Japanese High School Mathematics Teacher Competence in Real World Problem Solving

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ABSTRACT

This study attempts to qualitatively assess Japanese public high school mathematics teachers' competence in real world problem solving with test questions taken from American pre-calculus and first year calculus textbooks. The problem answers and the interview reveal these teachers to be highly capable in dealing with real world problems. Although the sample is small, the uniformity of preparations of Japanese high school teachers may allow findings to be indicative of a large population.

1. INTRODUCTION

Education researchers have attempted to measure teachers' subject competence with various objective markers, such as the number of college course credits earned in subject area they teach (Begle, 1979), standardized test scores (Strauss and Sawyer, 1986), teacher's degree level (Goldhaber and Brewer, 1996), or teachers' degree level and teacher certification status (Linda Darling-Hammond, 2000).

There is another type of study that attempts to measure teachers' mathematical competence directly with interviews and test questions chosen by the researcher. Liping Ma's study (1999) is well recognized cross-cultural comparative study of this type. Ma and her collaborators used interview questions developed by Ball (1988) to measure mathematical competence of elementary school teachers in China and the US. The present study utilizes the test-and-interview method and attempts to qualitatively assess the mathematical competence of Japanese public high school mathematics teachers in solving real world problems.

2. METHOD

Site of the Study

The site of the study is K.High School, a public high school in a suburban town in the greater Tokyo area that provides academic courses for approximately 800 students from tenth through twelfth grade. At the time of the study, it belongs to a high school district that has seven public high schools in total. Japanese high schools are essentially ranked in each district according to the number of students and level of prestige of the colleges into which their graduates are accepted. In the school year previous to this study, 37% of K.High School graduates entered four year colleges, 15% entered two year junior colleges, 28% entered vocational training schools, 4% were employed, and 16% were preparing for next year's college entrance examinations. None of the colleges that those graduates entered is regarded as an upper level college in Japan. The school principal regards the academic level of K.High School as being around the middle in its high school district. K.High School's profile described by the principal and shown in the above data is very close to the profile of Otani High School that was described by Rohlen (1983) as one of the most typical high schools in Japan in his book entitled "Japan's High Schools". Rohlen cites an Otani teachers' comment, "Our students are neither very smart nor particularly slow". The K.High School principal evaluated his students as being at this level adding that the students needed "step-by-step instruction" to understand their grade level mathematics.

It should be noted that in the Japanese public school system, the level of teacher quality of a school does not correspond to the academic level of the school. As described by Kinney (1998), public school teachers in Japan are employed by the local government, not by an individual school, and they are subject to scheduled rotational transfers to any school in the district. The school a teacher will be transferred to is determined not based on seniority or performance, but on the job rotation system designed to give every teacher working experience in a wide range of schools at every academic level, and to give every school equal access to the best teachers.

Problems Given to the Japanese High School Teachers

To investigate Japanese high school mathematics teachers' competence in real world problem solving, a problem set was prepared. Upon reviewing five American textbooks that focus on mathematical modeling from Algebra through Calculus, three problems were selected from an American pre-calculus high school textbook entitled

“Mathematics: Modeling Our World (Pre-Calculus)” (COMAP, 2000), which sets mathematical modeling as a core pedagogical tool, and three problems were selected from an American calculus textbook entitled “Calculus” (Anton, 1999), which also has a strong emphasis on modeling. The problems were chosen so that the problem set could measure mathematical skills at a level inclusive of pre-college mathematics and the first year college calculus, mathematical reasoning ability, and the ability to apply mathematical knowledge in a real world situation. Problems were also selected so that the problem set contained (1) a modeling problem for which a model was constructed which included parameters such as those in the real world situation (problems [1] and [2]), (2) a modeling problem for which a model was constructed after simplifying a real world situation ([3]), and (3) a problem asking for proofs for basic theorems in modeling ([4]).

These problems were put in the form of written, pencil and paper test problems with some numbers being replaced with variables, and then translated into Japanese. To avoid giving the impression to teachers that they were being tested on their mathematical ability and knowledge, which could have been off-putting making it difficult to get their cooperation, the problems were put into the form of a questionnaire requesting teachers’ solutions to the problems.

The questionnaire written in Japanese was sent to the principal of K.High School approximately a month before the interview was conducted. The principal of the school then passed out copies to his mathematics teachers and requested their cooperation. How much cooperation would be given to this study was left to individual teachers. The following is a translation of the problems given to the teachers:

[1] A bank account earns compound interest with $100r$ % ($0<r<1$) a year, which is deposited at the end of the year. The account was opened with a dollars at the beginning of a year. If constant amount b dollars are withdrawn from the account at the beginning of every year starting on the second year, how much money needs to be deposited at the beginning of the first year so that this withdrawal can be continued forever? Assume there is no other deposit except earned interests. What do you think the result means?

