MATHEMATICAL MODELS IN THE CONTEXT OF SCIENCES

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Abstract. This paper show a research which determine a classification and characterization of mathematical models in the engineering careers, as well as a definition about "mathematical model" and "mathematical modeling"; in the same way, it shows the cognitive and thinking skills elements that interfere in building a mathematical model in engineering.

Introduction.

This research influences university studies where mathematics is not a goal themselves. Due to the mathematical richness that prevails in engineering, the project comes particularly to these studying areas. The mathematization of the phenomena and problems that come up in the working field of the future engineer is a cognitive conflict point since he studied mathematics and engineering separately, so that when he uses both knowledge areas, these cognitive areas are separated and he has to integrate them in order to mathematize the problem to be resolved [1]. The intention is that the students build mathematics for all their future life. On the other side, the mathematical model is one of the subjects that appear in the hidden curriculum of university careers, since it is supposed that the graduate must know how to model, but in many study plans and programs the term "mathematical modeling" is not mentioned at all. Inside the goals of study programs of other curricula, it is said that the student must know how to model problems from other areas of knowledge, and this term is included in the subject programs in very few curricula. But, in no case it is said how to include mathematical modeling in mathematics courses, neither how to make students model situations of other areas or problems from the daily life [2,3]. In fact, there is no engineering subject that comes to work mathematical models. Besides, the mathematics teachers feel that this point is concerned by the teachers of engineering courses, while the latest presuppose that the mathematics teachers are the ones who have to teach the students how to model engineering phenomena [4]. This conflict point, about how to model problems from the working and professional life mathematically, from *Mathematics in the Science Context* theory, is considered to be attended in an interdisciplinary way. The problem is tackled by the mathematics teacher, who incorporates *Mathematics in the Sciences Context* theory to his lessons. To come to an end, it is pretended to have the necessary indicators that interfere in the mathematical model, to incorporate it consciously and efficiently in the mathematics courses, so that the students are trained to establish the mathematical model. That is to say, it is necessary to know which knowledge and skills are necessary so that the student learn how to model, besides from the knowledge areas used in the context process. It is clear that he has to know about the mathematics concepts, as well as the context discipline concepts, but what else does he have to know and dominate?

The research problem

We want to know the cognitive elements and thinking skills that interfere in building a mathematical model, as well as, classifying and characterizing the mathematical models in engineering. To tackle the research problem, we have the following research questions [5]: What is a mathematical model?, What is mathematical modeling?, What cognitive elements must the student know to build the mathematical model of a scholar engineering problem?, What thinking skills are essential to build the mathematical model of a scholar engineering problem?, How are the mathematical models characterized and classified?

The objective. The objective of the research is to determine how to classify and characterize mathematical models in the engineering careers, as well as how to define the “mathematical model” and the “mathematical modeling”; in the same way, to know the cognitive and thinking skills elements that interfere in building a mathematical model in engineering.

The theoretic framework. The theory in which this research is based is *Mathematics in the Sciences Context* [1,3,6,7,8], so I am going to explain you briefly what *Mathematics in the Sciences Context* is. This theory takes mathematics learning and teaching in engineering careers as a system which includes the student, the teacher and the mathematical knowledge, considering the interactions between the student and the teacher, all included in the learning environment where there are social, economical, political and human relations aspects. This systemic look makes five phases of the *Mathematics in the Sciences Context* theory, as we will see later.

Mathematics in the Sciences Context is based in three paradigms: Mathematics are supporting tools and educational subjects. Mathematics have a specific function in each educational level. Knowledge is born integrated. The educational philosophic assumption of this theory is that the student is trained to transfer mathematics knowledge to the areas which require it, so that they apply it in their working and professional life. The five phases of the theory are: The Curricular developed since 1984. The Didactic started since 1987. The Epistemological tackled in 1988. The Teachers Training defined in 1990. The Cognitive studied since 1992.

This research presents incidence in the didactic phase which has a didactic proposition which is called *Mathematic in Context*, through which contextualized events are worked, that is, problems and projects in the context of other student's knowledge areas, in the future professional and working activities and, in the daily life. Due to the globalization and the great competitiveness we live nowadays, it is important that the training received by the student permits him to incorporate efficiently and effectively to his working activity in the national and international fields. This makes the educational system incorporate the contextualized events from the early student's training.

