MATHEMATICAL MODELLING: FROM CLASSROOM TO THE REAL WORLD

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Abstract

In this article we approach, in the Linear Programming discipline, the Mathematical Modeling as a methodological option for teaching and learning mathematical contents. The Mathematical Modeling activities, developed with the students from an Information Systems Course, allowed for establishing a relationship between mathematical contents approached in the discipline with some problems related to the students' reality. In this article we have, as an objective, to analyze the possibility of pedagogical contributions when using such association between curricular contents and the application of Mathematical Modeling, with the support of technology, in daily life student situation, especially when such situations are related to their current or future professional activities. As main results we highlight, on one hand, the perceptions by the students about the relevance of the discipline, being it for their intellectual formation or for their professional valorization and for the technology applicability, through the use of specific software, in a mathematical discipline taught within a data processing field Course. And, on the other hand, the collaborative environment built among the students which contributed for their interaction in the group assignments and in the moment for academic and professional experiences exchange.

Keywords: Mathematical Modeling, Linear Programming, Collaboration.

1. Introduction

It is quite usual for students to show difficulties when it comes to learning disciplines like Mathematics, even for those who attended exact sciences courses. Such difficulties, on one hand, may become worse if the students are not able to envision an usage for what they are studying and, usually, they study only for getting ahead on the course. On the other hand the student involvement may occur in a deeper way if the studied contents in the subjects linked to mathematics were directly connected to the subjects of the chosen course. We have noticed the existence of these two situations in the disciplines that we teach.

Most of the students would rather attend mathematic classes which do present some level of connection to reality. For such students this connection would allow for more meaningful and less stressing learning. By approaching the perspectives that guide the concepts of mathematic literacy extrapolating the traditional concept related to the abilities for calculation and problems solution and enlarging the horizons of its meaning, Jablonka (2003) says that bringing to the classroom the mathematic used on working environment is one of the ways for associating the out of school mathematic with the curricular contents and, consequently, of showing the mathematical practical utility. This association, by contributing for the meaningfulness of the teaching activities for the students, on top of reducing what is known as mathematical anxiety towards learning the concepts and handling numbers and algorithms, allows for the relationship between the academic and professional learning, valorizing the diversity of culture (mathematic) present on the working places.

However, many times, relating mathematical techniques used routinely in the work place to curricular mathematic is quite hard because, on top of the rigidity that usually characterizes the courses curriculum, the meaningful concept, in this case, must be relativized in a way that, what may be meaningful for one student may not be for another. Besides, most of the time, the association between mathematic and reality demands more effort and commitment from the students than traditional classes centered on teacher lectures. It also

demands available time from the students for research and for other tasks away from the classroom. This situation is worse for night class, where most of the students do work during day time and do not have available time for out of class activities, required for tasks of such nature.

With the challenge of helping students on the understanding of the Linear Programming discipline, the first author of this article has conducted a pedagogical experience using Mathematical Modeling on her classes for the Information Systems Course students on the night shift of a private school in Campinas, SP, Brazil. These students, usually, work professionally in activities related to the data processing field.

So, based on such experience, we have as the objective, to evaluate the possibilities for the teaching and the learning the mathematical contents in graduation courses, when the Mathematical Modeling is used with the support of technology, based on problems related to the students daily life, mainly when such situations are related to their professional activities, current or future.

Next, on this article, right after some reflections over Mathematical Modeling, we approach the methodology used in the study, we present the built environment and discuss the results reached.

2. Some reflections over Mathematical Modeling in the classroom

The idea of creating models for understanding and studying a wide variety of phenomenon is very old, since the man across time has used real world representations in order to get a solution for the built model. The validation of such models is done through analysis, reflections and discussions over the reached results.

Mathematical Models are representations, in mathematical terms, of the aspects of interest from the problem being studied and can be formulated "[…] using numerical expressions or formulas, diagrams, graphs or geometric representations, algebraic equations, tables etc" (BIEMBENGUT; HEIN, 2000, p. 12). We highlight that one single model, with minor adaptations, can represent many applications. This is very useful in the professional modeling as well as in the classroom modeling, because it allows for the utilization of a single model to solve different situations.

