MATHEMATICAL MODELLING, THE SOCIO-CRITICAL PERSPECTIVE AND THE REFLEXIVE DISCUSSIONS

Jonei Cerqueira Barbosa
State University of Feira de Santana, Brazil
http://www.uefs.br/nupemm
joneicb@uol.com.br

Abstract: This paper is part of a wider study on the students' practice at the micro-social level of modelling-based classrooms by seeking to gain insights about their discursive practices. In particular, the aim of this study was to analyse the production of reflexive discussions in a mathematical modelling environment. The importance of this topic is related to the possibility of understanding how the socio-critical perspective works in action. Following the qualitative approach, a group of students was filmed during a modelling task. The findings suggest two ways of producing reflexive discussions: debating the influence of the criteria in building the mathematical models, and comparing the different models constructed by students. These results provide subsides for teachers who want to follow the socio-critical perspective.

Introduction

The mathematical modelling environment may carry out different goals (Niss, Blum & Galbraith, 2007). It's possible to associate them with the perspective debate, which represents a wider way of conceptualizing the modelling, including the didactical interests. In Kaiser and Sriraman (2006) and Kaiser, Sriraman, Blonhoj and Garcia (2007), there is a clarification of the perspectives presented in the literature. One of them is the "so-called" socio-critical modelling, according to what is addressed in Barbosa (2006).

The formulation of the socio-critical perspective is based on the implications from the studies about the relation among mathematics, mathematical models and society. For example, Skovsmose (1994, 2006) has

pointed out how mathematics has become a strong argument for making decisions in society, and the mathematical processes used in this are hidden. As the mathematical models are not neutral, they depend on their building processes, which include how the modeller understands the problem-situation, and how he/she projects mathematical conceptions into the situation. Following this argument, the biased nature of the mathematical models brings a closer relationship between the mathematical model and the criteria used to construct it.

In society, the mathematical results are usually presented as neutral descriptions from reality. This has increased in the age of technology, which has reduced the importance of mathematical skills and knowledge for people in general. According to Jablonka and Gellert (2007), the mathematisation of society and the demathematisation of people are two connected processes. As people may not be skilled enough to discuss a mathematical argument, then they have to accept it. It means that the social groups which control the mathematical arguments could have more power than those who don't.

This analysis brings implications for modelling in mathematics education. If we assume that education should go beyond the work training (D'Ambrósio, 2007), then the modelling environment should address the nature and the role of mathematical models in society. In Barbosa (2006), I have pointed out the presence of studies in literature which have this focus for modelling.

Formulating the socio-critical perspective for modelling is useful to support classroom experiences and vice-versa. One focus of the research is to study how this perspective works at the classroom level in relation to teachers' and students' actions. The main point in the classroom is that socio-critical perspective has to address the nature and the role of mathematical models in society.

In Barbosa (2006), I have developed an approach to analyse the students' modelling. By focusing on discursive activity, we can understand how people participate in social settings through discourses. When invited to solve a modelling problem, students may develop many kind of discourses. In Barbosa (2006), taking Skovsmose (1990) in consideration, I have examined three kinds of discussions:

- mathematical: refers to the ideas belonging to the pure mathematics field.
- technological: refers to the techniques of building the mathematical model.
- reflexive: refers to the nature of the mathematical model, the criteria used in its construction and the consequences of this criteria.

The reflexive discussions may be associated with interests in socio-critical perspective. It doesn't mean that the other discussions don't have a place in the modelling environment, but they are re-positioned. The reflexive discussions involve the other discussions, since one should always analyse criteria used in the model.

However, students can't be "compelled" to produce reflexive discussions, but their production has to make sense to the students. In particular, *this paper is meant to examine the situations in modelling-based lessons which bring up the reflexive discussions*. This study calls for theorizing the socio-critical perspective for modelling at the social-micro level, besides providing subsidies for teachers who want to put this perspective in action inside the classroom.

Methodology

The research setting was a modelling course for future mathematics teachers at the State University of Feira de Santana, in the Brazilian Northeast. The students were invited to solve some modelling problems in groups. One of them was based on a newspaper article that informed them about the possibility of having electrical energy rationed, because the level of the water

reservoir was low at the hydroelectric power station. In this paper, I am going to focus on the students' group composed of Ana, Beatriz, Maria and Tereza.

The study followed a qualitative approach, because this research methodology is more appropriate for capturing the meanings of discourses produced by students (Bodgan & Biklen, 1998). The data was collected through observation by a research assistant, and filming was a form of registration. The data was analysed and inspired by the Grounded Theory (Charmaz, 2006).

Data

The students were invited to analyse a newspaper article published in November, 2007, which reported on a reduction in volume of the Sobradinho Lake, due to the lack of rain. The water of this lake is used to produce electrical energy for 75% of the population of the Brazilian Northeast.

