

Using the Model-Strategy-Application Approach to Developing Pre-Service Teachers' Knowledge and Assessing Their Progress in Math Methods Courses

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ABSTRACT

The purpose of this research was to contribute to a better understanding of how to enhance pre-service teachers' knowledge of mathematics teaching through Model, Strategy, and Application (MSA). 153 pre-service teachers from six math methods courses (K-8) at two universities participated in this study. Both qualitative and quantitative methods were used to measure teacher learning and understanding of the MSA approach and to identify relationships between components of the MSA model. The results of this study show that using the MSA model can enhance pre-service teachers' knowledge of mathematics teaching. It can be used as a measurable and practical model in assessing pre-service teachers' learning progress in models, strategy, and application. In addition, the findings indicate that the three components of the MSA model are all interrelated and equally important; ignoring one or the other will result in the lack of proficiency in teachers' knowledge of mathematics teaching.

INTRODUCTION

In recent years, many studies have focused on building mathematics teachers' content knowledge, pedagogical knowledge, and pedagogical content knowledge (Shulman, 1987; Fennema, 1992; Ma, 1999; Hill & Ball, 2004; An, Kulm, & Wu, 2004). However, research has paid insufficient attention to systematically designing a repertoire of measurable and practical models of knowledge of mathematics teaching that allow pre-service teachers to not only build conceptual understanding and procedural skills, but also to be able to make connections between mathematics concepts, procedures, and applications. In order to teach effectively, pre-service teachers need to develop the knowledge of creating various representations to unpack abstract mathematics content and make meaningful and visible to students the ideas behind the concepts and procedures (National Research Council [NRC], 2001); they also need to perform computations efficiently and know when and how to apply computations and various models in real world situations. To achieve the goals above, this study explored an effective approach that comprises: 1) creating various visual and meaningful mathematics *models* to convey abstract math ideas through conceptual understanding; 2) developing various *strategies* to achieve fluency in computations; and 3) developing strategic competence in word problem *applications*. In the Model-Strategy-Application (MSA) approach, the three aspects of it form the critical components of teachers' knowledge of mathematics teaching in this study.

The research questions in this study were: How do math methods courses build a measurable and practical model that can be used to identify pre-service teachers' knowledge and develop their conceptual understanding, procedural fluency, and strategic competence in word problem applications? Is pre-service teachers' knowledge improved by implementing such a model? What are the relationships among the three components of the MSA model?

THEORETICAL FRAMEWORK

The Needs in Making Changes for Pre-Service Teachers' Learning

Evidence from the TIMSS, PISA, and other national and international reports indicate that teaching is one of the major factors related to students' mathematics achievement (Stigler & Hiebert, 1999). According to the National Council of Teachers of Mathematics (NCTM, 2000), "Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). However, many recent studies have revealed that mathematical education in teacher preparation, "has not provided them with appropriate or sufficient opportunities to learn mathematics." As a result, many pre-service teachers have "weak understandings of the concepts" and "have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations" (NRC, 2001, p.372). Researchers have indicated that pre-service teachers' knowledge in mathematics content has a great impact on their future teaching (Brown, Cooley, & Jones, 1990; Adams, 1996; Wolf & Reardon, 1996). To improve pre-service teachers' knowledge, math methods courses play a vital role in providing opportunities for pre-service teachers to acquire and enhance their knowledge of teaching; therefore, it is essential to find an effective model for math methods courses.

Why the MSA Approach?

In the report of *Adding It Up*, the NRC (2001) found that "real progress toward mathematical proficiency to be woefully inadequate" (p.11) and "school mathematics in the United States does not now enable most students to develop the strands of mathematics proficiency in a sound fashion" (p. 10). According to NRC (2001), mathematics proficiency has five components: Conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. In this study, model-strategy-application, which makes up the core of the five components of mathematics proficiency. Figure 1 shows that to achieve effective teaching, a teacher must focus on creating a variety of visual *models* to aid in addressing and proving mathematical concepts, building *strategies* for procedural and

computational fluency, and **applying** strategies to solve real-life word problems.

