APPLYING PASTORAL METAMATISM OR RE-APPLYING GROUNDED MATHEMATICS

Allan Tarp, MATHeCADEMY.net, Denmark, Allan.Tarp@gmail.com

*When an application-based mathematics curriculum supposed to improve learning fails to do so, two questions may be raised: What prevents it from improving learning? And is 'mathematics applications' what it says, or something else? Skepticism towards wordings leads to postmodern thinking that, dating back to the ancient Greek sophists, warns against patronizing pastoral categories, theories and institutions. Anti-pastoral sophist research, identifying hidden alternatives to pastoral choices presented as nature, uncovers two kinds of mathematics: a grounded mathematics enlightening the physical world, and a pastoral self-referring mathematics wanting to save humans through 'metamatism', a mixture of 'metamatics' presenting concepts as examples of abstractions instead of as abstractions from examples; and 'mathematism' true in the library, but seldom in the laboratory. Also 'applying' could be reworded to 're-applying' to emphasize the physical roots of mathematics. Three preventing factors are identified: 'ten=10'-centrism claiming that counting can only take place using ten-bundles; fraction-centrism claiming that proportionality can only be seen as applying fractions; and set-centrism claiming that modelling can only take place by applying set-based concepts as functions, limits etc. In contrast, an entailing factor is grounded mathematics presenting itself as modelling the natural fact many, counted by bundling&stacking predicted by a recount-formula T = (T/b)*b that can be re-applied at all school levels.*

Applying Mathematics Improves Learning– or Does it?

The background of this study is the worldwide enrolment problem in mathematical based educations (Jensen et al 1998), and 'the relevance paradox formed by the simultaneous objective relevance and subjective irrelevance of mathematics' (Niss in Biehler et al 1994: 371). To improve learning it has been suggested that applications and modelling should play a more central role in mathematics education. However, when tested in the classroom the result is not always positive: 30 years ago the pre-calculus course at the Danish second-chance high school changed from being application-free to being application-based by replacing e.g. quadratic functions with exponential functions. Still student performance deteriorated to such a degree that at the 2005 reform the teacher union and the headmasters suggested that pre-calculus should no more be a compulsory subject. Thus two questions can be raised: Why did this application-based curriculum not improve learning? And is 'mathematics applications' what it says, or something else that might make a difference? Postmodern thinking, dating back to the ancient Greek sophists, has identified the hidden power in fixed wordings.

Anti-Pastoral Sophist Research

Ancient Greece saw a struggle between the sophists and the philosophers as to the nature of knowledge. The sophists warned that to protect democracy people should be enlightened to tell choice from nature in order to prevent patronization presenting its choices as nature. To the philosophers, seeing everything physical as examples of meta-physical forms only visible to them, patronization was a natural order when left to the philosophers (Russell 1945).

The Greek democracy vanished with the Greek silver bringing wealth by financing trade with Far-East luxury goods as silk and spices. Later this trade was reopened by German silver financing the Italian Renaissance; and by silver found in America. Robbing the slow Spanish silver ships returning on the Atlantic was no problem to the English; finding a route to India on open sea was. Until Newton found out that when the moon falls to the earth as does the apple, it is not obeying the unpredictable will of a meta-physical patronizor only attainable through faith, praying and church attendance; instead it is following its own predictable physical will attainable through knowledge, calculations and school attendance.

This insight created the Enlightenment period: when an apple obeys its own will, people should do the same and replace patronization with democracy. Two democracies were installed, one in the US, and one in France. The US still has its first republic; France now has its fifth.

The German autocracy tried to stop the French democracy by sending in an army. However, the German mercenaries were no matches to the French conscripts only too aware of the feudal consequences of loosing. So the French stopped the Germans, and later occupied Germany.

Unable to use the army, the German autocracy used the school to stop the enlightenment spreading from France. Humboldt was asked to create an elite school, and used Bildung as counter-enlightenment to create the self-referring Humboldt University (Denzin et al 2000: 85).

Inside the EU the sophist warning is kept alive in the French postmodern or post-structural thinking of Derrida, Lyotard and Foucault warning against patronizing categories, discourses and institutions presenting their choices as nature (Tarp 2004).

