

# **MATHEMATICAL MODELLING IN A EUROPEAN CONTEXT – A EUROPEAN NETWORK PROJECT**

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The study reported in this presentation is part of the research in the Comenius-Network-Project “Developing Quality in Mathematics Education II”. One main focus of this network-project is to develop learning materials for maths education containing realistic and perhaps “European” contents. The solving of these tasks shall, aside from encouraging other competencies, also further pupils’ modelling activities, so that the main focus of the research group is to analyse modelling processes in maths education in different European countries. The first step is to develop a checklist which helps teachers to identify good modelling tasks. This presentation reports about a preliminary individual study, which shall test this checklist for teachers.

## Introduction

“Developing Quality in Mathematics Education II” (DQME II) is a Comenius-Network, funded by the EU (LifeLongLearning Programme). It is a continuation and expansion of the associated project “Developing Quality in Mathematics Education”. In each of the no longer four but now eleven participating countries, there are groups consisting of members of universities, teacher education institutions and schools. This is a special feature of this project and promotes a strong connection between theory and practice, as well as between the research on and development of mathematics education. One main area of the work in the project is the development and evaluation of learning material by the research group and the implementation of it in the classroom by the teachers.

For the evaluation of the learning material the research group decided to focus on mathematical modelling processes. One main focus is to develop a checklist which shall help teachers to identify “good” modelling tasks. This resulted from the request of the participating teachers, to find out how they can identify a good modelling task. This will be detailed in this paper.

Integrated in this description will be the results of testing the checklist at a teacher training conference. Those teachers who tested it think that such a checklist may be helpful, especially for teachers at the very beginning of their teaching careers.

## Identifying a “good” modelling task? – Theoretical Basis

The theoretical basis of the work will be mathematical modelling processes. The three objectives the research group will deal with are the following:

- 1) Clarifying learning objectives, which are necessary for the development of mathematical modelling competence,
- 2) Identifying which quality criteria need to be fulfilled for learning materials and teaching methods that further mathematical modelling competence,
- 3) Developing and running a pilot study for cross-cultural comparison of exemplary learning materials.

The first two objectives of the research group will lead to a checklist, which shall help teachers to identify good modelling tasks and lessons for classroom practice. The third objective includes the studies which will be done during the project.

The basis of our first discussion during the first meeting was the complex model of the mathematical modelling process by Blømhoj & Jensen (2006)

The model shows one ideal way of how to work on a real life situation mathematically. It should be a motivating situation that leads to a “Domain of inquiry”. This has to be systemised, idealised and simplified so that a mathematical view of this “System” is possible. This can now be translated into mathematical representations so that calculations can be done (Mathematical system).

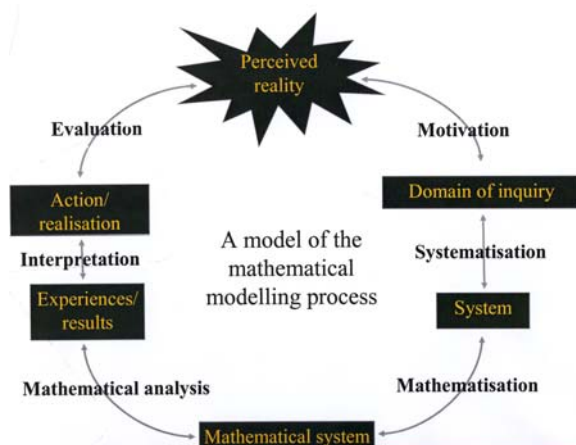


Figure 1: A model of the mathematical modelling process

Analysing the mathematical results with mathematical methods leads to “Results”, which can be interpreted according to the “Domain of inquiry”. This “Realisation” or insight can now be evaluated and validated in accordance with the perceived reality, which was the starting point of the whole process. If the results are satisfying, the process is finished; if not, you have to start over again. According to the results of Borromeo Ferri (2006), students do not follow this modelling process in a linear way, but you can find all stages in a complete and finished modelling process. Therefore, by analysing videos of pupils doing maths, we can observe the stages of the modelling process, but not necessarily in the order the modelling circle shows.

There are two different purposes for using modelling tasks in maths lessons. The first purpose is to impart knowledge and competence about mathematical modelling. The second purpose is to impart knowledge about mathematical contents. These two purposes can, of course, not be divided,

but the teacher can focus on one. Both purposes should have the background of integrating mathematics with reality, as well as getting the pupils involved in and motivated to deal with the real life situation. Especially this last point is important: at a first glance, to motivate the pupils to deal with the real life situation mathematically and, at a second glance, to make them work on these questions independently. This follows the idea of the constructive learning theory: “Learning is an active, autonomous construction of knowledge” (cp. Leuders, 2001). Leuders concludes different criteria for a constructive learning ambiance from this idea.