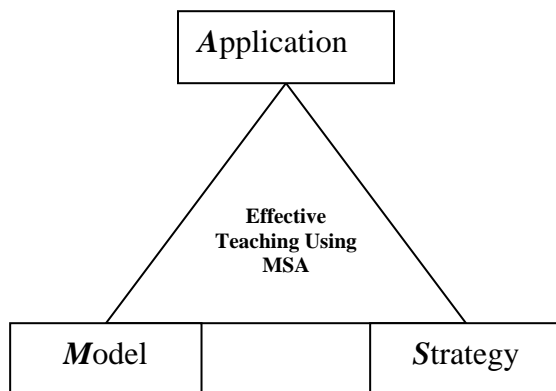


Figure 1: The MSA approach

These three components are interrelated in a triangular network in which the model and strategy components form a foundation for application.

Building Conceptual Understanding. In building conceptual understanding, many studies have addressed using various visual models and manipulatives (Thompson & Lambden, 1994; Hiebert et al., 1997), but too little of the extant content probes the connection between visual representations and abstract thinking (NRC, 2001; An, 2004), which results in students associating mathematics with fun but without an understanding of mathematical ideas. When using a manipulative, teachers help students “to see its relevant aspects and to link those aspects to appropriate symbolism and mathematical concepts and operations” (Fuson & Briars, 1990; NRC, 2001, p. 353). The link between concrete manipulatives and abstract mathematical ideas is mathematics representations, appropriate configurations that represent mathematical ideas (Cuoco, 2001; Goldin, 2003). Teachers’ knowledge of representations has helped them teach mathematics effectively (Wu, 2004). To build such knowledge, pre-service teachers must learn how to create various models and representations that make sense and meaning out of abstract mathematical ideas from a specially designed MSA approach in math methods courses.

Developing Procedure and Computation Fluency. Recent studies show that the importance of developing procedural fluency in mathematics has been largely ignored by research in mathematics education (NRC, 2001). In general, when cross-national studies in mathematics achievement have included samples of U.S. students, the findings have shown that U.S. students show consistently lower performance than their counterparts (e.g., Stevenson et al., 1990, Lapointe et al., 1992), especially in computation skills (Cai, 2000 & 2001). According to the NRC (2001), procedural fluency refers to the ability to perform procedures flexibly, accurately,

efficiently, and appropriately. To achieve procedural fluency, students must develop the ability to decompose numbers naturally, use number relationships to solve problems, make sense of the numbers, and learn flexible and intuitive thinking (Howden, 1989; Sowder, 1992; Reys & Nohda, 1994). In May 2005, NCTM issued a new position statement calling for improving student computational skills. To achieve this goal, classroom teachers play a key role in applying their knowledge by providing different strategies for computation skills and reinforce students' computation skills in their daily teaching. In order for pre-service teachers to acquire such knowledge, math methods must be considered essential courses, where pre-service teachers not only acquire conceptual understanding but also learn how to help children grasp appropriate strategies to solve problems flexibly and accurately, and achieve procedural fluency effectively.

Developing Strategic Competence in Application: Word Problems. Strategic competence refers to the ability to formulate, represent, and solve mathematical problems (NRC, 2001). The connections between mathematics and real world situations help students dispel the notion that mathematics is just about the numbers. In addition, it encourages students' creativity. There are three levels of strategic competence: a) creating word problems according to learners' experience (Wiest, 2000); b) building representations that capture the core mathematical elements (NRC, 2001), which requires students to use representations to convey their mathematical ideas, in word problems; and c) solving problems strategically and flexibly; that is, students must not only be able to solve problems using different approaches but must also be able to solve nonroutine problems (NRC, 2001). Developing students' strategic competence can enhance their conceptual understanding as they construct their own word problems; it can also reinforce their procedural fluency for those effective procedures (Stiegler & Jenkins, 1989) as they try different ways to find solutions. Therefore, strategic competence reflected in mathematical applications in word problems is critical in mathematical learning, and for assessing students' conceptual understanding and procedural fluency.

METHODOLOGY

Subjects. The participants were 153 pre-service teachers from six math methods courses (K-8) at two universities in the Multiple Subject Credential Program in the West and East coast of the U.S. The criteria of inclusion for subjects were: (a) to be taking math methods courses in spring 2005; (b) to have had at least two math content courses at the college level before taking methods courses.

Procedures. This was an ongoing project for a two-semester period: (a) In Fall 2004, a pilot study was conducted in two math methods courses in order to test the validity and reliability of the instruments; (b) In Spring 2005, the 153 participants were assessed using a pre-test and post-test. During the semester, the MSA approach was taught in all content areas. Participants explored different ways to create pictures to represent their understanding of mathematical concepts, learned various strategies to develop fluency in procedures, and designed word problems that relate to children's real life experiences while applying strategies to solve word problems. Multiple assessments were given during the semester to gauge the participants' understanding of the three components of the MSA approach in order to monitor students' progress and reinforce their knowledge of mathematics teaching.