Derrida recommends that patronizing categories, called logocentrism, be ‘deconstructed’:

> Derrida encourages us to be especially wary of the notion of the centre. We cannot get by without a concept of the centre, perhaps, but if one were looking for a single ‘central idea’ for Derrida’s work it might be that of decentring. It is in this very general context that we might situate the significance of ‘poststructuralism’ and ‘deconstruction’: in other words, in terms of a decentring, starting with a decentring of the human subject, a decentring of institutions, a decentring of the logos. (Logos is ancient Greek for ‘word’, with all its connotations of the authority of ‘truth’, ‘meaning’, etc.) (..) It is a question of the deconstruction of logocentrism, then, in other words of ‘the centrism of language in general’. (Royle 2003: 15-16)

As to discourses Lyotard coins the term ‘postmodern’ when describing ‘the crisis of narratives’:

> I will use the term modern to designate any science that legitimates itself with reference to a metadiscourse (..) making an explicit appeal to some grand narrative (..) Simplifying to the extreme, I define postmodern as incredulity towards meta-narratives. (Lyotard 1984: xxiii, xxiv)

Foucault calls institutional patronization for ‘pastoral power’:

> The modern Western state has integrated in a new political shape, an old power technique which originated in Christian institutions. We call this power technique the pastoral power. (..) It was no longer a question of leading people to their salvation in the next world, but rather ensuring it in this world. And in this context, the word salvation takes on different meanings: health, well-being (..) And this implies that power of pastoral type, which over centuries (..) had been linked to a defined religious institution, suddenly spread out into the whole social body; it found support in a multitude of institutions (..) those of the family, medicine, psychiatry, education, and employers. (Foucault in Dreyfus et al 1982: 213, 215)

In this way Foucault opens our eyes to the salvation promise of the generalized church: ‘you are un-saved, un-educated, un-social, un-healthy! But do not fear, for we the saved, educated, social, healthy will save you. All you have to do is: repent and come to our institution, i.e. the church, the school, the correction center, the hospital, and accept becoming a docile lackey’.

To Foucault, institutions building on discourses building on categories build upon choice, so they all have a history, a ‘genealogy’ that can be uncovered by ‘knowledge archeology’.

The French skepticism towards words, our most fundamental institution, is validated by a ‘number&word observation’: Placed between a ruler and a dictionary a so-called ’17 cm long stick’ can point to ‘15’, but not to ‘stick’; thus it can itself falsify its number but not its word, which makes numbers nature and words choices becoming pastoral if hiding their alternatives.

On this basis a research paradigm can be created called ‘anti-pastoral sophist research’ deconstructing pastoral choices presented as nature by discovering hidden alternatives. Anti-pastoral sophist research doesn’t refer to but deconstruct existing research by asking ‘in this case, what is nature and what is pastoral choice presented as nature, thus covering alternatives to be uncovered by anti-pastoral sophist research?’ To make categories, discourses and institutions anti-pastoral they are grounded in nature using Grounded Theory (Glaser et al 1967), the natural research method developed in the American enlightenment democracy and resonating with Piaget’s principles of natural learning (Piaget 1970).

A Historical Background

The natural fact many provoked the creation of mathematics as a natural science addressing the two fundamental human questions ‘how to divide the earth and what it produces?’

Distinguishing the different degrees of many leads to counting that leads to numbers.

1.order counting counts in 1s and creates number-icons by rearranging the sticks so that there are five sticks in the five-icon 5 if written in a less sloppy way.

2.order counting counts by bundling&stacking using numbers with a name and an icon, resulting in a double stack of bundled and unbundled, e.g. T = 3 5s + 2 1s = 3)2) if using cup-writing leading to decimal-writing separating the left bundle-cup from the right single-cup: T = 3.2 5s = 3.2*5. The result can be predicted by the ‘recount-formula’ T = (T/b)*b iconizing that counting in bs means taking away bs T/b times, e.g. T = (4*5)/7*7 = 2*7 + 6*1 = 2.6*7 = 2)6).

3.order counting counts in tens, having a name but not an icon since the bundle-icon is never used: counting in 5s, T = 5 1s = 1 5s = 1.0 bundle = 10 if leaving out the decimal and the unit.

In Greek, mathematics means knowledge, i.e. what can be used to predict with, making mathematics a language for number-prediction: The calculation ‘2+3 = 5’ predicts that counting on 3 times from 2 will give 5. ‘2*3 = 6’ predicts that repeating adding 2 3 times will give 6. ‘2^3 = 8’ predicts that repeating multiplying with 2 3 times will give 8. Also, any calculation can be turned around and become a reversed calculation predicted by the reversed operation: In the question ‘3+x = 7’ the answer is predicted by the calculation x = 7–3, etc.

Thus the natural way to solve an equation is to move a number across the equation sign from the left forward- to the right backward-calculation side, reversing its calculation sign.

3+x = 7	3*x = 7	x^3 = 7	3^x = 7
x = 7–3	x = 7/3	$x = \sqrt[3]{7}$	$x = \log_3(7)$

In Arabic, algebra means reuniting, i.e. splitting a total in parts and (re)uniting parts into a total. The operations + and * unite variable and constant unit-numbers; ∫ and ^ unite variable and constant per-numbers. The inverse operations – and / split a total into variable and constant unit-numbers; d/dx and √ & log split a total into variable and constant per-numbers:

Totals unite/split into	Variable	Constant
Unit-numbers $, m, s, ...	T = a + n *T – n = a*	T = a * n *T/b = a*
Per-numbers $/m, m/s, m/100m = %, ...	ΔT = ∫ f dx *dT/dx = f*	T = a ^ n $\sqrt[n]{T} = a$, $\log_a T = n$

In Greek, geometry means earth measuring. Earth is measured by being divided into triangles, again being divided into right-angled triangles, each seen as a rectangle halved by a diagonal.