The mathematic and the reality can be connected through modeling. This interactive connection is done by using known mathematical process, with the objective of studying, analyzing, explaining, forecast real daily life situations around us (CAMPOS, 2007).

The Mathematical Modeling, in a direction, on top of being an important applied mathematical tool for solving real problems, also creates the need for data gathering and for simplification of real situations. In this same direction Mathematical Modeling favors the construction of an environment where the students can perform simulations and analogies, considering that the same model can be useful in the representation of many different situations, helping the students in the identification of applications in other areas of knowledge and under different environments.

In a second direction, the Mathematical Modeling identify itself with a pedagogical perspective focused in the citizenship construction and a social/political conscience of the student, who aims at valorizing their individual abilities required for an effective participation in a democratic society and, similarly to the thinking of Skovsmose (1996), emphasizing the critic evaluation of the practices that involve mathematic, taking into consideration the cultural environment to which all the students belong. This identification comprehend the central core of a mathematical literacy turned towards social changes, as proposes Jablonka (2003), aimed at the formation of a critical citizen, with a power of arguing and, as said by Jacobini and Wodewotzki (2006) interested in questioning the social issues relevant to the community.

In a third direction, we see the role of data processing technology as an indispensable actor for working with mathematical modeling, being it as a operational supporting tool or as a instrument that comes for contributing for overcoming many challenges usually found in traditional classroom, such as students lack of interest or lack of required abilities for the working environment. We highlight that we are not alone in valorizing data processing technology role considering that currently, most of the researchers interested in the Mathematical Modeling considers indispensable, in their studies, the presence of such technology.

3. Methodology

Aiming at creating conditions for analyzing the association among curricular contents and application of Mathematical Modeling with the support of technology in the students daily life situations, the first author of this paper conducted, during the second semester of 2007, a pedagogical experiment on the Linear Program discipline, which is part of the Information Systems Course at a Private College in Campinas, Brazil, where the students were encouraged to work on real situations. In this experience, as previously said, we have tried to emphasize the construction of knowledge and to make the students more critical and with a higher arguing skill.

Linear Programming discipline is taught on the third year of Information Systems Course and, at that point, the students usually are already working, not as interns, but rather as regular employees. This way, the time spent with school activities is very low, mainly for mathematical disciplines which are considered as supporting subjects to the ones specific to the students' formation. This situation gets worse as the students are not able to visualize immediate usage for the activities currently being developed at the Companies they are working for. We believe that this is the main reason for the difficulties the students face at the discipline of Linear Programming and such situation generate discomfort for the students as well as for the teacher. Many times the students are dependent only on this discipline to graduate from the Course and, consequently, get better opportunities at the Company. On top of this, the requirement for linear algebra knowledge do established discomfort moments for many students that, by the time they attended such discipline, not only presented difficulties in understanding the concepts, which are in general abstract, but also, some of them, have failed in this discipline.

Lately it has been part of the evaluation process of this discipline, the development of assignments involving practical applications of the subjects studied. At such assignments the students (groups of two) chose problems related to what is being taught at the discipline, search for data, model the problems, that is, try to mathematically represent it, solve the problems using software required that are available, analyze and validate the solution found. It is worth to highlight that most of the doubles did not validate their solutions, as it will be approached in the coming section. Since most of the students work professionally it is quite usual that they gather data from the Companies they are working for and, next step, present the mathematical formulation in the Linear Programming representation. We have found, many times, students not interested in working with real problems from their daily life due to the complexity demanded for data and information gathering, as well as the mathematical representation, they would rather working with the problems available on the text books.

In any situation, gathering real data or using data from text books, it is required from the students to use the software to solve the problem represented as Linear Programming. They may ask for a license from a software supplier for using a specific tool, such as LINGO – Language for Interactive Optimizer or use the resources available at the Microsoft Office Excel.

By the end of the semester we request the students to present to their classmates the final assignment developed. The doubles presented the problem, the Linear Programming

formulation, justifying the variables, the objective function and the restrictions. In the sequence, they presented the solution they arrived at and its interpretation and, at last, some simulations related to the theory taught in classroom. In some cases, simplifications and analogies were presented. It is worthwhile to highlight that some of the assignments envisioned Integer Programming or Non-linear Programming for treating its applications.

