A Comparative Study on Mathematical Modelling Competences with German and Chinese Students

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Abstract: During the last ten years both in Germany and in China mathematic education focuses on promoting students to develop their mathematic modelling competences. In our study we have investigated modelling competence level of students from grade 9 to 11 (15- 17 years old) while dealing with the special real situation "peeling a pineapple". This study analyzed which level related to the modelling cycle the German and Chinese students have reached, and if there are some gender differences during doing mathematic modelling process. Moreover this study explored the mathematic modelling competences related to different grades, and some diversities between German and Chinese students in relation to the modelling competences.

1. Introduction

During the last ten years it has been a central aim for mathematic education to help students to develop abilities to recognize the relation between the real world and mathematics, and to identify and understand the role that mathematics plays in the world. (KMK, 2003; YZZ, 2003, NCTM 2006) Such abilities are related to the development of mathematic modelling competences, such as analyzing, assimilating, interpreting and validating a problem. In our study we have investigated modelling competencies activated by student when dealing with an authentic problem from the real world. (Maass, 2006)

2. Theoretical Framework

Following several research works "mathematical modelling competencies" have been defined as "the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation."(Werner Blum, Peter L. Galbraith, Hans-Wolfgang Henn & Morgens Niss, 2007, p.12) Similarly, Blum (2002) defined modelling competence as the ability to structure, mathematize, interpret and solve problems and, in addition, the ability to analyse or compare models by investigating the assumptions being made, checking properties and scope of models etc. In our study we have divided mathematical modelling competences into different levels as follows:

- Level 0: The student has not understood the situation and is not able to do sketch or write anything concrete about the problem.
- Level 1: The student only understands the given real situation, but is not able to structure and simplify the situation or cannot find connection to any mathematic ideas.
- Level 2: After investigating the given real situation, the student finds a real model through structuring and simplifying, but does not know how to transfer this into a mathematical problem (word problem).
- Level 3: The student is able to not only find a real model, but also translate it into a proper mathematic problem, but cannot work with it clearly in the mathematic world.

- Level 4: The student is able to pick up a mathematic problem from the real situation, work with this mathematic problem in mathematic world, and have results.
- Level 5: The student is able to experience the mathematic modelling process and validate the solution of a mathematic problem in relation to the given situation.

Based on a concrete problem situation we will show different actions in relation to different levels. The levels defined above are basing on the traditional modelling cycle which is used e.g. by Blum and Leiß (Blum/Leiß, 2005). We can link the five levels above with the steps in this modelling cycle. Level 0 corresponds the Situation before step1(Understandig the task). Level 1 is between Step one and two. Level 2 corresponds to Step 2. Level 3 corresponds to the mathematical modell. Level 4 is comparable to the "mathematical result" and level 5 means that the student has make a full modelling cycle comparable to Step 6.

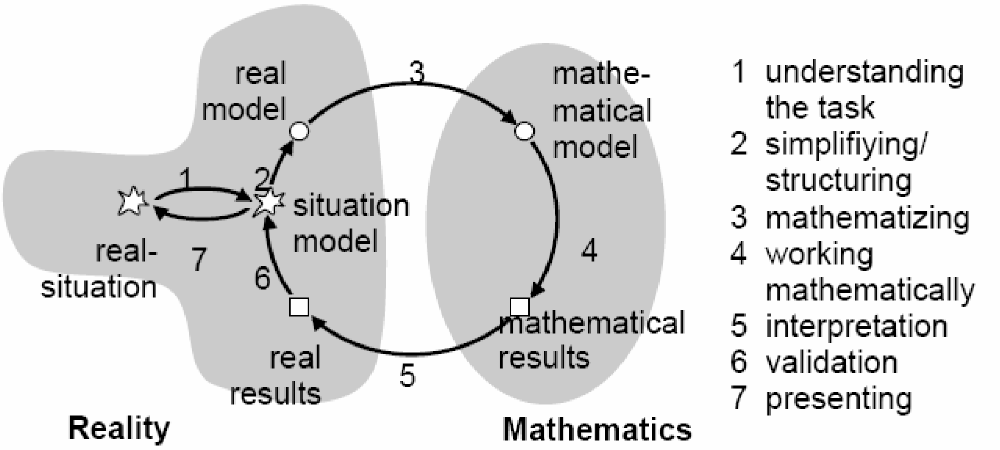

Fig.1 Taken from Blum/ Leiß 2005

Of course we know by the reserach of Borromeo-Ferri (Borromeo-Ferri, 2006), that most of the students do not pass the circle consecutive, but they jump e.g. directly from the real situation to the mathematical model. But we assume, that the farther you will end in the modelling cycle you have to negotiate more cognitive obstacles.