Mathematic in Context considers nine steps [1,2]: 1. Books analysis of the other subjects studied by the student. 2. Event context establish. 3. Determination of the variables and constants of the event. 4. Inclusion of the mathematical topics and concepts necessary for the development of the mathematical model and its solution. 5. Determination of the mathematical model. 6. Mathematical solution of the event. 7. Determination of the solution required by the event in the disciplines context field. 8. Interpretation of the solution in the event terms and the disciplines context area. 9. Presentation of the descontextualized mathematics in the classroom so that the student knows that it is applied in other knowledge fields and that he develops the skills given by the formal mathematics.

We can see that a the central element is building the mathematical model. It is central in the sense that without this element the *Mathematic in Context* is not obtained, neither the resolution of the event [2,3,7,8]. To resolve contextualized events it is necessary to build a mathematical model that describes the event, as we saw in step five [5,9]. In general, to speak about *Mathematic in Context* is to develop the mathematics courses for the requirements and rhythm indicated by the engineering courses. In fact, *Mathematic in Context* strengthen the cognitive reorganization of mathematical concepts and processes [7,8,10,11,12,13]. When we use *Mathematic in Context* we work with groups of three students in the classroom, taking into account the Vygotsky [14] socialization knowledge. To tackle the contextualized events, Polya heuristics [15] are considered, as well as metacognitive elements, thinking skills and students' beliefs [16,17]. Through *Mathematic in Context*, the traditional educational knowledge paradigm is changed. Now we are speaking about teaching with integrated knowledge, linking mathematics concepts with the other subjects studied and presenting them at the rhythm and times required by the student [2,3]. It is necessary to mention the profile of the mathematics teacher that works with *Mathematics in the Sciences Context* theory: If he is a mathematician he must learn the engineering knowledge areas, while if he is an engineer he must be more prepared in the mathematics knowledge [4].

Working methodology

Since, it is desired to characterize and classify mathematical models used in engineering, the methodology used will be the engineering text analysis, such as the analysis of some engineering investigation projects and engineering text books. We have to work with some particular engineering areas, so, this text analysis comes to electronic engineering and its adjacent fields particularly. As known, the text analysis is a methodology which works to detect certain elements related with the teaching and learning of sciences. It depends on what is persecute, to look in the correct way those texts. The text analysis will be done in an implicit and explicit way. Thus, for engineering mathematical models, we mainly look for: 1.- Establishing problems to be tackled. 2.- How the established problems are represented mathematically. 3.- How the concepts of engineering subjects are described mathematically. Also, it is necessary to consider a sample of engineering students. They will have resolve some problems, in order to identify the cognitive elements and thinking skills that interfere in the process of building the mathematical model of the problem. We selected 21 students, 3 of each semester, from the third to the ninth of the Communications and Electronic Engineering career from the National Politechnical Institute of Mexico.

The text book sample

For the text book analysis it was considered the classification established by the "Asociación Nacional para Universidades e Instituciones de Educación Superior en México" about the engineering careers subjects. This classification defines 5 subject blocks: the basic sciences, the engineering basic sciences, the engineering specialization sciences, the social and humanistic sciences and, the economical and administrative sciences. It is clear that the first three blocks are the important ones for the present research. Inside the basic sciences are physics and chemistry as basic engineering sciences foundations, while mathematics are supporting tools for them, without forgetting the thinking skills development that mathematics offer to the future engineer [1,4,8]. The electric circuits, electromagnetism, computation, basic electronics and basic communication subjects form the basic engineering sciences, which are the foundations of communications, electronics, control, acoustic, robotics, telephony, and computation, the engineering applying areas. This classification, for electronic engineering and its adjacent fields, are called engineering cognitive stages; as you can see in table 1, the first line includes these cognitive stages and their subjects are in the corresponding column.