- The pupils have the opportunity to work on their own and to think independently from teachers and other pupils.
- The pupils have the opportunity to use their previous knowledge and experiences and to connect these with the new knowledge. (prior knowledge)
- There are possibilities for the pupils to interact, to argue and to agree on one solution. (negotiation)
- There are opportunities for pupils to realise that learning is a process, which makes it possible to solve problems. (student centeredness)

(My translation based on Leuders 2001)

These four points fit the modelling process very well. Being involved in a real life subject motivates you to work and think independently. Further, the pupils have to use their prior knowledge about mathematical and real life contents. In order to convince others of their mathematical model and their solutions, it is essential to let the pupils argue. After that they can reflect on the whole process of solving a modelling problem as a learning process.

To make the ideal model of a mathematical modelling process a bit clearer for teachers, it was simplified into four categories:

- Motivation,
- Systematisation and Mathematisation,
- Doing the mathematics and
- Interpretation and Validation.

These resulting topics were then filled with criteria (descriptors), which describe what the learning objectives mean in detail. I added a third column, which describes possible activities of pupils according to the descriptors. The first two columns of the following table contain the first proposal for a checklist.

Learning objectives	Descriptors	Possible pupils' activities relating to the descriptors
Motivation	<ul style="list-style-type: none"> <li>• Engagement (personal and societal)</li> </ul>	<ul style="list-style-type: none"> <li>• They develop their own questions</li> <li>• They can obviously identify with the task/ the real life content of the task</li> </ul>

	<ul style="list-style-type: none"> <li>Teaching purpose</li> </ul>	<ul style="list-style-type: none"> <li>Not really identifiable by pupils' activities.</li> <li>Control by the teacher at the end of the lesson: Did the pupils learn mathematical contents? And/or did they improve their mathematical modelling competence?!</li> </ul>
	<ul style="list-style-type: none"> <li>Authenticity</li> </ul>	<ul style="list-style-type: none"> <li>The given situation is really a real life situation</li> </ul>
	<ul style="list-style-type: none"> <li>Linking existing math. knowledge</li> </ul>	<ul style="list-style-type: none"> <li>They use, e.g. mathematical formulas, without looking them up in the maths book</li> </ul>
	<ul style="list-style-type: none"> <li>Challenging</li> </ul>	<ul style="list-style-type: none"> <li>They do not find a solution in five minutes but go on trying to find a solution</li> </ul>
Systematisation & Mathematisation	<ul style="list-style-type: none"> <li>Is data needed?</li> </ul>	<ul style="list-style-type: none"> <li>Pupils estimate or look in a book or the internet for data</li> </ul>
	<ul style="list-style-type: none"> <li>Abstraction</li> </ul>	<ul style="list-style-type: none"> <li>Pupils create a mathematical model for the real life situation</li> </ul>
	<ul style="list-style-type: none"> <li>Assigning variables</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Making assumptions</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Simplifying</li> </ul>	<ul style="list-style-type: none"> <li>They simplify the real life situation to create a mathematical model</li> <li>They simplify their mathematical model to make the handling easier</li> </ul>
	<ul style="list-style-type: none"> <li>Representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>They use mathematical symbols etc. to describe the real life situation</li> </ul>
Doing the mathematics	<ul style="list-style-type: none"> <li>Formalizing and analyzing the math problem</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Using data</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Approximation and estimation</li> </ul>	<ul style="list-style-type: none"> <li>They have to approximate and estimate data for their calculation</li> </ul>
	<ul style="list-style-type: none"> <li>Use of Information and Communication Technology (ICT)</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Use known algorithms</li> </ul>	<ul style="list-style-type: none"> <li>They use, e.g. mathematical formulas, without looking them up in the maths book</li> </ul>
	<ul style="list-style-type: none"> <li>Mathematical common sense</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Proof (validation of the math used)</li> </ul>	<ul style="list-style-type: none"> <li>Self-explanatory</li> </ul>
	<ul style="list-style-type: none"> <li>Use of math. representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>They use mathematical symbols etc. to describe the real life situation</li> </ul>
Interpretation &	<ul style="list-style-type: none"> <li>Validation of the</li> </ul>	<ul style="list-style-type: none"> <li>They question if their solution is</li> </ul>