3. A Comparative Study

3.1 Study Design

3.1.1 A Real Situation

We give the following real situation (peeling a pineapple):

Every April is season for pineapples. While we buy a pineapple, sometimes the salesman will peel it for us, especially in China. This is an artistic peeling process, after peeling very nice spirals will leave behind.

Please think about it, why does the salesman peel the pineapple in this way? Is it easy for peeling? Or do they aovid lossing too much pulp? Or are there some other reasons? Please explain it mathematically.

3.1.2 Testee Students

We have chosen more than 1,000 students respectively from grade 9 to grade 11 in Germany and China. The distribution of the testee students is as follows:

(Total Number)		Grade 9	Grade 10	Grade 11
German testee students (from South Germany)	Female	64	76	53
	Male	81	71	83
Chinese testee students (from Shanghai)	Female	103	124	106
	Male	103	129	115

3.2 Study Implementation

The testee students have been shown a 90-second video about peeling a pineapple. Then they received a work sheet with explanation of peeling pineapples and some questions like:

“Please think about it mathematically, why does the salesman peel the pineapple in this way?”

The students could work for 25 minutes to 35 minutes. At the end of the lesson the students should present their results to their classmates.

3.3 Result Analyzing

According to level criterion on mathematic modelling competence, the students’ solutions have been carefully rated, classified and registered. Using some statistical tools the classified information has been analyzed in response to the following research questions:

> Which level related to the modelling cycle the German and Chinese students have reached? Does the mathematic modelling competence relate to different grades? Are there some gender differences when doing mathematic modelling process? Are there some diversities between German and Chinese students in relation to the modelling competences?

4. Mathematics behind the problem

4.1. A first solution

Before we present some students solution which are correspondent to our levels we want to show one possible mathematical modelling of pineapple peeling. The question was why the fruitseller peels the pineaapple in this special way. A first aproach is to consider the pineapple as a cylinder. The black dots are very regular, see fig. 2 left. You can see the spirals in a good way. The cylinder on the right side looks nearly like the peeled pineapple.

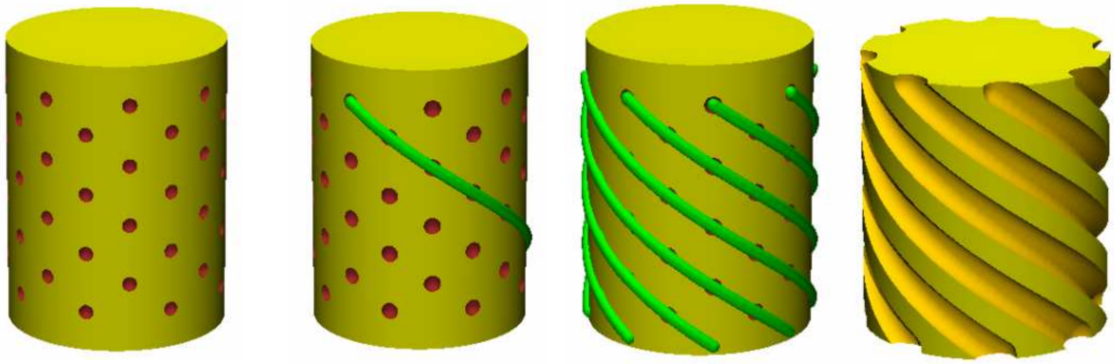

Fig. 2 the developement of the mathematical model

In figure 2 we have transformed the real situation into a mathematical situation. Our next step is only mathematically: we spread the curved wall to a rectangle. And now we see that the spirals transformed into straight lines. We know that a straight line is the shortes conjunction between two points. But we have to make clear while in this case the diagonal lines are the optimum.We assume that the dots are building a square grid. So we can calculate that the length of the diagonal d is $\sqrt{2}$ s. This means that the length of the peeling way is more than 40% longer if you take the vertical way or the horizontal way instead the diagonal way. (see Figure 3, left)

4.2. A students solution

Here we want to show you two students solutions the left one is related to level 4 and the rigth one to level Zero. The student is able to translate the real model into a mathematical model. He found out that the diagonal way is longer and he calculated this and got a mathematical result but finally there is no feedback to the real world. So the student miss level 5.