Recounting the height h and base b in the diagonal d produces three per-numbers:

sinA = height/diagonal = h/d, tanA = height/base = h/b, cosA = base/diagonal = b/d.

Also a circle can be divided into many right-angled triangles whose heights add up to the circumference C of the circle: C = 2 * r * (n*sin(180/n)) = 2 * r * π for n sufficiently big.

However, needing the Arabic numbers, Greek geometry turned into Euclidean geometry, freezing the development of mathematics until the Enlightenment century:

> The enthusiasm of the mathematicians was almost unbounded. They had glimpses of a promised land and were eager to push forward. They were, moreover, able to work in an atmosphere far more suitable for creation than at any time since 300 B.C. Classical Greek geometry had not only imposed restrictions on the domain of mathematics but had impressed a level of rigor for acceptable mathematics that hampered creativity. The seventeen-century men had broken both of these bonds. Progress in mathematics almost demands a complete disregard of logical scruples; and, fortunately, the mathematicians now dared to place their confidence in intuitions and physical insights. (Kline 1972: 399)

The success was so overwhelming that mathematicians feared that mathematics (called geometry at that time) had come to a standstill at then end of the 18th century:

> Physics and chemistry now offer the most brilliant riches and easier exploitation; also our century's taste appears to be entirely in this direction and it is not impossible that the chairs of geometry in the Academy will one day become what the chairs of Arabic presently are in the universities. (Lagrange in Kline 1972: 623)

But in spite of the fact that calculus and its applications had been developed without it, logical scruples soon were reintroduced arguing that both calculus and the real numbers needed a rigorous foundation. So in the 1870s the concept 'set' reintroduced rigor into mathematics.

Mathematics Versus Metamatics

Using sets, a function is defined 'from above' as a set of ordered pairs where first-component identity implies second-component identity; or phrased differently, as a rule assigning exactly one number in a range-set to each number in a domain-set. The Enlightenment defined function 'from below' as an abstraction from calculations containing a variable quantity:

> A function of a variable quantity is an analytic expression composed in any way whatsoever of the variable quantity and numbers or constant quantities. (Euler 1748, 1988: 3)

So where the Enlightenment defined a concept as an abstraction from examples, the modern set-based definition does the opposite; it defines a concept as an example of an abstraction. To tell theses alternatives apart we can introduce the notions 'grounded mathematics' abstracting from examples versus 'set-based metamatics' exemplifying from abstractions, and that by proving its statements as deductions from meta-physical axioms becomes entirely self-referring needing no outside world. However, a self-referring mathematics soon turned out to be an impossible dream. With his paradox about the set of sets not being a member of itself, Russell proved that using sets implies self-reference and self-contradiction known from the classical liar-paradox 'this statement is false' being false when true and true when false:

Definition $M = \{A \mid A \notin A)\}$, statement $M \in M \Leftrightarrow M \notin M$.

Likewise, without using self-reference it is impossible to prove that a proof is a proof; a proof must be defined. And Gödel soon showed that theories couldn't be proven consistent since they will always contain statements that can neither be proved nor disproved.

Still, set-based mathematics soon found its way to the school even if it creates syntax errors:

A formula containing two unknowns is a function, e.g. y = 2*x+3 = f(x) where f(x) = 2*x+3 means that 2*x+3 is a calculation containing x as the variable number. A function can be tabled and graphed, both describing if-then scenarios 'if x = 6 then y = 15'. But writing f(6) = 15 means that 15 is a calculation containing 6 as the variable number. This is a syntax error since 15 is a number, not a calculation, and since 6 is a number, not a variable. Functions can be linear, quadratic, etc., but not numbers. So a function cannot increase or decrease.

Set-based metamatics defines a fraction as an equivalence set in a product set of two sets of numbers such that the pair (a,b) is equivalent to the pair (c,d) if a*d = b*c, which makes e.g. (2,4) and (3,6) represent then same fraction ½. However, this definition conflicts with Russell's set paradox, solved by Russell by introducing a type-theory stating that a given type can only be a member of (i.e. described by) types from a higher level. Thus a fraction defined as a set of numbers is not a number itself, making e.g. the addition '2+3/4' meaningless.

Wanting fractions to be 'rational' numbers, set-based mathematics has chosen to neglect Russell's type-theory by accepting the Zermelo-Fraenkel axiom system making self-reference legal by not distinguishing between an element of a set and the set itself. But removing the distinction between examples and abstractions and between different abstraction levels means hiding that historically mathematics developed through layers of abstractions; and that mathematics can be defined through abstractions in a meaningful and uncontroversial way.

