

FORMATTING REAL DATA IN MATHEMATICAL MODELLING PROJECTS¹

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Abstract:

Skovsmose (1994) proposes the thesis of *the formatting power of mathematics*, according to which part of our reality is projected by means of mathematical models. If we intend to develop mathematical modelling projects in our classrooms, within the Critical Mathematics Education perspective, it is important to bring the “formatting power of mathematics” concept to educational practice. In this paper, I present an experience that took place during a Mathematics course proffered to undergraduate geography students at the Federal University of Minas Gerais (UFMG), Brazil. I focus on the presentation of a mathematical modelling project developed by a group of students enrolled in the course. The objective is to analyse students’ handling of the data gathered during the development of the project. The group decided, without much justification, to adopt a periodic function to model the data. However, the data did not seem to fit a periodic function. The students then proceeded to re-group the data in such a way that they seemed to fit a mathematical model represented by a periodic function. I analyse whether or not students’ procedures can be understood as an example of the *formatting power of mathematics*.

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1. MATHEMATICAL MODELLING PROJECTS AND THE FORMATTING POWER OF MATHEMATICS: INTRODUCING THE PROBLEM

Mathematical modelling has stood out among current perspectives in mathematics education. In general terms, it can be understood as the utilization of mathematics to resolve real problems. When applied in the classroom, this approach takes on special forms, depending on the educational context, the professionals involved, and the profile of the students, among other factors.

Bassanezi (2002), for example, understands mathematical modelling – whether as a scientific method or a teaching and learning strategy – as the “art of transforming problems from reality into mathematical problems and resolving them through interpretation of their solutions in the language of the real world” (p. 16). For Barbosa (2001), “modelling is a milieu of learning in which students are invited to question and/or investigate, by means of mathematics, situations with reference in reality” (p. 31).

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In my classes, when I am the teacher and propose the development of mathematical modelling projects, I seek to put into practice an understanding of mathematical modelling as

an approach, by means of mathematics, to a non-mathematical problem based in reality, or to a non-mathematical situation based in reality, chosen by groups of students in such a way that questions of Critical Mathematics Education form the basis for the development of the work (ARAÚJO, 2002, p. 39).

Within this perspective, there are some explicit characteristics of the milieu of learning that I seek to put into effect when I propose the development of mathematical modelling projects, including working in groups, and basing the work on Critical Mathematics Education.

According to Skovsmose (1994), the main concern of Critical Mathematics Education is the development of *mathemacy*, which is an extension to mathematics of the problematizing and liberating conception of education proposed by Freire (1970). A similar concept – *matheracy* – has also been discussed by D’Ambrosio (1999). In *mathemacy*, the objective is not to merely develop the ability to carry out mathematical calculations, but also to promote the critical participation of students/citizens in society, discussing political, economic, and environmental issues in which mathematics serves as a technological support. In this case, critique is directed to mathematics itself, as well as to its use in society, the concern thus extending beyond the teaching and learning of mathematics.

Taking account of mathematical modelling, within the Critical Mathematics Education theoretical perspective, demands special care with respect to the role of mathematics (or the role of mathematical models). According to Skovsmose (1994), “mathematics not only creates ways of describing and handling problems, it also becomes a main source for the reconstruction of reality.” (p. 52). The author defends the thesis that mathematics is used to format reality: *the formatting power of mathematics*. According to this thesis, part of our reality is projected by means of mathematical models. One example of this is the Human Development Index (HDI): based on mathematical models, a number from zero to one is associated with every city, country or any other locale. Based on this index, governments or international institutions, for example, decide how to distribute funds to achieve a given objective. A city with an HDI near 1, for example, because of their relatively high rating, might not be selected to receive funds that could resolve some of their problems. Thus, mathematical models are used to create a “real situation” that did not exist before. Critical Mathematics Education questions this power with which mathematics is imbued.