Validation	solution mathematically	possible <ul style="list-style-type: none"> <li>• They recalculate to make sure that the solution is correct</li> </ul>
	<ul style="list-style-type: none"> <li>• Validation of the solution in the 'real world'</li> </ul>	<ul style="list-style-type: none"> <li>• They question, if their mathematical solution fits to the real world</li> <li>• They search for other possibilities to otherwise depict the situation in the real world</li> </ul>
	<ul style="list-style-type: none"> <li>• Are the results good enough?</li> </ul>	<ul style="list-style-type: none"> <li>• Pupil question: Are the results good enough in relation to the real life situation?</li> </ul>
	<ul style="list-style-type: none"> <li>• Or is another cycle needed?</li> </ul>	<ul style="list-style-type: none"> <li>• Do they modify their first mathematical model?</li> </ul>

*Table 1: Results of the first meeting of the research group*

The questions to be answered are:

- Are the criteria manageable for the teacher in terms of formulation and expenditure of time?
- Is it possible to identify a “good” modelling task with these criteria?

The approach for a first answer to these questions was as follows: The task the teacher wanted to use was analysed by the teacher with the help of the developed descriptors before giving a lesson to find out if the task is (in his opinion) a modelling task. Then a lesson with this task was held by the same teacher and analysed by me according to the whole modelling circle.

The result is a comparison of what the teacher decided about the task according to the developed criteria and with what the pupils really did according to the model of the mathematical modelling process. This approach was developed after what Wittmann (1995) calls “design science”. According to this theoretical part a first definition of a “good” modelling task in this paper could be: If it is possible to identify nearly all modelling activities described in the modelling circle (according to the descriptors and possible pupil activities) in the pupils’ work with one special task, this task is a “good” modelling task.

### Identifying a “good” modelling task? – Practical realisation

#### *Modelling Task- Swimming Pool*

The original version of the used task was as follows:

Despite the rather cool weather during winter, small outdoor swimming pools are popular among private house owners in Sweden. Imagine a swimming pool that is circular with a radius of 2.75 meters and a depth of 1.18 meters. The distance between the water surface and the pool edge is 0.06 meters. Every spring the pool is filled through two water pipes, each of them delivering 20 liters of water per minute. The water cost 2 Euro per cubic meter.

Questions

- How much water is here in the swimming pool? Answer in the unit cubic meters.
- How much does it cost to fill the swimming pool?
- How long does it take to fill the swimming pool?
- How many humans should be in the swimming pool at the same time in order for the water to pour over the edge? Find out the average volume for an average person yourself.

(From Matte Direkt (Mathematics Directly) Grade 9, 2003, p. 53; translated by Thomas Lingefjärd)

In the study, this task was used in class 8 of a German Gymnasium. They had dealt with calculating volumes during the previous lessons, so the tasks fit to the current content. The teacher decided to delete the questions, add a picture (Figure 1) and led the pupils find their own questions corresponding to the real life situation he gave them. His tasks for the pupils were the following:

- Think out at least 2 meaningful mathematical questions to the text and answer them using a calculation!
- Reach an agreement in your group, which one of your questions should be written on the blackboard!
- Hence solve the questions of the other groups!



Figure 2: [www.badepool.eu](http://www.badepool.eu)

*Using the checklist*

Upon first looking at the task, including the mathematical questions, the teacher decided that this is a good modelling task. Then he used the checklist to evaluate this first thought. For this we translated the English version into German and added a scale: 1 for not existent and 4 for existent. I have chosen the scale from one to four to make it clearer to the teacher that a “good” modelling task is if more items are answered with 4. After testing the checklist this one time, we realised that this is not a good idea, because it is not clear what 2 or 3 means. Some of the teachers at the mentioned teacher training conference think that this scale might be helpful to identify single modelling competences. But on the whole, it is perhaps better to add a scale from 1 to 2 so that a descriptor can be found or not; or to leave this scale out and let the teacher answer questions.

You find the answers of the teacher in the following table.