[2] Digitalis, a medicine used to treat a heart disease, needs to be prescribed in a way that the amount of the medicine in the blood is kept in a certain range. When the amount of the medicine in the blood was measured in a certain patient, it became $100r$ % ($0<r<1$)

every twenty four hours. To this patient, the medicine is given every twenty four hours, *a* milligrams of the medicine is given at first and the constant amount of *b* milligrams of the medicine is given after that. What is the value of *b* in order for the amount of the medicine in the patient's blood to be always kept more than *s* milligrams and less than *t* milligrams? (Assume $0<s<a<t$) What do you think the result means?

[3] A rent-a-car company that owns 700 cars is planning to improve the business in Orlando and Tampa. The company had to transport empty cars between the two cities to meet the demand of the two cities. The company wants to know how many cars should be placed in each city to reduce the transportation of empty cars as much as possible. According to the investigation of the business so far, 60% of the cars rented at Orlando were returned to Orlando and 40 % of the cars to Tampa, while 70% of the cars rented at Tampa were returned to Tampa and 30 % to Orlando. To construct a model of this phenomenon, assume that all 700 cars are rented every day and are returned on the same day and that the result of the investigation holds under this assumption. What advice do you give to the company based on the results?

[4] Prove: (1) If a data set (X_n, $\ln Y_n$) has a "linear model" i.e. there exist m and b such that y=mx+b, then a data set (X_n, Y_n) has an "exponential model" i.e. there exist c and a such that $y=ce^{ax}$. (2) If a data set ($\ln X_n$, Y_n) has a "linear model" i.e. there exist m and b such that y=mx+b, then a data set (X_n, Y_n) has a "logarithmic model" i.e. there exist b and c such that $y=b+c\ln X$. (3) If a data set ($\ln X_n$, $\ln Y_n$) has a "linear model" i.e. there exist m and b such that y=mx+b, then a data set (X_n, Y_n) has an "n-th degree function model" i.e. there exists c such that $y=cx^n$.

Interview Procedure and Profiles of Study Participants

An interview was scheduled by the principal and conducted at his office. On the day of the interview, four mathematics teachers including the principal participated. Three teachers were absent. The written answers of six out of seven mathematics teachers that had been submitted in time for the interview were reviewed. The seventh teacher who did not submit his answer on the interview day sent his written answer later to the author.

To elicit teachers' thoughts, the interview was conducted as a group discussion with the author as facilitator. The opinions of the three teachers who missed the interview were collected by the principal and relayed to the author over the telephone.

Those who participated in the group interview are (the number following the degree is the years of teaching mathematics) Mr.O: Principal of K.High School, BA in engineering, 26; Mr.W: Chairman of mathematics department, BA in physics, 23; Mr.Y: BA in mathematics, 25; Mr.D: BA in mathematics, 20. Those who missed the group interview are Mr.M, Mr.T and Mr.N: all three teachers hold BA in mathematics, 10plus.

3. FINDINGS

For problem [1], all teachers formulated a correct recurrence equation. All of them succeeded in expressing the general term in terms of *n* by solving a characteristic equation except for Mr.Y who closed a parenthesis at a wrong place. Mr.T succeeded up to this point but then failed to consider the limit of the general term to reach the final answer. All others successfully obtained the final answer.

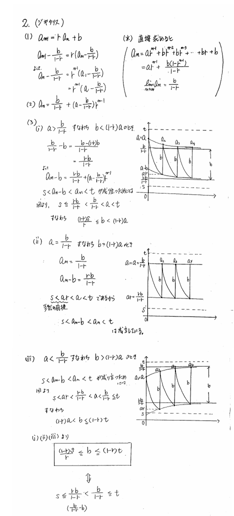

Figure 1. Mr.W's solution to [2].

For problem [2], all formulated a correct recurrence equation except for Mr.Y who incorrectly multiplied *n*th term by 1-*r* instead of *r* to get (*n*+1)th term. Mr.T again succeeded up to this point, but then carelessly omitted *b/(*1-*r)* in the last stage of the process of obtaining an expression of the general term in terms of *n*. Then Mr.W and Mr.D carefully examined the behavior of the sequence while all others used only the limit value of the general term of the sequence to obtain the final answer.

Here, Mr.W drew graphs of the putative changes in the amount of medicine in the blood (Figure 1). In the interview, he said, "I needed to draw graphs to make myself understand. As you can see, not *b* but *b/(1-r)* is the total amount in the blood. You can also find this by calculating the sum of the infinite series". Mr.D, on the other hand, successfully used the common, basic calculus technique of checking the difference between two consecutive terms to examine whether the amount of medicine in the blood was monotonically increasing or decreasing (Figure 2).