Mathematics Versus Mathematism

Traditionally, both 2+3 = 5 and 2*3 = 6 are considered universal true statements. The latter is grounded in the fact that 2 3s can be recounted to 6 1s. The first, however, is an example of 'mathematism' true in a library, but not in a laboratory where countless counter-examples exist: 2weeks + 3 days = 17 days, 2m + 3cm = 203cm etc. Thus addition only holds inside a bracket assuring that the units are the same: 2m + 3cm = 2*100cm + 3cm = (200 + 3)cm = 203cm.

Also, adding fractions without units is an example of mathematism:

Inside the classroom	20% (20/100) + 10% (10/100)	= 30% (30/100)
Outside the classroom e.g. in the laboratory	20% + 10%	= 32% in the case of compound interest = b% (10<b<20) in the case of the total average

Mathematics Modelling in Primary School

Having learned how to assign numbers to totals through counting by bundling&stacking, a real-world question as 'what is the total of 2 fours and 3 fives' can lead to two different models.

Model 1 says: This question is an application of addition. The mathematical problem is to find the total of 2 and 3. Applying simple addition, the mathematical solution is 2 + 3 = 5, leading to the real-world solution 'the total is 5'. An answer that is useless and incorrect because it has left out the unit.

Model 2 says: This question is a re-application of recounting. The mathematical problem is to find the total of 2 4s and 3 5s. Re-applying recounting to find a mathematical solution, the units must be the same before adding so we recount the 2 4s in 5s, predicted by the recount-formula T = (2*4)/5*5 = 1 5s + 3 1s, giving the total of T = 4 5s + 3 1s = 4.3 5s, which leads to the real-world solution 'the total is 4 fives and 3 ones'. A prediction that holds when tested:

IIII IIII + IIIII IIIII IIIII -> IIIII III + IIIII IIIII IIIII -> IIIII IIIII IIIII IIIII III = 4)3) = 4.3 5s

This example shows that applying mathematism may lead to incorrect solutions when modelling addition problems. Whereas applying grounded mathematics creates the categories 'stack' and 'recounting', and allows practicing recounting by asking e.g. 3 fours = ? fives.

Recounting a stack in a different bundle-size is a brilliant example of a modelling process:

	Problem		Solution
Mathematics	T = 3*4 = ?*5	→	T = (3*4)/5*5 = 2.2 5s
	↑		↓
Real-world	IIII IIII IIII -> 5s		IIII IIII IIII -> IIIII IIIII II

Rephrasing the problem to 'what is the total of nines in 3 fours and 2 fives' introduces integration already in primary school. As a matter of fact, the core of mathematics can be introduced as application of recounting, using 1digit numbers alone (Zybartas et al 2005).

However, this is impossible in a 'ten=10'-curriculum that by presenting 10 as the follower of nine introduces at once the number ten as the standard bundle-size, a pastoral choice hiding that also other numbers can be used as bundle-size. 10 simply means bundle, i.e. 1.0 bundle if not excluding the unit. Thus counting in 7s, 10 is the follower of 6, and the follower of nine is 13.

With ten as bundle-size, recounting problems disappear, and all numbers loose their units, which creates the basis for teaching mathematism where 3 + 2 IS 5 without discussion.

Thus in primary school an application-based curriculum using recounting to learn the modelling process is prevented by a pastoral choice, 'ten=10'-centrism, hiding that also other numbers can be used when counting by bundling&stacking. And prevented by mathematism claiming that 3 + 2 IS 5.

Mathematical Modelling in Middle School

Middle school introduces fractions as rational numbers and allows them to be added without units in spite of the fact that fractions are multipliers carrying units: 1/3 of 6 = 1/3*6.

The real-world question ‘what is the total of 1 coke among 2 bottles and 2 cokes among 3 bottles?’ can lead to two different models.

Model 1 says: This question is an application of adding fractions. The mathematical problem is to find the total of 1/2 and 2/3. Applying simple addition of fractions, the mathematical solution is 1/2 + 2/3 = 3/6 + 4/6 = 7/6, leading to the real-world solution ‘7 out of the 6 bottles are cokes’. An answer that is meaningless and useless because it has left out the unit: we cannot have 7 cokes if we only have 6 bottles; and we do not have 6 bottles, we only have 5.

Model 2 says: This question is a re-application of adding stacks by integrating their bundles. The mathematical problem is to find the total of 1/2 of 2 and 2/3 of 3. Re-applying integration, the mathematical solution is T = 1/2 * 2 + 2/3 * 3 = 3 = 3/5 * 5, giving the real world solution ‘the total is 3 cokes of 5 bottles’. A prediction that holds when tested on a lever carrying to the left 5 units in the distance 2 and 7.7 units in distance 3, and to the right 6 units in distance 5.