Data Collection and Instruments. Data on using the MSA approach were collected via two tests. In addition, data were collected from multiple assessments during the semester. The MSA questions in pre- and post-tests were adapted from a study of school-based professional development (Wu & An, 2005). The pre-and post-tests included a set of ten questions on number concepts, fractions, decimals, and percents. Each problem consisted of three parts: design a visual *model* for the problem using a representation; use computational *strategies* to solve problems; and create a word problem to represent a real-world *application*.

Data Analysis. A qualitative data analysis method was used to measure teacher learning and understanding of the MSA approach (Lincoln & Guba, 1985) and to identify the emerging themes of the effective approaches from various assessments during the semester. The open-ended responses from pre-service teachers on tests were coded, categorized, and compared for data analysis. A Paired T-Test was used to determine the statistical significance of the results on students' learning progress by comparing the means between pre- and post-tests. The Pearson Correlation Test was used to identify relationships between any two variables among the three components of the MSA model. The reliability and validity of the study were ensured by using triangulation of data, member checks, and peer examinations. The combination of both quantitative and qualitative methods is supported by numerous studies (Cronbach et al., 1980; Fielding & Fielding, 1986). Specifically, the inquiry nature and the complexity of pre-service teachers' knowledge of MSA require an in-depth qualitative study to validate, explain, illuminate, and interpret its characteristics and growth.

RESULTS

The Impact of the MSA Model on Knowledge and Achievement

The analyses of students' pre- and post-tests show that the pre-service teachers in this study made substantial progress in their knowledge in conceptual understanding, procedural fluency, and application using the MSA approach. The results from Tables 1- 3 (**will show in full paper later**) reflect the impressive growth of pre-service teachers' knowledge in the three MSA areas.

Procedural Fluency. The results from Table 1 show that the MSA approach helped the pre-service teachers enhance their scores by more than 20% in their knowledge in the areas of fraction operations, decimal multiplication and division, and changing fractions to percents. For example, in the pre-test, only 39% of students could calculate $24.275 \div 1.25$ correctly, while in the post-test 86% of students performed the calculation correctly; in the pre-test only 46% students knew how to change $7/40$ to a percent, while in the post-test 89% of students were able to perform the computation correctly.

Conceptual Understanding. Table 1 indicates that the pre-service teachers made great progress in all areas, especially in the areas of fraction addition and subtraction, fraction multiplication, decimal addition and subtraction, decimal division, and changing decimals to fractions and percents. More than a 60% increase in their scores in the above areas is observed. For example, in the pre-test only 4% of students knew how to create visual models for fraction addition like $11/12 + 5/7$, while in the post-test 82% of students were able to create appropriate visual representations for fraction addition. Similar growth patterns are also found in fraction subtraction, multiplication, decimal addition and subtraction. However, although pre-service teachers demonstrated progress in fraction division (0% in the pre-test vs. 56% in the post-test), decimal multiplication (1% in the pre-test vs. 39% in the post test), and changing fractions to percents (1% in the pre-test vs. 53% in the post-test), the growth in these areas is less than 60%, which illustrates the challenges and difficulties the pre-service teachers are facing in these areas.

Applications using Word Problems. Table 1 shows that the pre-services teachers gained notable growth with more than a 60% increase in their scores in applications, in the areas of fraction addition and division, decimal multiplication and division, changing fractions to percents, and changing decimals to fractions and percents. For example, 97% of students created word problems for fraction division illogically, with misconceptions, in the pre-test, while in the post-test only 26% of students still had difficulties in creating a correct and logical word problem. In

the pre-test, only 7% of students created decimal division word problems correctly. In contrast, 82% of students were able to create word problems in decimal division correctly in the post-test.

The Effect of the MSA Model on Learning Progress

A Paired Sample T-Test was employed to compare the changes and their statistical significance in the pre- and post-test scores. The SPSS output in Table 2 shows that the *p* value .000 in Table 2 is less than 0.01, which indicates a statistical significant difference between the pre- and post-test mean scores on the pre-service teachers’ learning progress. Table 2 shows the results of three comparisons: 1) overall result between pre- and post- tests; 2) the results of comparisons in university 1; and 3) the results of comparisons in university 2.