Learning objectives	Descriptors	Answers of the teacher
Motivation	<ul style="list-style-type: none"> <li>• Engagement (personal)</li> <li>• Engagement (societal)</li> <li>• Teaching purpose</li> </ul>	<ul style="list-style-type: none"> <li>• 2</li> <li>• 2</li> <li>• -</li> </ul>

	<ul style="list-style-type: none"> <li>• Authenticity</li> <li>• Linking existing math. knowledge</li> <li>• Challenging</li> </ul>	<ul style="list-style-type: none"> <li>• 4</li> <li>• 4</li> <li>• 3</li> </ul>
Systematisation & Mathematisation	<ul style="list-style-type: none"> <li>• Is data needed?</li> <li>• Abstraction</li> <li>• Assigning variables</li> <li>• Making assumptions</li> <li>• Simplifying</li> <li>• Representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>• 3</li> <li>• 1</li> <li>• 1</li> <li>• 3</li> <li>• -</li> <li>• -</li> </ul>
Doing the mathematics	<ul style="list-style-type: none"> <li>• Formalizing and analyzing the math problem</li> <li>• Using data</li> <li>• Approximation and estimation</li> <li>• Use of Information and Communications Technology (ICT)</li> <li>• Use known algorithms</li> <li>• Mathematical common sense</li> <li>• Proof (validation of the math used)</li> <li>• Use of math. representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>• 1</li> <li>• 4</li> <li>• 4</li> <li>• 1</li> <li>• 1</li> <li>• 4</li> <li>• 1</li> <li>• -</li> </ul>
Interpretation & Validation	<ul style="list-style-type: none"> <li>• Validation of the solution mathematically</li> <li>• Validation of the solution in the 'real world'</li> <li>• Are the results good enough?</li> <li>• Or is another cycle needed?</li> </ul>	<ul style="list-style-type: none"> <li>• 2</li> <li>• 4</li> <li>• yes</li> <li>• no</li> </ul>

Table 2: Answers of the teacher

The reason the teacher did not answer all of the questions is that he did not understand all of the questions. On the whole, he answered 17 questions. For 8 of them he decided that the asked descriptor is existent in the task (answer: 3 or 4). For 9 descriptors he decided that they are not existent in the task (answer: 1 or 2). As a result of the answers on the checklist, it is not really clear now, if the task is a “good” modelling task or not. Is it good, since half of the answered questions are positive, or is it not good, since the other half of the answered questions are negative. How many answers have to be positive to decide that the task is a good modelling task? Or: How many descriptors for each learning objective have to be answered positive? Before modifying the checklist, I tested it at a teacher training conference with 37 teachers. I gave them some modelling tasks, which they had to evaluate with the checklist. Some results from the questionnaire the teachers filled out which were important for me, are that they think a checklist to identify modelling tasks is helpful, especially for beginners and that they would prefer a list with criteria describing modelling tasks. So in terms of formulation the checklist has to be modified as well as the evaluation mode for the checklist.

*What did the pupils do with that task?*

In groups, the pupils developed the following questions concerning the above mentioned task<sup>1</sup>:

- *How much does it cost if you fill up the whole pool?*
- *How much water (in cubic metres) can you fill in the pool?*
- *How long does it take to fill up the whole pool?*
- *How much water are you allowed to fill in the pool?*
- *How much canvas cover do you need for the pool, if it is cylindrical?*
- *How much does it cost if you fill 170 litres in the pool?*
- *How long does it take to clean the pool if one pump can pump 175 litres per hour?*
- *How many people have to go into the pool so that it overflows?*

The videotaped group of pupils, consisting of five boys, started with question 2: How much water can you fill in the pool?

To evaluate if the pupils are doing mathematical modelling, we tried to find a relation between the actions and discussions of the pupils and the descriptors in the way the description of the pupils' activities in table 1 show.

The authenticity of this task can be evaluated by having a look at advertisements for such swimming pools. The producers of such pools do not tell the user how much water has to be filled into them (Modelling Circle → Perceived reality). By letting the pupils develop their own questions, they got personally involved in the task. (Descriptors → Engagement). This task is challenging, because especially this group tried to find a “difficult” question, which not only the other pupils, but also they themselves had to answer. (Modelling Circle → Motivation)

They simplified the pool by assuming that the “sides of the pool are straight.” (Modelling Circle → Systemisation; Descriptors → Simplifying) Then they began discussing the formula to calculate the volume of the pool for the highest possible water level: “R to the power of 2 times pi times 1.18-0.06.” (Descriptors → Linking existing mathematical knowledge; Modelling Circle → Motivation) And they also started mathematising because of calculating the volume using the formula. They used their calculator to get a result, which was in cubic metres (Modelling Circle → Mathematical System). The scale unit “cubic metres” was not helpful for them to validate their result, so they calculated how many litres their cubic metres are. Then, they compared it with a cube with a 1m edge length and

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<sup>1</sup> The teacher did not comment on or change the questions before the pupils worked on them.

decided that their solution is possible. (Modelling Circle → Interpretation and Validation; Descriptors → Validation of the solution mathematically and in the “real world”)

Thus we find Motivation, a Domain of Inquiry, Systematisation, a System, Mathematisation, a Mathematical System and Analysis as well as the Validation of the results in the work of the students. According to this analysis the task is a “good modelling task” because all stages of the modelling circle can be found.