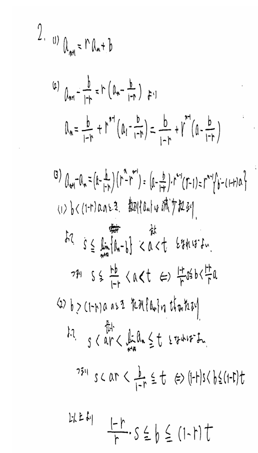

Figure 2. Mr.D's solution to [2]

It is possible that the fact that Mr.W was trained as a physicist in college and Mr.D was trained as a mathematician may have led them both to their respective solutions. If that is true, having teachers who majored or minored in physics may provide members of mathematics departments with a grater range of opportunities "to discuss alternate ways of approaching problems"- which is an idea proposed by Usiskin (2002) as to what every mathematics teacher needs to do.

For problem [3], all teachers formulated the correct system of recurrence equations. All succeeded in expressing the general term in terms of *n* by solving its characteristic equation and reached the final answer except for Mr.T who unsuccessfully attempted to construct the matrices solution.

Only Mr.W further went on to extrapolate general cases of problem [3] in which the ratio is *p: q* instead of 60%: 40% and in which three cities were involved instead of two (Figure 3). In the interview, Mr.W explained how he became curious about the further investigation saying, "The reversed ratio of the answer 40%:30% appears in the given condition. I started wondering why it was this way". Then he explained what he had found saying "I tried to solve the problem by replacing numbers with variables, and I found out what was happening. If the numbers of the cars that are not returned to the city where cars were initially placed are the same in both cities, it is the same as the situation with no movement. This is the stable condition". Mr.W's approach to the problems is a good illustration of Usiskin's (2002) other ideas as to what every mathematics teacher needs to do, which are "to see how problems and proofs can be extended and generalized" and "to know the wide range of applications of the mathematical ideas". It should be noted that Mr.W's knowledge in linear algebra enabled him to come up with a way to construct and solve a generalized version of [3].

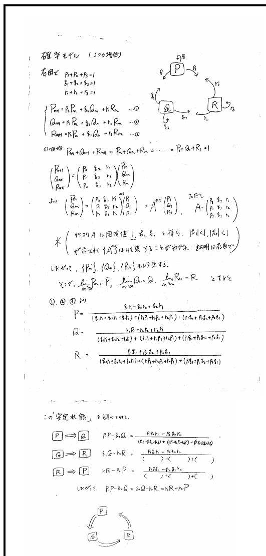

Figure 3. Mr.W's solution to a generalized version (three cities) of [3].

No teachers had trouble in constructing proofs for problem [4] except for Mr.T who inadvertently put an extra *m* in an equation. Although some teachers simply wrote "easily shown by substitution" or did not even bother writing it, they correctly stated the process of the proofs in the interview.

In sum, although other teachers' answers are not as comprehensive and mathematically sophisticated as Mr.W's, one can conclude from the analysis of the answers and results of the interview that all teachers are sufficiently mathematically competent for learning and teaching mathematical modeling.

According to Mr.O, who has observed his teachers in class, Mr.W has consistently conducted what Mr.O calls a “perfect” lesson with a lot of helpful comments which enable students to overcome difficulties and lead them to understanding. From his answers to the problems, it was clear that Mr.W had sound mathematical knowledge and skills. In addition to this, the discussion revealed that Mr.W’s attitude in pursuing a comprehensive understanding of the problems was more intense than that of other teachers. Mr.W felt compelled both to solve and to understand the problems. He enlisted the full range of his mathematical knowledge and skills where others did not. It is as though uncertainty stimulated Mr.W to apply more mathematics where other colleagues applied less.

To be an excellent mathematics teacher, mathematical competence and persistence to pursue comprehensive understanding seem to be pre-requisites. A teacher who has had rich experiences struggling to understand mathematics until the feeling of uncertainty finally disappears will take students’ understanding seriously and make a great effort to lead students to the acquisition of mathematical knowledge with thorough understanding.

4. DISCUSSION

The present study showed that K.High School mathematics teachers, each of whom holds either a BA in mathematics, physics or engineering, possessed solid foundation of mathematical knowledge and skills. In their written answers to mathematical modeling problems, concept of which they were not familiar with, most of the teachers demonstrated strong mathematics skills, reasoning ability and the ability to apply mathematical knowledge in formulating equations to express real world situations, in deriving solutions from them and in constructing proofs. Although the sample is small, the uniformity of preparations of Japanese high school teachers may allow findings to be indicative of a large population.

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