Engineering Cognitive Stages	Knowledge Areas
Basic Sciences	Physic, Chemistry
Engineering Basic Sciences	Electric circuits, Electromagnetic, Computation, Basic Electronic, Basic Communication
Engineering Specialization Sciences	Electronic, Communications, Control, Acoustic, Robotic, Telephony, Computation

TABLE 1. ENGINEERING COGNITIVE STAGES

Mathematical Models Characterization

To start, mathematics in engineering is a language, since almost everything said in engineering can be represented through mathematical symbols [1,2,3]. Even more, representing it through mathematical terminology and since mathematics are used in engineering, it helps the engineering to have a science character from one side, and from the other, it facilitates its communication with the engineering scientific community [1,2,3]. Inside the engineering knowledge, there are engineering problems, also, there are engineering objects that are represented mathematically for a better use or reference, and there are also situations that can be described through the mathematical symbols. These cases will permit to characterize the mathematical models. Examples of each case are presented bellow. **a) Problems.** We want to know the charge of a condenser (capacitor), which capacity is C. This is connected in series with a resistor with resistance R to battery terminals which provide a constant tension V, this layout can be represented by the following linear differential equation: $Rq'(t)+(1/c)q(t)=V$. That is to mention that under the term problem are included the phenomena presented in engineering as the charge of a condenser, the free fall of a body, the movement of a pendulum, etc. **b) Objects.** Consider an electrical signal of the sinusoidal alternating type, the signal is the engineering object which is represented through the function: $f(t) = A \sin(t+\propto)$. **c) Situations.** The charge condenser $q=q(t)$ is totally discharged at the beginning of the problem. This situation may be represented mathematically, taking into account that at the beginning of the problem $t=0$ and the charge is a time function, as: $q(0)=0$.

The mathematical model concept

From the three mentioned cases, the ones that characterize the models are the objects and the problems, so the definition is: *A mathematical model is a mathematical relation that describes engineering objects or problems.* The mathematical relations may be from an equation, equations system, to a distribution probability, etc. [5]

Mathematical models classification

In the same way that any object can be classified in different ways, it has been detected, through the analysis that there are at least two classifications for engineering mathematical models. The first is structured according to the use of the given model by engineering, while the second classification is done according to the knowledge blocks that the student has to study.

I. Engineering objects

When the mathematical models describe *engineering objects*, there are mathematical relations where the engineer has to do mathematical operations, and there are mathematical relations where the engineer does not have to do mathematical operations, so these originate dynamic or static type models [5].

The ***dynamic models*** are mathematical relations that due to the engineering requirements need mathematical modifications constantly, thus mathematical operations are done with them. An example of a dynamic model is an electrical signal of the sinusoidal alternating type, which is modeled mathematically by a real function of a real variable: $f(t) = A \sin at$. If its amplitude wave and frequency are changed, the new function is $g(t) = B \sin(bt)$, where $B=kA$ and $b=ca$, then, the original function was modified by multiplying it by a constant "k" and its composition with a linear function: $h(t)=ct$.

So, $(k)\,[f(t)] = kA \sin at = B \sin at$

and $(f \circ h)(t) = f[h(t)] = A \sin a[h(t)] = A \sin act = A \sin bt$

Then, $(k)\,[f \circ h](t) = (k)f[h(t)] = kA \sin a[h(t)] = kA \sin act = B \sin bt = g(t)$

The ***static models*** are mathematical relations which describe an engineering object as if it was a "nickname", that is, nothing else is done mathematically. An example of a dynamic model is the impulse function in electronic engineering, which is modeled mathematically by the Dirac delta. $\delta(t) = \begin{cases} 0, t \neq 0 \\ \infty, t = 0 \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

They only use this definition and the ones of displacement

$$\delta(t\text{-}a) = \begin{cases} 0, t \neq a \\ \infty, t = a \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t\text{-}a)\, dt = 1$$

and they do not do any mathematical operation. As it can be observed from static and dynamic models classification, it is according to the use given in engineering, because it is obvious that a given model could be dynamic in any engineering specialty, while in another it could be static. So, from this point, it is important to know which type of engineering the teacher is working on.

II. Engineering problems

On the other side, when mathematical models describe *engineering problems*, the difficulty of each problem is different. It is not the same if we have a problem that includes a data collection; a problem which includes single elements; a problem that has many elements where each one is a single element, which are called complex elements; a problem which includes many complex elements; a problem that includes many single and complex elements and combinations of these; etc. as the following examples [5].

(a) If we are in a laboratory and we want to know what is the relation among the experimental data of resistance (R), voltage (V) and current (I), we establish a mathematical relation which is the Ohm law, V=RI. This is a problem which includes single elements, which are: voltage, current and resistance, because they do not need another element to be determined. Another examples of these kind of problems are the engineering phenomena as the condenser charge, the free fall of a body, a pendulum movement, etc. In general, these are mathematical relations that originate laws or theorems of the physic and chemistry, which are a foundation of electronic engineering and its adjacent fields and which are basic sciences, as we saw en table 1. We classified these as ***first generation models***, they are the most simple models.