The project evaluation represented 20% of the final students grade and, for such evaluation, it was taken into consideration: a) the complexity of the problem chosen; b) the mathematical formulation as well as the justification for the objective function, the restrictions and the variables; c) software simulations for theory checking and sensitivity analysis.

4. The modeling environment built during the second semester of 2007

At the beginning of the semester the students are informed that they will be asked to perform a practical assignment involving real problem, whose solution is obtained through the content studied within the Linear Programming discipline. They are also informed that this assignment will be required by the end of the course and that it must be presented the their peers. It is up to the student to choose the problem and, at this moment the difficulties begin for, in general, the students are not used to the idea of creating their own problems since they are, in general, formulated and proposed by the teacher. Considering that in this phase of the assignment, the students find themselves lost as to which path to take, they start their search using the internet and/or through text books recommended by the teacher. This phase is very interesting because it entices the need for researching through many sources and not just the traditional didactic material provided by the teacher.

In the modeling environment, referred to by this article, the students had the opportunity to know many Linear Programming application examples in real situations, with the proper mathematical formulation. Many of these examples illustrated applications in the industry and, some of them referred to situations experienced by the teacher as a consultant in a Optimization Consulting Company. Such problems were related to optimized forest planning, optimized poultry production integrated planning. During the presentation of such problems it was highlighted that simplifications were required in order to reach a solution through mathematical treatment. We took advantage of this moment to show the students that, many times, the same mathematical tool is used to solve different problems, for instance, the model used to solve the feed formulation problem can be adapted for solving mixing problems for juice, steel, etc.

During the subject for the assignments choosing phase, many students have opted for working on projects related to real problems, most of them directly related to situations originated within their working places. Usually, in the companies, the professional activities related to optimization problems, solved through Linear Programming resources, are found in departments involving production control and planning. So, within the working environment, the requirement for the knowledge of such resources, although superficial, are justified, mainly due to the application of theoretical concepts in the decision making process. Reciprocally, at school, the possibility of practical application constitutes itself in an important motivational factor in the process of teaching and learning. About the relationship between professional mathematic and classroom mathematic D'Ambrósio (2002), properly, reminds us that the real life facts help us on the knowledge acquisition.

However, due to the complexity involving these problems, some students would soon give up such choice, and rather follow the constructions of simpler problems which would require less effort from them. The students that kept their interest in the problems from their working places, in order to present them as Linear Programming, had to do simplifications in the data gathering phase, as well as during the restrictions formulation for the construction of the mathematical model. This was meaningful for them since they could realize the real

requirement to do simplifications on the original conditions that surround the problem of interest in order to obtain a possible and viable solution.

The discussions in the learning environment, build within the classroom, produced, in most cases, simplifications and reformulations and, in some other cases, abandonment of the subject chosen and exchanging for another. Later the students were oriented on using a software for solving the problems. Due to its data loading simplicity, part of the students chose the software named LINGO – Language for Interactive General Optimizer. Due to acquaintance the remaining students chose the Microsoft Spreadsheet Software Excel.

The students, in general, presented some difficulty during the results interpretation provided by the software, as in identifying the optimal solution and the value of the objective function, in the understanding of the meaning of the slack and surplus variables, as well as the dual price. This problem was partially solved when, based on the classroom discussions, the students listed the solutions representations found with the concepts studied during the course. As an example we can mention the pedagogical moment when the students noticed the application of many concepts seen in classroom, as those related to the slack and surplus variables and also the Sensitivity Analysis Theory. The usage of software packages to perform many simulations which contributed to the understanding of the theory studied on top of allowing for software packages comparison.

The difficulties in understanding the concepts related to the Sensitivity Analysis Theory were partially solved as the students, based on the simulations performed with the support of LINGO or Excel, could perceive the practicality of such theory. This was thoroughly discussed during the presentations from the doubles, when the results from the simulations could be compared to those found in the theory.

