly aware of three gaps in the current design that pertain to mathematical knowledge for teaching: engaging with school mathematics, engaging the mathematics education literature and engaging the practice of mathematics teaching. I discuss each of these further below.

- *Engaging with school mathematics*. Students need to revisit school mathematics in various ways and for various reasons. Some aspects, such as functions and sequences and series, must be revisited *before* proceeding with advanced mathematics. Other aspects, such as limits and asymptotes, emerge *within* advanced mathematics and thus are inevitably explored in more depth. But there are other topics such as integers, and proportional reasoning that are assumed to be in place and are not given explicit attention from a mathematical point of view. So

students may "know" the content in a procedural way but can't deal with partially correct ideas. While some might argue that methodology courses should deal with these issues, our experience suggests that our students' difficulties in these areas are fundamentally mathematical and need to addressed separately from, but obviously related to, the teaching of the topics. Thus we believe there is a need to open up a space for students to study school mathematics in depth – not simply to revisit school mathematics as adults but to prepare them to do the complex work of teaching where, for example, they learn about typical difficulties that school learners have, where these stem from and how they might be addressed.

- *Engaging the mathematics education literature.* Mathematics teachers in South Africa know very little about the findings of mathematics education research, yet there is knowledge from the field that is helpful for their own learning and essential for teaching. For example, teachers should be aware of process/object conceptions of functions and algebraic objects (e.g. Sfard & Linchevski, 1994) to understand the origins of learners' difficulties but also to reflect on their own learning of these aspects. Similarly they should gain access to the literature on difficulties in learning the limit concept. They should be familiar with the literature on learners' conceptions of variable and understanding of equality. These issues are not merely about teaching methodology – they lie at the centre of the knowledge that teachers need before they begin to design tasks and plan lessons. They need to be engaged, at least in an introductory manner, at the same time as dealing with the relevant mathematics. It is this knowledge that we argue comprises part of the professional knowledge that distinguishes mathematics teaching from other mathematically intensive professions.
- *Engaging the practice of mathematics teaching.* There needs to be a clear distinction between revisiting school mathematics, engaging the appropriate mathematics education literature and preparing students for the practice of mathematics teaching. It seems that in pre-service teacher education, particularly at secondary level, revisiting school mathematics is too often taken as a proxy for learning to teach it. We have often found ourselves in a methodology class where we are planning to work on the teaching of a particular topic but end up in extended discussions about the mathematics itself and don't get to focus on the teaching of it. By contrast in other sessions we have deliberately focused on helping students deepen their understanding of a topic and then we are surprised when they are unable to plan a coherent lesson on the topic during the school practical sessions. This suggests that we need to pay more attention to the work of didactic transposition (Chevallard, 1992) or recontextualising (Bernstein, 1996). For us this is perhaps the kernel of methodology courses but it must follow from the two points mentioned above if it is to be done successfully.

An emerging curriculum framework

Although our conceptualising of the B.Ed mathematics programme was considered fairly radical in our University at the time, based on what we have learnt, the gaps we have identified, and the growing body of literature on mathematical knowledge for teaching, we are beginning to consider a far more radical curriculum framework – one that might no longer be structured around mathematical domains.

Within this framework we distinguish three types of mathematics courses. I will refer to them as Mathematics content courses, Mathematics for Teaching courses, and Mathematics Methodology courses. Mathematical and teaching objects should be present in all three courses but this will happen in different ways. A brief description of each type of course is given below:

Mathematics courses - students learn mathematics in investigative ways, focusing on the key ideas of the domain, and on appropriate mathematical practices. In most cases the domain content will be new and would be considered as advanced mathematics. Typical examples would be courses in calculus, linear algebra, modelling and statistics. Such courses must balance the tension between coverage of the necessary content and depth in key aspects.

Mathematics for Teaching courses – students focus on key aspects of school mathematics with the intention to broaden and deepen their understanding of it. This will also include how learners come to understand the content and their typical misconceptions; the development of the mathematical ideas in the school curriculum; and how text books approach the content. The focus is on enabling the student teacher to relearn school maths in a deeper and more connected way and to give thought to how learners might encounter the ideas. Typical mathematical issues that need to be explored include: multiplication of two negative numbers, proportional reasoning, word problems, notions of variable, ways of understanding the equal sign, and triangle- versus unit-circle approaches in trigonometry. In dealing with these issues, students would draw on ideas from advanced mathematics that they have learned in the Mathematics courses.

Mathematics Methodology courses – students focus on the practice of teaching and the work of transposing their knowledge for the classroom, including preparing lessons, planning sections of work, studying the actions of an experienced teacher. To do this students need to draw on the knowledge they have gained in the Mathematics for Teaching course. For example, they will draw on what they have learnt about various conceptions of variable and then design activities and lessons that engage these conceptions and misconceptions. Methodology courses will also introduce student teachers to issues such as the practical and theoretical aspects of assessment, groupwork and language specifically with reference to school mathematics.

These courses will then be structured into an overall framework that reflects a shift in the objects of attention over the four years, possibly with the main shift taking place between second and third year. In the first two years we suggest that the focus is *mathematics for teachers*. This implies that attention is primarily given to developing students' own mathematical abilities, expanding their mathematical imagination, and exploring their own mathematical identity. It would be characterised by learning and re-learning mathematical content; learning to participate in a mathematical discourse community; the deliberate use of technology for particular goals; and a limited focus on teaching. In the third and fourth years the focus shifts to *mathematics for teaching*, and with it a focus on the practice of teaching and on the learning of mathematics by others. While students will still be learning new mathematics, greater attention will be paid to connecting and applying known mathematics, to flexible use of technology in learning mathematics, to understanding how others learn mathematics and to the design and thoughtful execution of teaching sequences.

Conclusion

In this paper I have described a pre-service mathematics teacher education programme for secondary teachers where students who would not normally be considered "mathematical" in university terms, are prepared to become competent and confident mathematical thinkers, and ultimately well-equipped secondary mathematics teachers. I have focused on the mathematical components of the programme, identified important issues that have emerged in the implementation and made suggestions for a revised curriculum framework for the programme. The notion of mathematics for teaching has been central to the initial conceptualisation of the programme and I have made suggestions for ways in which appropriate and necessary mathematical knowledge can become even more central in the design of the programme.

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