Sharing problems asking ‘the boys A, B and C paid $1, $2 and $3 to a pool buying a lottery ticket. How should they share a 300$ win?’ can lead to two different models.

Model 1 says: This question is an application of fractions. The mathematical problem is to split a total of 300 in the proportions 1:2:3. Applying simple addition of fractions gives the answer: since boy A paid 1/(1+2+3) = 1/6 of the ticket he should receive 1/6 of the win, i.e. 1/6 of $300 = 1/6*300 = 50; likewise with the other boys. So the real-world solution is: boy A $50, boy B $100, and boy C $150. Of course such questions can only be answered after fractions and its algebra has been taught and learned.

Model 2 says: This question is a re-application of recounting. The mathematical problem is to recount the win in pools, i.e. in 6s, which then can be paid back to the boys a certain number of times. Since 300 = (300/6)*6 = 50*6, the boys are paid back 50 times. So the real world solution is A: $1*50 = $50, B: $2*50 = $100, and C: $3*50 = $150.

Trade problems as ‘if the cost is 2$ for 5kg, what then is the cost for 14kg, and how much can I get for 6$?’ can lead to three different models.

Model 1 says: This question is an application of proportionality, fractions and equations. The mathematical problem is to set up an equation relating the unknown to the 3 known numbers. Applying proportionality, fractions and equations we can set up a fraction-equation expressing that the cost c and the volume v is proportional, c/v = k. Hence c1/v1 = c2/v2, or 2/5 = x/14 and 2/5 = 6/x. Now the x can be found by solving the equations, or by cross multiplication. So the real-world solution is 5.6$ and 15 kg. Of course such questions can only be answered after fractions and proportionality and equations has been taught and learned.

Model 2 says: This question is an application of linear functions. The mathematical problem is to set up a linear function expressing the price y as a function of the volume x, y = f(x) = m*x + c, given that the points (0,0) and (5,2) belongs to the graph of the function. The mathematical solution first finds c by inserting the point (0,0) in the formula: f(0) = m*0+c = 0, so c = 0; then we find m by inserting the point (5,2) in the formula: f(5) = m*5 = 2, so m = 2/5 = 0.4. Hence the linear formula is f(x) = 0.4*x. To answer the questions we insert the points (14,y) and (x,6) into the function: f(14) = 0.4*14 = y, and f(x) = 0.4*x = 6. Solving these equations give y = 5.6 and x = 15. So the real-world solution is 5.6$ and 15 kg. Of course such questions can only be answered when general functions, linear functions and equations has been taught and learned.

Model 3 says: This question is a re-application of recounting. The mathematical problem is to recount the 14kg in 5s and the 6$ in 2s since the cost is 2$ per 5kg. Thus the mathematical solution is 14kg = (14/5) * 5kg = (14/5) * 2$ = 5.6$, and 6$ = (6/2) * 2$ = (6/2) * 5kg = 15kg.

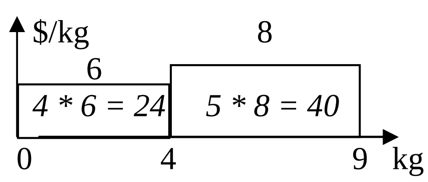
The examples show that many problems in middle school are re-applications of recounting from primary school; unless they are presented as being only solvable by applying fractions, proportionality and equations, in which case the modelling has to wait until these subjects has been taught and learned, which also excludes students unable to learn ungrounded mathematics.

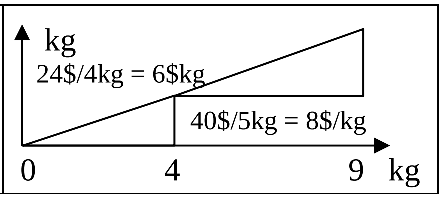
Again, unreflectively applying mathematism may lead to incorrect solutions. Whereas using grounded mathematics creates the category fraction as an abstraction from per-numbers coming from double-counting in two different units 3 1s = (3/5)*5, and 2$/5kg = 2/5 $/kg.

Adding numbers with units also occurs when modelling mixture situations, generalizing primary school's integrating stacks to middle school integral and differential calculus.

Thus asking 4 kg at 6$kg + 5kg at 8$/kg = 9 kg at ? $/kg can be answered by using a table or a graph, realizing that integration means finding the area under the per-number graph; and vice versa, that the per-number is found as the gradient on the total-graph

4kg at 6$/kg	= 4 * 6 = 24
5kg at 8$/kg	= 5 * 8 = 40
9kg at ?$/kg	= 9 * ? = 64
	? = 64/9





Mathematical Modelling in High School

High school introduces set-based functions as its basis. Thus a quantity growing by a constant number IS an example of a linear function; and a quantity growing by a constant percent IS an example of an exponential function; and both ARE examples of the set-based function concept.