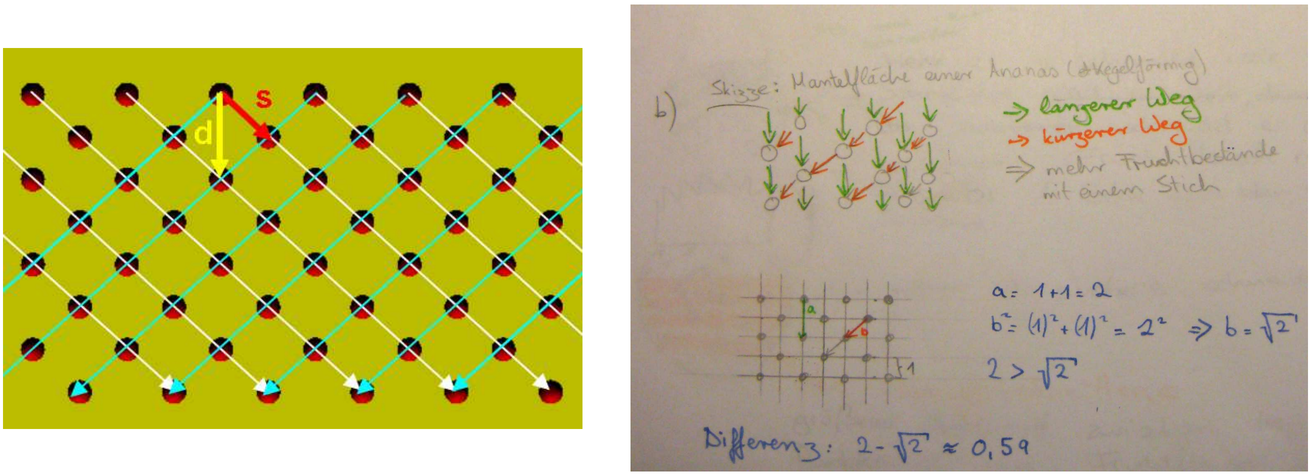

Fig.3: The grid made on the spread wall (left) , a students solution corresponds to level 4 (right)

5. Discussion

5.1. First results

We found some interesting results. First the general performance of german and chinese students is nearly the same, but for chinese students, the development of mathematic modelling competence depends strongly on different grades. From grade 9 to grade 11 the students have developed their competences better and better, while among German students there have not been obvious differences between grade 9 and 10, but in grade 11 there is a big leap. In addition we have found gender differences especially for chinese students when dealing with mathematic modelling problem. For German students we found that the boys will develop their mathematic modelling abilities very strong in the 11th grade. (see table 1)

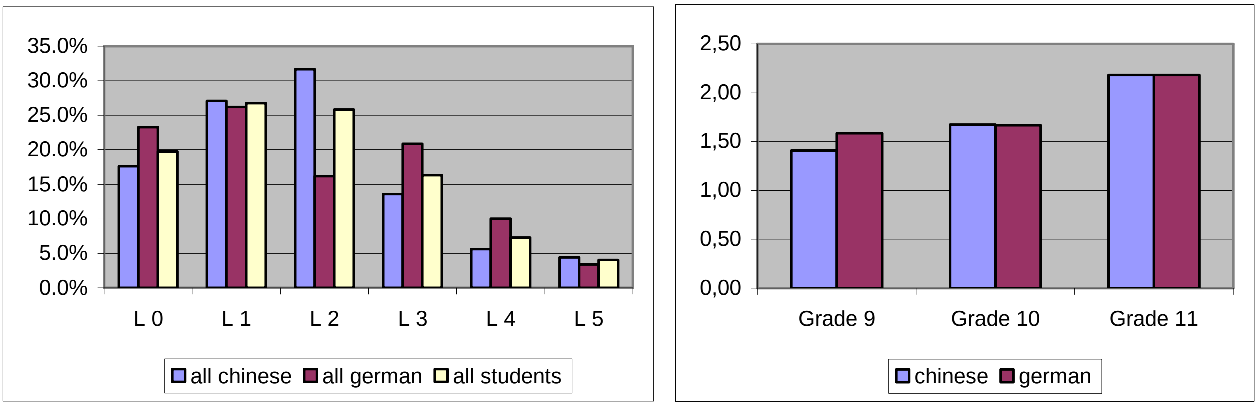

Fig. 4: Percentage of students who reach level x The average level the students reach

This table shows the results which depended on gender and grade.

Table 1

		Grade 9	SD		Grade 10	SD		Grade 11	SD
	N	Mean	SD	N	Mean	SD	N	Mean	SD
Chinese girls	**103**	**1,63**	1,27	**124**	**1,71**	1,09	**106**	**2,30**	1,30
Chinese boys	**103**	**1,18**	1,20	**129**	**1,64**	1,15	**115**	**2,07**	1,49
German girls	**64**	**1,78**	1,34	**76**	**1,83**	1,46	**53**	**2,09**	1,18
German boys	**81**	**1,43**	1,45	**71**	**1,49**	1,37	**83**	**2,21**	1,50

5. 2 Have a closer look

Let´s have a look on the perfomance of each country.