### Conclusions/ Outlook

When we now compare what the teacher said before using the task and what the pupils really did with this task, we can answer the questions:

- Are the criteria manageable for the teacher in terms of formulation and expenditure of time?

According to the questionnaires the teachers at the teacher training conference filled out, the checklist is too long and time consuming, therefore, it has to be shortened.

- Is it possible to identify a good modelling task with these criteria?

It is obvious that the first proposal of the checklist is not that helpful for identifying a “good” modelling task, because according to the analysis of what the pupils did the task is a “good” modelling task.

I already discussed the problem of the scale from 1 to 4 in the above text. Another view on the checklist must include the difference of what the teacher said about the task and of what the pupils really did. One reason could be that some descriptors describe common mathematical competencies like the ones marked in red in the following table. They are not explicitly indicators for modelling tasks.

Learning objectives	Descriptors	Answers of the teacher
Motivation	<ul style="list-style-type: none"> <li>• Engagement (personal)</li> <li>• Engagement (societal)</li> <li>• Teaching purpose</li> <li>• Authenticity</li> <li>• Linking existing math. knowledge</li> <li>• Challenging</li> </ul>	<ul style="list-style-type: none"> <li>• 2</li> <li>• 2</li> <li>• -</li> <li>• 4</li> <li>• 4</li> <li>• 3</li> </ul>
Systematisation & Mathematisation	<ul style="list-style-type: none"> <li>• Is data needed?</li> <li>• <i>Abstraction</i></li> <li>• Assigning variables</li> <li>• Making assumptions</li> <li>• Simplifying</li> <li>• Representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>• 3</li> <li>• 1</li> <li>• 1</li> <li>• 3</li> <li>• -</li> <li>• -</li> </ul>
Doing the mathematics	<ul style="list-style-type: none"> <li>• <i>Formalizing and analyzing the math problem</i></li> <li>• Using data</li> </ul>	<ul style="list-style-type: none"> <li>• 1</li> <li>• 4</li> </ul>

	<ul style="list-style-type: none"> <li>• Approximation and estimation</li> <li>• Use of Information and Communications Technology (ICT)</li> <li>• Use known algorithms</li> <li>• Mathematical common sense</li> <li>• Proof (validation of the math used)</li> <li>• Use of math. representation(s)</li> </ul>	<ul style="list-style-type: none"> <li>• 4</li> <li>• 1</li> <li>• 1</li> <li>• 4</li> <li>• 1</li> <li>• -</li> </ul>
Interpretation & Validation	<ul style="list-style-type: none"> <li>• <i>Validation of the solution mathematically</i></li> <li>• Validation of the solution in the 'real world'</li> <li>• Are the results good enough?</li> <li>• Or is another cycle needed?</li> </ul>	<ul style="list-style-type: none"> <li>• 2</li> <li>• 4</li> <li>• yes</li> <li>• no</li> </ul>

Table 3: Analysis of the answers of the teacher

If you work on a mathematical problem not connected to real life, you also use your mathematical competencies like using known algorithms or using ICT. Leaving aside these non-explicit descriptors leads to the relation that 7 are judged positive and 5 negative by the teacher. This is also not convincing that this task is a good modelling task. So we should have a look at the negatively judged descriptors marked in italics in the table. The main questions resulting are: Why does the teacher think that the given task did not include “Abstraction”, “Formalizing and analyzing the mathematical problem” and “Validation of the solution mathematically”? My assumption is that this is connected to the group of pupils you want to present the task to. How good is this group in abstracting or structuring real life situations to make them fit into a mathematical model? Is it then necessary for this group to formulate a mathematical problem? And is a deeper analysis of the solution necessary? This has to be discussed in detail during the oral presentation of this paper.

On the whole some of the descriptors must be formulated more explicitly and an evaluation tool for the usage of the table has to be developed. So the next steps will be modifying and the checklist, testing the modified checklist with the same teacher as well as using the same task and the modified checklist with different teachers.

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