Thus the real-world problem '200$ + ? days at 5$/day is 300$' leads to two different models: model 1 seeing the question as an application of linear functions; and model 2 seeing the question as a re-application of abbreviating a statement to an equation: with constant change, the terminal number T is the beginning number b added with the change m a certain number of times x: T = b + m*x. Inserting T = 300, b = 200 and m = 5 and using the Math Solver on a Graphical Display Calculator, the solution is found as x = 20. This prediction can be tested when graphing the function y = 200 + 5*x and observing that tracing x = 20 gives y = 300.

Cumulating a capital C by a yearly deposit p and interest rate r leads to two different models: model 1 seeing the question as an application of a geometric series; and model 2 setting up two accounts, one with the amount p/r from which the yearly interest p/r*r = p is transferred to the other, which after n years contains the cumulated capital, and the cumulated interest p/r * R where by 1+R = (1+r)^n. With C = p/r * R, a beautiful a simple ratio appears: C/p = R/r.

Two different models come out of the real-world problem 'Out driving, Peter observed the speed to be 6, 18, 11, 12 m/s after 5, 10, 15 and 20 seconds. What was the speed after 6 seconds? When was the speed 15m/s? When did he stop accelerating? When did he begin to accelerate again? What was the total distance traveled from 7 to 12 seconds?

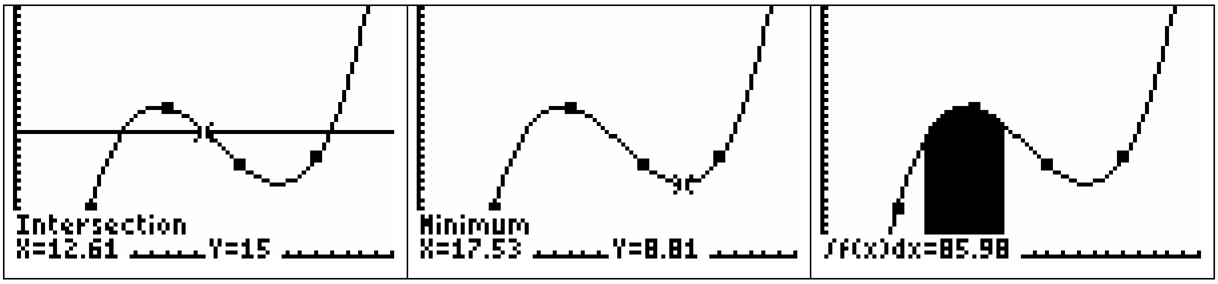
Model 1 says: This question is an application of matrices and differential and integral calculus. The mathematical problem is to set up a function expressing the distance y as a function of the time x, given that the function's graph contains the points (5,6), (10,18), (15,11), and (20,12). Applying matrices to solve 4 equations with 4 unknowns, the mathematical solution is y = 0.036 x^3 – 1.46 x^2 + 18 x – 52. Now the point (6,0) is inserted in the function to find y = 11.22. Inserting the point (x,15) in the function leads to a 3rd degree equation.

To solve this equation we guess a solution in order to factorize the 3rd degree polynomial to y = 0.036(x – 7.07)(x – 12.61)(x – 20.87). To find the turning points we must find the zeros of the derivative y' = 0.108 x^2 – 2.92 x + 18, i.e. x = 9.51 and x = 17.53, as well as the signs of the double-derivative y'' = 0.216 x – 2.92 changing sign from minus to plus in x = 13.52. Finally the distance traveled from 7 seconds to 12 seconds comes from the integral:

$$\int_7^{12} (0.036\, x^3 - 1.46\, x^2 + 18\, x - 52)\, dx = \left[\, 0.009\, x^4 - 0.49\, x^3 + 9\, x^2 - 52x \,\right]_7^{12} = 85.98.$$

Of course, this must wait until after matrices, polynomials and calculus has been taught.

Model 2 says: This question is a re-application of per-numbers. The mathematical problem is to find a per-number formula f(x) from a table of 4 data sets. On a Graphical Display Calculator Lists and CubicRegression does the job. Tracing x = 6 gives y = 11.22. Finding the intersection points with the line y = 15 using Calc Intersection gives x = 7.07, 12.61 and 20.87. Finding the turning points using Calc Minimum and Calc Maximum gives a local maximum at x = 9.51 and y = 18.10, and a local minimum at x = 17.53 and y = 8.81. The total meter-number from 7 to 12 seconds is found by summing up the m/s*s, i.e. by using Calc ∫f(x)*dx, which gives 85.98.