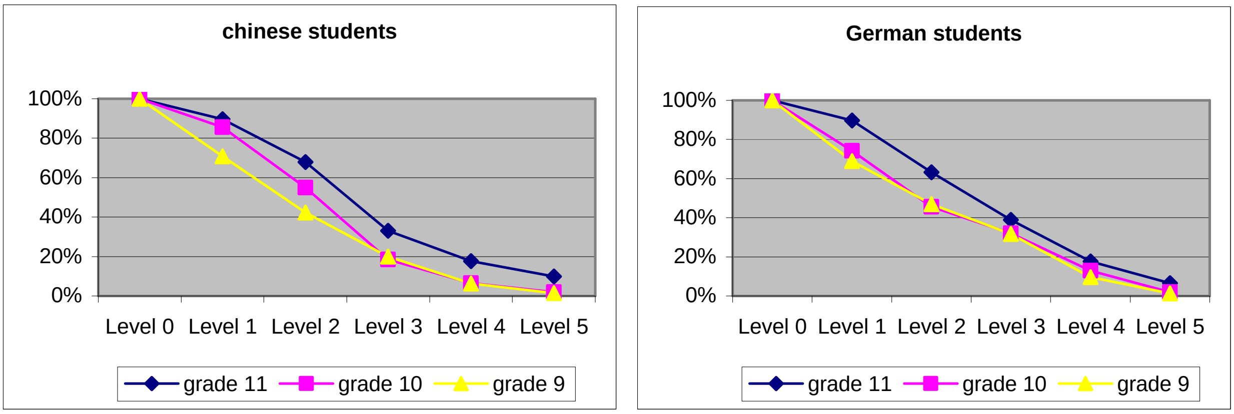

Fig. 5. :Percentage of chinese students who reach level x Percentage of german students who reach level x

These two diagrams in figure 5 look nearly similar. This seems also a signal that between the chinese and german students performance is not a high difference. For example we see that the 11th grade students (♦ - line) have a higher perfomance in each country. To proofing this we made also an unpaired t-test.

Table 2

		df	t	Two tail P-value	
Grade 9	German- chinese	349	.441	.223	n.s.
Grade 10	German- chinese	394	.058	.954	n.s.
Grade 11	German- chinese	355	.013	.899	n.s.
all	German- chinese	1102	.445	.657	n.s.

So lets have a closer look to the male and female students of each country.

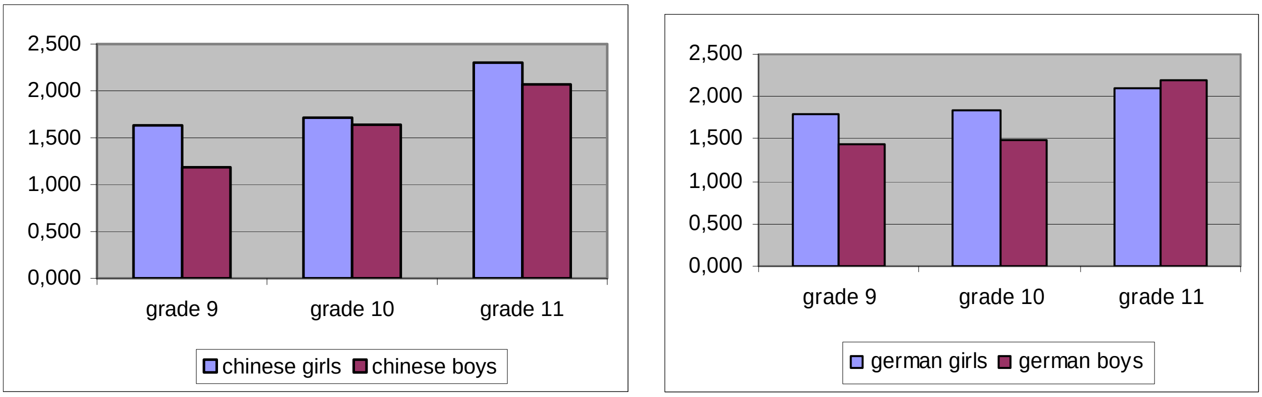

Fig. 6: The average level the chinese students reach The average level the german students reach

Also these two diagrams in figure 6 look nearly the same. We can see that in the 9th grade and the 10 th grade the girls perform better in each country (of course not significantly) but in the 11th grade the male german student perform better than the female.
In the diagrams we see in a good way that every year the students perform better and better. Before we have a special look on the development of the performance in the different grades we want to have a deeper look to the different levels.