This discussion has been carried out by Skovsmose (1994) in social terms; that is, he discusses using mathematical models to build part of reality, but does not elaborate on how the concept of the formatting power of mathematics can be applied in educational contexts. However, since I intend to develop mathematical modelling projects in my classrooms within the Critical Mathematics Education perspective, I believe it is important to bring the “formatting power of mathematics” concept to educational practice.

Milanezi (2007) tried to extend the formatting power concept to the educational context in a study of the power of mathematics in decision-making in Military Schools. However, rather than restricting her discussion to the use of mathematical models to project part of reality, she also considered the formatting power of the discipline of mathematics itself to analyse “situations in which mathematics, in some way, influences the school reality and the relationships underlying it.” (p. 42).

Skovsmose (1994) himself acknowledges that this concept, as well as others presented in his work, “are not immediately operational in relation to particular educational situations.” (p. 74).

In this paper, I intend to discuss the formatting power of mathematics in an educational context: I present an experience that took place during a mathematics course proffered to undergraduate geography students at the Federal University of Minas Gerais (UFMG), Brazil. I focus on the presentation of a mathematical modelling project developed by a group of students enrolled in the course. The objective is to analyse students’ handling of the data gathered during the development of the project. I analyse whether or not students’ procedures may be understood as an example of what Skovsmose (1994) calls the *formatting power of mathematics*.

In the next section, I describe the context in which the mathematical modelling project was developed.

2. CONTEXT OF THE STUDY

The experience I describe here took place in the first semester of 2006 in a Mathematics course offered to undergraduate students of the geography program at UFMG. I was assigned by the Mathematics Department to teach the course. The mathematics contents planned for the course included functions, derivatives, and notions of integral. However, in the interest of developing the classes in accordance with the pace of the students, not all of this content was covered in 2006.

The main activities that took place during that semester were lectures, activities with computers, and the development of mathematical modelling projects. These activities were carried out in such a way that they were not entirely separate and disconnected from one another, and I, as the teacher, tried to stimulate students to establish relationships among them. To address the objective of this paper, I will focus on the discussion about the modelling projects.

The development of the modelling project in the mathematics course began with the discussion of a text (ARAÚJO, 2006). In this text, I presented my understanding of mathematical modelling and suggestions for topics that should be considered in the “research proposal” to be written by the groups. At the same time, students were asked to think about themes for their projects and about the formation of groups to develop them.

In the following class, themes and groups were defined through a long process of negotiation. In the first semester of 2006, each group ended up with approximately seven members, and the themes chosen were the following: the transposition of the São Francisco River (two groups formed, one to address physical aspects and the other social aspects); physical impacts of the implantation of hydroelectric dams; socio-cultural aspects of the Linha Verde (Green Line) freeway construction project in Belo Horizonte; Campus 2000: consequences for transportation in the UFMG; climate myths; and solar energy.

Once the themes had been defined, each group elaborated a work plan, which I evaluated and returned to the group. In this evaluation, I encouraged them to describe in detail all the steps to be followed during the development of the project, as well as the definition of the focus of the research. I also sought to raise questions regarding how mathematics would be used in the project.

After the projects had been approved, the groups began to carry them out, holding meetings during and outside of class. They presented partial progress reports each month, and based on

these reports, each group received guidance and suggestions - my own as well as from the entire class - regarding how to proceed. During each of these advisory sessions, I sought to take into account the concerns of Critical Mathematics Education.

At the end of the semester, all the groups made an oral presentation of their project to the class (which were videotaped), and handed in a written version of the project. One project, in particular, attracted my attention because of the group's careful treatment of the mathematical information. This project is considered in greater detail in the section that follows.

3. THE PROJECT "TRANSPOSITION OF THE SÃO FRANCISCO RIVER: PHYSICAL ASPECTS"

The theme of the group's project was "physical aspects of the transposition of the São Francisco River". The group's choice of theme portrays, at the same time, the relation with their field of interest, geography, and their interest in a controversial subject, the transposition of the São Francisco River². The objective of the project was to analyse whether or not the rainfall in a given region along the course of the river would be sufficient to compensate for the amount of water that would be diverted as a result of the transposition.