Change Equations

Solving any change-equation dy/dx = f(x,y) is easy when using technology. The change-equation calculates the change dy that added to the initial value gives the terminal y-value, becoming the initial y-value in the next period. Thus is dy = r*y, r = ro*(1 – y/M) is the change-equation if a population y grows with a rate r decreasing in a linear way with the population having M as its maximum. A spreadsheet can keep on calculating the formula y + dy -> y.

The Grand Narratives of the Quantitative Literature

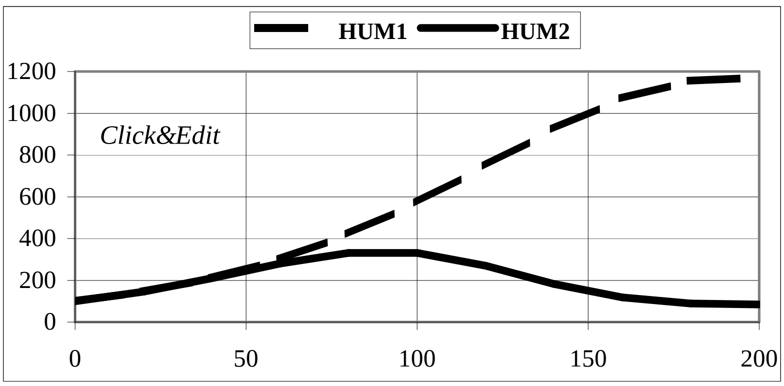
Literature is narratives about real-world persons, actions and phenomena. Quantitative literature also has its grand narratives. That an infinity of numbers can be added by 1 difference if the numbers can be written as change-numbers is a grand narrative: The y-change dy can be recounted in the x-change dx: dy = (dy/dx)*dx = y'*dx. So ∫y'dx = ∫dy = Δy = y2 – y1:

Since x^2 = (x^3/3)', then ∫x^2 dx = ∫(x^3/3)' dx = 7^3/3 – 2^3/3 if summing from 2 to 7.

In physics, grand narratives can be found among those telling about the effect of forces, e.g. gravity, producing parabola orbits on earth, and circular and ellipse orbits in space. Jumping from a swing is a simple example of a complicated model. Physics' grand narratives enabled the rise of the Enlightenment and the modern democracy replacing religion with science.

In economics, an example of a grand narrative is Malthus' 'principle of population' comparing the linear growth of food production with the exponential growth of the population; and Keynes' model relating demand and employment creating the modern welfare society. As are the macroeconomic models predicting effects of taxation and reallocation policies. Also limit-to-growth models constitute grand narratives predicting the future population depending on different assumptions as to e.g. food and pollution: without pollution only food will restrict human growth, but including pollution the population level might be different:

Time	HUM1	HUM2
0	100	100
20	145	145
40	210	210
60	301	280
80	421	331
100	572	330
120	747	269
140	925	182
160	1071	118
180	1156	89
200	1171	84



An Applied Ethnography Observation

Working together with Mogens Niss 30 years ago, we discussed ways to include applications and modelling in the Danish mathematics curriculum. Niss stayed at the university, I chose to become an applied ethnographer in the classroom. By replacing functions with variables at the pre-calculus level, I designed and tested an application and modelling based curriculum that made all students learn everything. However, Denmark is the only country in the world still practicing oral exams in mathematics, and the ministry of education's examiner didn't like a function-free curriculum. Likewise, countless articles to the mathematics teachers' journal on the advantages of a set&function-free curriculum were neglected. It took 30 years before the Ministry of Education finally followed my advice: to improve learning, replace functions with variables and make the curriculum application and modelling based. However, care is needed since the phrasing 'apply mathematics' installs as self-evident that 'of course mathematics must be learned before it can be applied'. This contradicts the historical fact that mathematics was created as layers of abstractions coming from modelling real-world problems.

Factors Preventing an Application & Modelling Based Curriculum

Several factors preventing an application and modelling based curriculum have been identified.

1. When mathematism is applied instead of mathematics answers proven correct in a library may not hold in a laboratory. This makes mathematics totally self-referential and impossible to use as a prediction of real-world situations. Adding numbers without units in primary school and adding fractions without units in middle school are examples of mathematism.

Different examples of 'centrism' claiming to have monopoly in certain modelling situations prevent or postpone many fruitful modelling situations, and exclude many potential learners.

2. In primary school, 'ten=10'-centrism conceals that 'ten IS 10' is a pastoral choice hiding its alternatives, e.g. five=10 in the case of counting&bundling in 5s. This prevents modelling recounting-situations as e.g. 4 5s = ? 7s predicted by the recount-formula T = (T/b)*b.

3. In middle school, fraction-centrism presenting fraction without units is a pastoral choice hiding its alternative, per-numbers. This forces proportionality to become an application of fractions, and prevents it from being a re-application of recounting. Being defined as sets, fractions become an example of 'metamatism' merging metamatics with mathematism.