The Relationships between the Three Components of the MSA Model

The results of data analysis show that there are relationships and connections among the three components of the MSA model. In order to find out the significant relationships between the three components and draw a valid conclusion, a Pearson Correlation was employed separately for both pre- and post-tests. Table 3 shows the SPSS output of the Pearson Correlation for the pre-test, and Table 4 for the post-test. The results in Tables 3 and 4 indicate that the correlation between any two variables among MSA components is significant and that they are positively related, suggesting that conceptual understanding is associated with procedural fluency, thereby enhancing the skill necessary for applications.

The Learning Progress from the Examples on Pre- and Post Tests

The results of the analyses of the pre- and post-tests show that the MSA approach improved students’ knowledge in the three components in the areas of fractions, decimals, and percents. The following selected examples demonstrate the pre-service teachers’ learning progress in models, strategies and applications.

Examples of Misconceptions on the Pre-Test (more examples will show in the full paper later):

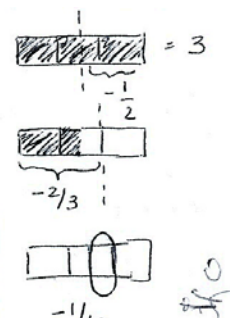
$3 - \frac{1}{2} - \frac{2}{3}$ $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3} = \frac{4}{6}$	$\frac{6}{6} - \frac{3}{6} - \frac{4}{6}$ $-\frac{1}{6}$		<p>If you have 3 apples, and you eat 1/2 of them, can you be left with enough to eat 2/3 of original #?</p>
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Figure 2. Misconception on fraction subtraction

The example in Figure 2 shows that the student made an error in converting 3 to the fraction 6/6. This error can be associated with a misunderstanding of the visual model. The student misunderstood $\frac{1}{2}$ as $\frac{1}{2}$ of three whole instead of one whole in the visual model, which led the student to create the word problem with misconceptions.

Examples of Learning Progress from the Post-Test:

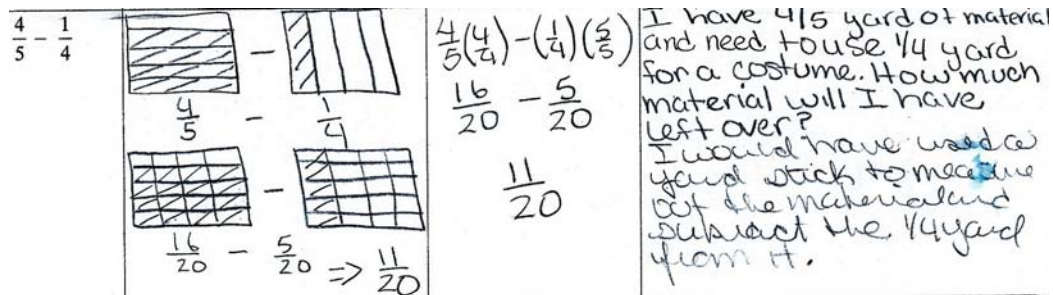


Figure 3. Learning Progress on fraction subtraction

The visual model in Figure 3 indicates that the student understood that it is necessary to change the unlike unit fractions $\frac{1}{5}$ and $\frac{1}{4}$ to a like unit fraction $\frac{1}{20}$ in order to subtract these fractions. The conceptual understanding of fraction subtraction guided the student to perform procedures of fraction subtraction correctly and directed the student to create a valid word problem in application.

In summary, the results of the analysis of the pre- and post-tests show that the MSA approach helped to improve students' achievement on the tests and enhanced their knowledge, which was reflected in their work. This study shows that the MSA model is an effective approach that provides pre-service teachers with a strong knowledge base on how to teach mathematics effectively by focusing on three critical components.

Conclusion

The results of this study show that the three components of the MSA model build upon each other; thus, ignoring one or another will result in ineffective teaching. Procedural fluency without conceptual understanding will yield non-meaningful and inappropriate strategies for solving applications; conceptual understanding without procedural fluency will yield strategic applications that are inefficient and inconsistent. Undoubtedly, this inquiry into the knowledge of effective teaching, along with the development of the three parts of MSA in each mathematics content area, especially the examination of the effects of this new model of teaching and the investigation of the measurable assessment model for pre-service teachers' knowledge, will be a significant step in the development of mathematics teacher preparation programs in the U.S.

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