(b) If we have a problem which includes single element relations, that is to say, first generation models, and we have to relate them to build a new mathematical relation which modeled the problem, we called it ***second generation model.*** I will show you in example from a text book about it [18].

Como ejemplo de un circuito *RLC* de segundo orden, considérese el circuito de una malla que se muestra en la figura 6-1.1. Aplicando la LVK (ley de voltaje de Kirchhoff) a este circuito, podemos escribir $Li'(t) + Ri(t) + (1/c)\int_{-\infty}^{t} i(\tau)d\tau = 0$ Si diferenciamos esta ecuación y dividimos todos los términos entre L, de manera que el coeficiente del término de la derivada de mayor orden se haga igual a la unidad, obtenemos $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = 0$

As we said, the text book analysis can be done explicitly and implicitly. So, in implicit way we can see several first generation models, as the Ohm law, Kirchhoff laws, the current in a condenser and a bobbin, and condenser voltage: $v(t)=Ri(t)$, $v_L(t)+v_R(t)+v_C(t)=0$, $i_L(t)+i_R(t)+i_C(t)=i$, $i_c(t)=q'(t)$, $Li_L'(t)=v_L(t)$, and $v_c(t)=q(t)/c$. These generate the integrated-differential equation, where each element added is a first generation model, so the equation is second generation model. These kind of problems are studied in the electric circuits subjects which are part of the engineering basic sciences, as shown in table 1.

(c) In the text books analysis we detected problems like the next one.

Empezaremos notando que los circuitos *RL* y *RC* de segundo orden deben contar con más de una malla o más de un par de nodos. Si éste no es el caso, habrá dos elementos que almacenen energía conectados en paralelo o en serie en el circuito. Éstos pueden obviamente sustituirse por un solo elemento que almacena energía equivalente; el circuito resultante puede tratarse entonces como uno de primer orden. Un ejemplo de un circuito que contiene dos inductores, en el que éstos no pueden reducirse en un solo inductor, se muestra en la figura 6-3.1. Para caracterizar este circuito, es necesario escribir dos ecuaciones diferenciales. Éstas se encuentran aplicando la LVK (ley de voltaje de Kirchhoff) alrededor de las dos mallas del circuito. De este modo, obtenemos $R_1i_1(t)+L_1\frac{d(i_1-i_2)}{dt}=0$ and $R_2i_2(t)+L_1\frac{d(i_2-i_1)}{dt}+L_2\frac{di_2}{dt}=0$ [18]

As you see, in this problem all the elements are included as in letter (b), but it also includes other elements which make to combine some of the second generation models. In this case, it was detected implicitly that we have an equations system; in fact, we have two electrical net works and they are modeled by equations system.

$$\begin{cases} R_1i_1(t) + L_1\frac{d(i_1 - i_2)}{dt} = 0 \\ R_2i_2(t) + L_1\frac{d(i_2 - i_1)}{dt} + L_2\frac{di_2}{dt} = 0 \end{cases}$$

This mathematical model is formed by two models of second generation, so it is called **third generation model.** These kind of problems are worked in electrical circuits subjects, which are in the engineering basic sciences group, but they are used and worked too in the control theory subjects which belongs to the group of engineering specialization sciences. We found this kind of models in engineering specialization subjects.

(d) When the student is working with professional problems, as well as his engineering specialization subjects, he requires models that describe problems where third generation models are included and other kind of complex elements which has to combine and to build new mathematical relations, these relations are named as **fourth generation models**. Sometimes, due to the problem complexity it is difficult to tackle it, so it requires to be modeled by simulation in the computer, so a mathematical models family is built on the same problem. These kind of problems are more common in professional field, that is to say, in applied engineering.
From the aforementioned, the existing correlation between the classification of these models and the cognitive engineering stages are observed, as demonstrated in Table 2.

COGNITIVE ENGINEERING STAGES	TYPES OF MODELS
Basic Sciences	First generation models
Engineering Basic Sciences	Second generation models
Engineering Specialization Sciences	Third generation models
Applied Engineering	Fourth generation models

TABLE 2. CORRELATION BETWEEN COGNITIVE STAGES AND MODEL TYPES

A summary of a mathematical models classification according to their characterization is presented in table 3.