This small report demonstrates the possibility for using mathematics (**quantity** of rainfall and diverted water) to discuss a problem from geography (quantity of **rainfall**) in a critical manner (questioning the environmental consequences). Thus, a mathematical modelling project was proposed that could be approached from a Critical Mathematics Education perspective.

In order to present the group's procedure for handling the data gathered during their project, in the next sub-section, I will first introduce the methodological approach used in the present study.

3.1. Methodological Aspects

The research was developed through a qualitative approach, which is characterized by: the natural environment as the source of data; being descriptive; greater interest in the process than in the final results; inductive data analysis; and the attribution of vital importance to the meaning given to the facts (BOGDAN & BIKLEN, 1994).

The methodological procedures included observation of some meetings of one of the groups (socio-cultural aspects of the Green Line); observation of every group during the partial reports and the final presentations; and analysis of the written version of the project handed in by the groups. All the observations were videotaped.

During data collection, I had no pre-established categories to be used and/or verified. I did not seek data that confirmed (or refuted) any *a priori* established theory. Indeed, I wanted to understand the facts in the way that they occurred. Lincoln and Guba (1995) introduce, therein, the idea of inductive data analysis. According to the authors,

the investigator typically does *not* work with either a priori theory or variables; these are expected to emerge from the inquiry. Data accumulated

² The São Francisco River is the most important river in the Brazilian Northeast, the driest region in the country. The fertile areas along the river contrast with the rest of the region, dominated by caatinga. For years, there has been talk in Brazil of diverting the waters of the river to other areas of the northeast. However, the river has been suffering from pollution and silting, and it is not known whether it would withstand such a procedure. It is also known that there are political interests involved. In summary, it is a very controversial topic.

in the field thus must be analyzed *inductively* (that is, from specific, raw units of information to subsuming categories of information) in order to define local working hypotheses or questions that can be followed up. (p. 203). (Emphasis in the original).

Taking this into account, during analysis of data derived from the videotapes, I selected *episodes*, which are small “clips” of video of the groups while they were developing activities. I identified episodes that I judged to be appropriate for contributing to understanding the questions that had arisen from the research. From the written work, I selected sections or excerpts that helped to deepen my understanding of the episodes.

In the following sub-section, I present some excerpts and one episode. They show how the group that studied “physical aspects of the transposition of the São Francisco River” proceeded in their treatment of the data they collected for their project.

3.2. Excerpts and one episode: some data

At the end of the written work, the group presented a report of the steps followed to develop the project. After they had defined the objective of their project, they began to seek a mathematical model to represent the quantity of water in the river in the region they had chosen (the region near the cities of Juazeiro, Petrolina, Cabrobó and Penedo). Their concern was made explicit in the following question, taken from the written report³: “*Which mathematical model to use?*”. The group provided the following answer to this question:

Excerpt 1:

“Laura had the idea to use an exponential function. However, the group analysed the idea and observed that it wouldn’t be possible to apply it, since we didn’t have data on how much water the pump would take from the river. To solve this problem, Bernardo and Elton had the idea to use a periodic function with constant [sic] to calculate how much the flow of the river would decrease at some points. Everyone in the group liked the idea and we decided to put it into practice.”

To put this idea into practice, the group searched for data on the Internet regarding the daily rainfall from 2000 to 2002, in the cities chosen. They used Excel software to organize and to manage data.

In their oral presentation of the project to the class, the group provided more details about how they treated data, as can be observed in the following episode:

Episode:

7:32 – 8:20⁴

[Elton summarises Israel’s talk, explaining that the soil in the region is crystalline, which makes it difficult for the water to penetrate the soil and form aquifers. Because of this, the flow of the São Francisco River is determined mainly by the amount of rainfall; that is, it is