4. In high school, set-centrism demands all concepts be defined as examples of the concept set. Solving equations by the set-based neutralizing method prevents equations from being solved by reversed calculation. Set-based functions and calculus prevent change situations from being modelled by re-applying per-numbers and using a Graphical Display Calculator.

Factors Entailing an Application & Modelling Based Curriculum

Two factors entailing an application and modelling based curriculum have been identified.

First, changing or deconstructing 'applying mathematics' to 're-applying mathematics' will signal that historically mathematics was created as an application modelling real-world problems, i.e. as a language describing and predicting the natural fact many. This distinction is useful when answering the question 'does 'applying mathematics' mean applying pastoral metamatics and mathematism, or re-applying grounded mathematics?'

Second, banning from school set-based metamatics and mathematism will bring back a new Enlightenment period with grounded mathematics. Thus in primary school 'ten=10'-centrism is banned by practicing counting by bundling&stacking in 5-bundels, 7-bundles etc. before finally choosing ten as the standard bundle-size. In middle school fractions should be seen as per-numbers, and should always carry units; and equations should be introduced as reversed calculations. In high school functions and equations should be seen as formulas with two or one unknown to be treated on a graphical display calculator using regression to produce formulas, describing per-numbers to be integrated to totals, or totals to be differentiated to per-numbers.

Conclusion

Now an answer can be given to the two initial questions. To improve learning, an application-based mathematics curriculum should be applying grounded mathematics rooted in real-world applications; and not be applying metamatism, a mixture of metamatics presenting concepts as examples of abstractions instead of as abstractions from examples, and mathematism true in a library but not in a laboratory and therefore unable to predict real-world situations.

Applying metamatism forces three cases of centrism upon mathematics as pastoral choices hiding their alternatives. The use of 'ten=10'-centrism hides that also other numbers than ten can be used as bundle-size when counting by bundling&stacking. Fraction-centrism hides that proportionality and many other applications of fractions can also be solved by instead re-applying recounting. And set-centrism hides that modelling change can take place without the use of set-based concepts as functions and limits. Finally the wording 'apply mathematics' installs as self-evident that 'of-course mathematics must be taught and learned before it can be applied', thus hiding that historically mathematics is rooted in the real world as a model.

So to improve learning, the wording 'apply mathematics' should be checked carefully. Does it imply that metamatism must be taught before applications? Or does it mean re-applying mathematics grounded in real-world problems rising from the two fundamental questions 'how to split the earth and what it produces' that provoked the creation of geometry as earth measuring, and algebra as reuniting constant and variable unit and per-numbers?

A grounded approach will respect the historical nature of mathematics as a natural science investigating the physical fact many. Here mathematics is created through its applications and then re-applied to similar situations. To avoid 'ten=10'-centrism in primary school, before introducing 3.order counting installing ten as the only bundle-size, 2.order counting is used to emphasize that mathematics is a language for predicting real-world numbers, and to allow the learning of 1digit mathematics. To avoid fraction-centrism in middle school, proportionality is based upon recounting and per-numbers, and fractions always carry units when added. To avoid set-centrism in high school, the Graphical Display Calculator is used when modelling change, both the standard linear, exponential and polynomial models and the more complicated models.

References

Biehler R, Scholz R W, Strässer R & Winkelmann B (1994) *Didactics of Mathematics as a Scientific Discipline*, Dordrecht: Kluwer Academic Press

Denzin N & Lincoln Y (2000) *Handbook of Qualitative Research 2nd ed.*, London: Sage

Dreyfus H L & Rabinow P (1982) 2. ed. *Michel Foucault, beyond structuralism and hermeneutics*, Chicago: University of Chicago Press

Euler L (1988): *Introduction to Analysis of the Infinite*, New York: Springer Verlag

Glaser B G & Strauss A L (1967) *The Discovery of Grounded Theory*, NY: Aldine de Gruyter.

Jensen J H, Niss M & Wedege T (1998) *Justification and Enrolment Problems in Education Involving Mathematics or Physics*, Roskilde: Roskilde University Press

Kline M (1972) *Mathematical Thoughts from Ancient to Modern Times*, New York: Oxford University Press

Lyotard J (1984) *The postmodern Condition: A report on Knowledge*, Manchester: Manchester University Press

Piaget J (1970) *Science of Education of the Psychology of the Child*, New York: Viking

Royle N (2003) *Jaques Derrida*, London: Routledge

Russell B (1945) *A History of Western Philosophy*, New York: A Touchstone Book

Tarp A (2004) *Pastoral Power in Mathematics Education*, Paper accepted for presentation at Topic Study Group 25. The 10th Int. Conf. on Mathematics Education, ICME 10, 2004, www.MATHeCADEMY.net

Zybartas S & Tarp A (2005) One Digit Mathematics, *Pedagogika (78/2005)*, Vilnius, Lithuania