The mathematical modeling concept

After the detections made in the analysis of the problems studied for this research, we defined a mathematical modeling concept. *The mathematical modeling is conceived as the cognitive process that has to be carried out to build the mathematical model of a problem or object of the context area.*

This cognitive process consists of three moments, which constitute the mathematical modeling indicators: 1) Identify variables and constants of the problem, it includes the identification of what changes and what remains constant. 2) Establish relationships between these, through the involved concepts of the problem, either implicitly or explicitly, whether they are from the mathematical or from the context area. 3) Validate the ”mathematical relation” that models the problem, which is made through going back and verifying that it involves all the data, variables and concepts of the problem; depending on the problem, some times the mathematical problem can be validated through seeing if the mathematical expression predicts the experimental given information; in other cases, to validate the model, it is necessary to give a mathematical solution to see that the involved elements are predicted.

MATHEMATICAL MODELS CHARACTERIZATION					
Engineering Objects Modeling		Engineering Problems Modeling			
The **classification** is in terms of the use given by engineering		The **classification** is in terms of the engineering cognitive stages			
Static Models	Dynamic Models	First G. Models	Second G. Models	Third G. Models	Fourth G. Models

TABLE 3. CLASSIFICATION OF THE MATHEMATICAL MODELS

An important point to mention is that the mathematical model is not unique, there are several mathematical representations that describe the same problem, this is the reason why its validation is necessary (third moment). The way to tackle mathematically the mathematical model, is neither unique, this element permits to verify the mathematics versatility as well as its consistency.

Cognitive elements which interfere in building a mathematical model.

The analysis of the instrumentation problems, for the student sample, of each cognitive electrical engineering stage permitted to detect the regularities reported in this work, which are independent from scholar levels and independent from the knowledge areas. It is important to mention that the small paper space does not permit to present the evidence of the research that makes these results. In order to tackle the mathematical modeling it is necessary to have the following cognitive elements: ▪The focus of the mathematical subjects and concepts from the context area [2]. Each mathematical subject and concept have several focuses, for example, the derivative is a quotient of differentials, it is a very particular limit, it is the inverse operation to integrate, it is a rate of change, it is the slope of the tangent straight line to the curve, etc. The knowledge of these focuses is necessary for modeling. ▪The contextualized transposition [19]. It is known that the fact of scientific knowledge suffers a transformation to become knowledge to teach, denominated didactic transposition. The knowledge taken to the classroom suffers another transformation to become an applied knowledge, which is named contextualized transposition, the student has to know it. ▪The conceptual handle of descontextualized mathematics [6]. It is important that the student knows that mathematics are universal in the sense that they are applied in several contexts. Inside *Mathematics in the Sciences Context* the conceptual mathematic is conceived as a point that if a person has the concept, it is because this knowledge can be transferred, because the different concept focuses are known, because the mistake control points of the concept are known, because the concept behavior standards are known when the parameters to conform it are moved, because it is possible to travel among the different representations of the concept, etc.

Knowledge abilities that interfere in building a mathematical model. In the same way that in the cognitive elements, through the analysis of the instrumentation problems of each cognitive stage of the electronic engineering, thinking skills are detected which go into action in the building of a mathematical model. So, for tackling the mathematical modeling it is necessary to develop in the student the following thinking skills: ▪Skill to identify the mistake control points. This skill is part of having conceptual mathematics, as we have mentioned. ▪Skill to translate from the natural language to the mathematical language or vice versa. We have a research about how to categorize the contextualized mathematics problems related to the translation from the natural to the mathematical language [20]. ▪Skills to apply heuristics. The heuristics, as strategies to tackle a problem, with the classification given by Nickerson [21], to the ones given by Polya [15]. ▪Skill to identify regularities. Among the basic thinking skills, this skill becomes notorious. ▪Skill to travel among the different representations of a mathematical element. The representations described by Duval [22] are considered: arithmetic, algebraic, analytic and visual, including the contextual representation of *Mathematics in the Sciences Context*. ▪Skill to make “considerations” or “idealize” the problem (when it is right). There are problems so complex that they must be idealized so that they can be mathematized, in other occasions, it is necessary to consider, how to control variables so that the mathematization can be done.

Conclusions

The mathematical models are a fundamental part of *Mathematics in the Sciences Context*, the classification and characterization of mathematical models, as well as the cognitive elements and thinking skills that had been detected give a knowledge source to teach models in the mathematics classroom.

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