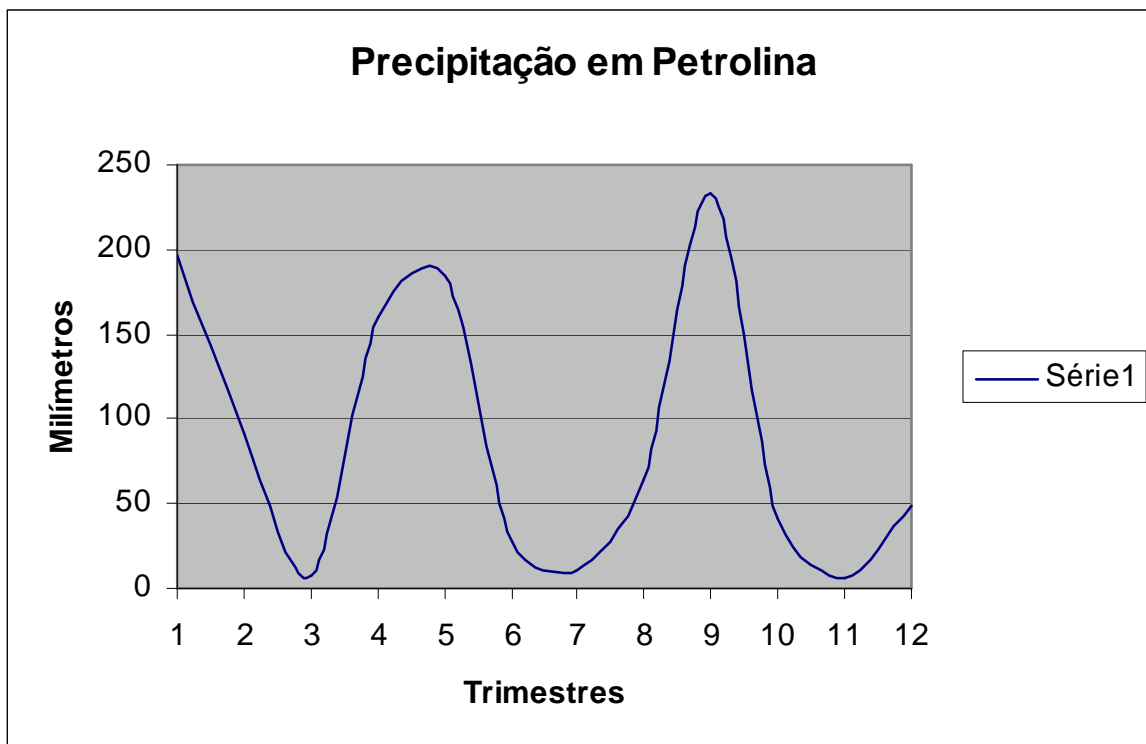
³ The complete reference of the group’s written work is not presented here to preserve their identity.

⁴ These times are relative to the video length of time.

the precipitation that will determine the flow. The group wanted to analyse the quantity of rainfall (which would determine the flow of the river) and predict what would happen with this flow following the transposition.]

8:22 – 9:30

Elton: *Then, we went and took here ... We have ... when we take a rainfall indicator of some city, as in this case, we have the general rainfall indicator. For example, 1000 mm per year, and so on. Then we took the rainfall indicator and divided it trimester by trimester, to know the influence that it had in each trimester. That is, how much it rained during each trimester. Then, doing like this, we took here, put, then ... added up... Here are ... At each four annn ... here [pointing to the graph (Graph 1)] it would be one trimester, two trimesters, three, four, that is, there are four trimesters here: it adds up to one year. Here we have three years represented on each one of these graphs. Then what did we do? Because if we had represented it monthly, then, during one year, we wouldn't have had a function ... Then, this function that seems ... that looks like a [pause] sine function, a periodic function ... If we had put just one year, then it wouldn't have had this aspect. Then, we did, we considered by trimester, so it would have this aspect. [Pointing to the graph].*



Graph 1: amount of rainfall in the region of Petrolina from 2000 to 2002, represented trimester by trimester.

9:31 [Some comments and laughs from the audience.]