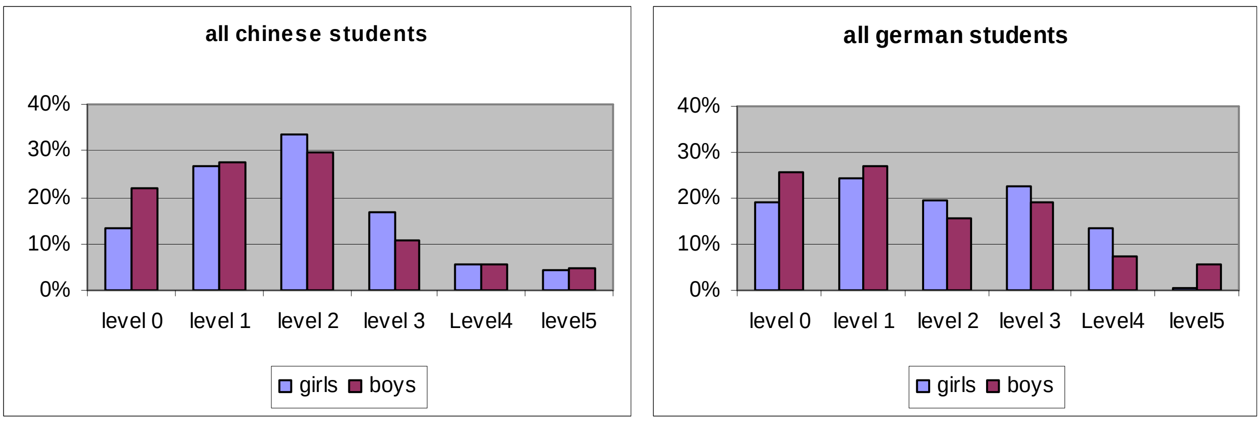

Fig. 7: Percentage of students who reach level x

First we see, that the german girls do not reach the level 5. But in China there are nearly no differences between the boys and girls. We see very clearly, that for the chinese students the level 3 is a barrier. Only 42 % of the students who reach Level 2 will reach level 3 or more. In germany more than 66% of the students who reached level 2 will reach level 3 or more. For the german students level 4 is the barrier. Remember level 3 means, that the student can calculate in the mathematical model found by himself, Level 4 means in addition that he can reasoning his calculation and find a result. This means for the shanghai students that they cannot orient themself in the mathematical model they found by themselves. The german students have problems to interpret their mathematical results and give them a meaning in the real world.

Table 3

		df	t	Two tail P-value	
Chinese Girls	11th vs.10th	226	3.64	.00003	v.s.
	11th vs.9th	207	3.75	.00002	v.s.
	10th vs. 9th	223	0.51	.609	n.s.
Chinese boys	11th vs.10th	240	2.50	.013	s.s.
	11th vs.9th	216	4.83	$2.5x10^{-6}$	v.s.
	10th vs. 9th	228	2.90	.004	v.s.
German girls	11th vs.10th	127	1.13	.262	n.s.
	11th vs.9th	115	1.33	.186	n.s.
	10th vs. 9th	138	.200	.842	n.s.
German boys	11th vs.10th	152	3.05	.003	v.s.
	11th vs.9th	162	3.33	.001	v.s.
	10th vs. 9th	150	0.26	.792	n.s.

Of course the chinese boys perform in the 9th and 10th grade very poor but they perform every year better and better. The differences between all grades are statistically significant and we have an effectsize from about 0.38. For the german boys and chinese girls we found that there is no significant difference between the 9th grade and the 10th grade but a very statistically significant diffebnrence between the 11th grade and the 10th grade($p<0,01$) with an effect size from nearly 0.5. Very interessting is also the situation of the german girls. Although they perform better and better every year the difference is not statistically significant.

Over all we see, that in grade 11 the student´s performance make a great leap forward. Although the mathematical knowledge to solve the pineapple tasks (e.g. Theorem of Pythagoras) was taught in grade 9 they cannot use this knowledge.

6. Conclusions

To use modelling tasks for a comparative study is a very fruitfully thing. And the results are really interesting. For example, we do not yet have an idea while the results of the chinese and german students are nearly the same. You must know that they got an completly different math education. The results show us also that chinese students have difficulties to work clearly in the mathematic world, although chinese math education traditional would be regarded that is good at training mathematics knowledge and skill. It’s also interesting why there are gender differences during working with mathematical modelling tasks. To investigate these could be the next project.

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