As mentioned before, by the end of the course the groups (two students) presented in the classroom the results found. This procedure was very significant because it allowed, from one side, each group to present to the class all the phases of the problem directly related to Linear Programming. And, on the other side, that all could visualize many application on different areas. In many occasions the teacher had to intermediate the discussion, relating the students presentations to what had been taught in the classroom. We can mention, as an example, the moment when some of the doubles, in their presentations, approached the necessity of doing simplification in the original problem such that it could be formulated as Linear Programming.

We have noticed in these presentations, as well as reading over the text provided, that few of the doubles went back to the real situation in order to validate the solution found. To be able to reach to a solution was already considered satisfactory in the student's point of view. As it happens in many modeling assignments, reaching to a result is the objective to be met. In those, the discussions, being it related to the reliability and the adaptability of the results obtained or related to the its meaning and its consequences (social, cultural, economical, environmental etc) to the daily live from where these problems were originated are, most of the times, put aside[1]. During the assignments presentations we have tried to approach such aspects.

Among all the assignments accomplished in the environment, we highlight initially what has been developed individually by a student, who had previously failed this discipline, and grabbed this opportunity, chose for the solution in a problem that he was facing at the Company he was working for. This problem was related to the wood boards bi-dimensional cutting process for furniture manufacturing. Using the subjects studied in the classroom and also some additional help from the teacher and with the usage of Microsoft Excel e could

[1] **These questions appropriately are argued in the scope of the Critical Mathematical Education. (SKOVSMOSE, 1996; 2007; JACOBINI; 2007).**

manage to reduce the loss on the wood cutting. The accomplishment of such practical assignment, beyond providing for the applicability of the school learning within his working environment, also contributed favorably for the student to overcome his difficulties with the discipline and, as a consequence, get approved in it. On top of that, the presentation of better solutions in his work place contributed to his professional growing.

Other doubles have also chosen their assignment based on the requirements of the company where one of the participants worked. The assignments from these doubles related to (1) the optimization of the production line on a car plant; (2) the expansion study on a textile company; (3) the optimization on the employee allocation on a call center company and (4) the optimization on the steel roll cutting process.

On top of theses students that have chosen for the development of projects directly related to their working environment, some others created fictitious problems, but related to their professional fields. We included in these cases the projects related to (1) the optimization on the number of internet users in order to reach the advertisement of a product; (2) the optimization on the allocation of projects from a software company and (3) the optimization on the employees hiring for a data processing company.

Other doubles would rather create models similar to the examples presented in classroom, but in a certain way related to their interest such as, for instance, the projects related to (1) the optimization on the manufacturing of a chemical fertilizer; (2) the optimization on the manufacturing of chocolate; (3) the dieting problem; (4) the optimization on the resources used in a farm and (5) the optimization on choosing a car based on its fuel consumption.

5. Results

The assignments developed allowed for minimizing the felling of lack of relevance with respect to this discipline, for they could visualize many applications where Linear Programming can help in the problem solving and in the decision making process for the problems from their work places or in their daily lives. Beyond that, there was an intense collaboration among the students allowing for a better interaction which is highly significant since meetings for exchange of ideas and experiences are very difficult considering they are already working professionally. The debates in this environment were about what the students have found in the text books and about their difficulties in the software results interpretation.

The usage of software packages in the solution of the problems brought about by the students allowed for a greater interaction among them, generated more knowledge e showed them the possibility of interaction between mathematic (through the contents related to Linear Programming), real problems and technology. We have considered meaningful the perception of such interaction since the students, usually, complain exactly about the class taught in the traditional form where the relation between what they learn and their real professional lives, in the data processing field, are not perceivable. We highlight the presence of the technology as an important tool for collaboration in the mathematical classroom, for it allows the treatment of real situations that involve different levels of algebraic complexity, mainly for the Information Systems Course students, where the usage of technology is their natural environment. Text books, such as Winston, Albright, Broadie (1997), Hillier e Lieberman (2006), Colin (2007), in their approach to Linear Programming associate examples of real applications and the usage of computational resources.

The mathematical modeling presents itself as a pedagogical strategy that complements this association between real application and the utilization of computational resources, as it provides for the construction of favorable environments for the students to choose their problems of interest, gather their own data, and participate on the investigations, analysis, discussions and reflections.