I would like to present some information from the article in a few words. At that time, the volume of water represented 15% of the capacity of the lake, which produced 450 megawatts/hour. According to the newspaper, the expectation was to operate at capacity the 13% next month. The lake produces 1,050 megawatts/hour at its fullest capacity.

Part 1

The students were invited to anticipate when the Sobradinho Lake would have zero capacity. The group composed of Ana, Beatriz, Maria and Tereza started by trying to structure the situation and listing the variables which interfere in the lake's volume: the use of the water by the population which lives near the lake, evaporation, and water provided by tributary rivers. However, they quickly noticed that data was not available in the newspaper article that considered these variables.

The students pointed out to the teacher that the results would be a bit different, because they couldn't collect important information. They had to use the variables provided by the article. The students' point was that different variables would bring different results.

Thus, students noticed that the results depended on the considered variables. This episode suggests that the students developed their own perceptions about the close relationship between chosen variables and models. This sort of discussion may be seen as reflexive, because it shows the interference of criteria – in this case, variables – in models.

Part 2

At the end of the task, every group was invited to present their results to the whole class. That group composed of Ana, Beatriz, Maria and Tereza showed their results in the form of a table (see table 1), which enabled them to conclude that the necessary time for the lake to become empty would be 8.5 months, and so the production of electrical energy would be interrupted.

Time (month)	Useful volume
1	15%
2	13%
3	11%
4	9%
5	7%
6	5%
7	3%
8	1%
9	-1%

Table 1. Tabular data

The criteria used by the students was that the volume of the lake was changing constantly 2% of its full capacity per month. In other words, the variation was constant.

However, another group found a different result. This one took the full volume of the Sobradinho Lake, which is 28,669 Hm^3 and calculated 15%, so that the students found the volume at that moment, 4,300.35 Hm^3. Then, they considered the variation as proportional to the volume each month. As they fitted an exponential equation, from a mathematical point of view, they found

that the lake would never become empty. However this would happen within 40 months, in practice.

Comparing the students' strategies, it is supposed that they framed the problem-situation with previous mathematical experiences. In this case, they used proportion and exponential function for that. In doing so they established real criteria as tied to the mathematical criteria. However, I am not going to expand this point here, because it is out of the focus.

The different results made the students surprised and they went to review the procedures used by them. From comparing their models, they re-examined the strategies used for building the results, which enabled them to notice the different criteria used by them. While the first group considered a monthly variation of lake volume as a constant, the second group took it as depending on the monthly volume.

In this case, students produced what we may recognize as reflexive discussions, because they related mathematical models to the criteria used to produce them. By working in different groups, the students had the opportunity to use different criteria and so produce different models to compare. The presentation of the results to whole class seems a crucial moment for the generation of this sort of discussion.

Discussion

The results found suggest that the students produced reflexive discussions through two situations: analysing the criteria to be used in building mathematical models; and comparing the models produced from different student groups. In the first situation, when students were discussing how to simplify the problem by defining hypothesis and choosing variables, they anticipated the influence of these criteria in the model. In the second situation, the students' group compared the results with those found from other groups, and they verified different mathematical models. Debating why different results

were produced, they perceived how different models were established according to the way that problem-situation was simplified. These findings have also been observed in a recent study conducted by Santos (2007).

The reflexive discussions are not clearly connected to the validation. This one seeks to verify the models in terms of the real data (Edwards & Hamson, 1990). In fact, the latter is more related to what I named previously technical discussions, because it addresses the building of a model whereas the reflexive discussions do the relation between criteria and results.

In episodes above, considering the concept of reflexive discussions found in Barbosa (2006), the data suggests that the genesis of reflexive discussions is related to the presence of the criteria and results (or mathematical models) in students' debates. In this way, the students had the opportunity to challenge the view that mathematical models are neutral descriptions of reality, and so put the socio-critical perspective in action at the classroom level.

Final remarks

These results suggest evidence regarding the production of reflexive discussions: analysing the influence of criteria in building mathematical models, and comparing different models. In both cases, there is the opportunity to analyse the relation between criteria and results, bringing to the classroom the implications from socio-critical perspective at a discursive level (Barbosa, 2006).

The focus of reflexive discussions is more on the link between criteria and results than validity. This is more appropriate to be found in technological discussions. The former is defined in terms of analysing how different criteria produce different results. Once every model is biased, then reflexive discussions might address the biases.

This study also brings a demand for a particular research agenda to the socio-critical perspective. For

example, questions about the relationship between reflexive discussions and other aspects in school settings need more attention. From the point of view of teachers interested in a socio-critical perspective, this study allows us to infer strategies to motivate the reflexive discussions in modelling environment, such as calling students to anticipate results according to the criteria used and to examine the different models produced in the classroom.

Acknowledgments

I thank the reviewers for their comments on an earlier version of this paper.

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