9:31 – 9:45

Elton: [laughing] *I mean ... not “so it would have” this aspect! We didn’t force it ... But, doing it this way, it was easier to visualize. That is ... it was possible, but it was a bit strange! This way, it was much easier to observe.*

These procedures are also described on the written version of the work:

Excerpt 2:

“[] these daily rainfall indicators were added and grouped trimester by trimester, adding up to twelve trimesters over these three years, as we can observe on the X axis of the graphs [...]. The Y axis, in turn, represents the absolute value of precipitation for each trimester.”

4. DISCUSSION

In summary, the group reported that, after agreeing on the objective of the research (to analyse whether the quantity of rainfall in a given region along the course of the river would be sufficient to compensate for the amount of water that would be diverted as a result of the transposition), they began to consider what mathematical model to use, and decided, without much justification, to adopt a periodic function. This choice may have resulted from the students’ knowledge regarding the behaviour of rainfall, but it may also have been influenced by the subject discussed in mathematics class, which would exemplify what Araújo and Barbosa (2005) call the inverse strategy in the modelling process. This possibility is reinforced by the fact that the group did not use this part of the study to arrive at their conclusions. In the written work, they just say that “from data about the annual flow of the São Francisco River, it is possible to conclude that, if the transposition were in fact carried out, according to data available from the Integration Ministry, it could cause environmental impacts, positive as well as negative.”

However, data that the group had gathered, related to the rainfall in a given region along the course of the river, did not seem to fit a periodic function. Thus, the solution that the group found was to re-group data in such a way that they seemed to fit a mathematical model represented by a periodic function.

Can we conclude that the group’s procedure for handling data gathered during the mathematical modelling project is an example of the thesis of the formatting power of mathematics? Taking into account that no part of reality was projected or reconstructed by the group when they treated the data as they did, we can readily conclude that the group’s procedure is not an example of this thesis.

However, a mathematical model was used to format the data gathered by the group. That is, the periodic model, chosen beforehand, formatted a set of information which, in principle, did not have a familiar form, if compared with the models that we had been studying in mathematics classes. Elton’s disconcertion as he unveiled the group’s procedure to his classmates, explaining that the group had not forced the results, shows that he had perceived some problem in their treatment of the data. I, as the teacher, could have taken advantage of this moment to exercise a *social imagination* (NEGT, 1964⁵ *apud* SKOVSMOSE, 1994):

⁵ NEGT, O. (1964) *Soziologische Phantasie und Exemplarisches Lernen*. Europäische Verlagsanstalt, Frankfurt am Main.

what would happen, in the real transposition of the São Francisco River, if data about precipitation in the region chosen by the group were really treated in the same way by the geographers who work on the project?

The *social imagination* is related to the concept of *exemplarity* presented by Skovsmose (1994). The author uses this concept with the intention of applying sociological discussions of Critical Mathematics Education to educational contexts. Based on Negt's ideas, Skovsmose (1994) presents three theses to explain exemplarity:

- 1) A small phenomenon can be a reflection of a larger complexity.
- 2) "It is possible to understand a social complexity by concentrating on a particular event." (p. 77).
- 3) The objective of education is not to transfer information to the students, but to stimulate them to try to change their own situation.

I believe that the discussion about the group's procedure while treating data gathered during the mathematical modelling project could be broadened to the social phenomenon "Transposition of the São Francisco River". In other words, the exemplarity of the project developed by the group could provoke a discussion among all the students regarding the formatting power of mathematics in society. Although this did not occur, I think that the three theses of exemplarity, in this situation, could potentially be:

- 1) The procedure of the geography students, using a mathematical model to format rainfall data in a given region, can be a reflection of data handling in larger social situations.
- 2) While discussing and understanding the formatting power of mathematics in the project developed by the group, the entire class could come to understand how this concept could be applied in more complex situations.
- 3) This discussion could lead students to re-think their responsibility as future geographers.

Therefore, although the treatment given by the group to the modelling project data is not an example of the general concept of the formatting power of mathematics, it represents one possible way of approaching the concept in a specific educational situation.

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