Blum (1995) shows five arguments favoring the inclusion of the usage of such strategy in the school environment: motivation, learning facilitation, preparation for the usage of mathematic in different areas, development of general abilities for exploration and understanding of the mathematic role in the society. In this same line, Zbiek and Conner (2006), highlight some objectives to be reached when working with the Mathematical Modeling in the classroom such as, prepare the students for working professionally with the modeling, motivate the students by showing them the applicability of the mathematical ideas in the real world and provide opportunities for the students to integrate it with other areas of the knowledge.

The student's statements, some of them shown below, valorize the professional opportunities provided by the work performed. They also show the importance, for teaching and for learning, of the learning environments construction based on this association between mathematical modeling centered on real problems related to the working world and the utilization of computational resources. Such statements also confirm that the curricular content involvement with a daily life mathematic (through mathematical modeling) helps not only to show the practical utility of mathematics and the relevance of its learning but, likewise, to reduce stress feelings and fear towards it.

"I have found very interesting the software Excel application for solving the problem, I have learned a little more about this software, I could not even think that it had such a tool".

"This subject was very important to me, for solving problems from my daily life and I am also foreseeing the application of it in many situations at the Company I work for".

"I have been applying Linear Programming for getting a better solution for the problem at the Company I work for".

"Among all of the Course subjects, this was the one that was worthwhile".

"I was very satisfied when I could solve my problem, I was afraid of this subject because everybody say it was very hard to get approved".

"Currently I make money out of Linear Programming".

6. Final Considerations

We have evaluated, from this experience, that Mathematical Modeling by providing the students with opportunities for identifying and studying problem-situation from their professional realities or interests and by creating opportunities for the construction of a more critical and reflexive knowledge, presents itself as an adequate pedagogical way for teaching and learning contents related to Linear Programming. We have also evaluated that experiences with Mathematical Modeling favor collaboration, between the participants of the groups (or doubles in this case) and among all the students when questions related to the software utilization arise or in the interpretation of the results obtained.

We also highlight the importance, in the created environment, of the interaction among the students and with the teacher, through e-mails exchange or through classroom talks, since this, by providing a closer proximity among all players contributes, on one hand, to the expectations and questions from the students to be readily debated and clarified. And, on the other hand, to facilitate the assignment execution and experiences exchange. As the student sees the teacher and his peers as collaborators, he (she) can see the classroom and the working place closing in and, as a consequence, he (she) associates the knowledge resulting from the pedagogical process with his (her) professional requirements. In summary, the student can see a practical meaning to what he (she) learns in school.

Finally we highlight the advances related to the knowledge of available resources on the software used (Excel and LINGO). Such advances are equally mentioned by researchers interested in Mathematical Modeling as a pedagogical strategy. At the work referred here, the students presented initially, some difficulties towards the software usage, since not specific activity had been previously taken place. As time went by, com classes being taught and

mainly with the collaboration among them, with some helping others, the difficulties were overcome. It has also contributed to the enhancement on the software usage the results from many researches over the internet, where user manuals were found by the students and made available to all participants. Borba, Malheiros and Zulatto (2007) highlighted the collaboration as part of the interactive process where teacher and students act as partners in the learning process.

We finalize this article with two considerations related to the Mathematical Modeling, environment built at the Linear Programming discipline. On the first we highlight, on one side, the perceptions by the students of the relevance of the discipline, being it as much for their intellectual formation as for their professional valorization, and the technology applicability, through the usage of specific software packages, in a mathematic discipline taught within a Information System field course. We believe that the course has awaken in the students the interest for learning and, despite the short contact with theory e with the applications presented in the classroom, has also collaborated in such a way that they can continue by themselves in the application and solution of other problems, from the simple daily life problem to more complex ones at their working places. And, on the other hand, the environment built among the students has contributed for the interaction in the assignments in a group and in the moments of professional and academic experiences exchange.

On the second, we highlight that the option for practices that differentiate themselves from the common way in a classroom, characterized mainly by predictable actions and done with the single intention of transmitting information intrinsic to the programmed content, requires a lot of effort and dedication from the teacher. So, jobs of such nature are incompatible with a docent agenda full of classes or many activities.

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