THE METHOD OF PROBLEM SOLVING BASED ON THE JAPANESE AND POLYA´S MODELS .
A CLASSROOM EXPERIENCE IN CHILEAN SCHOOLS. [1]

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".. Only major discoveries allow us to solve major problems ". There is, a little bit of discovery in the solution of any problem, and if any problem is solved and gets to excite our curiosity," this kind of experience, at a certain age, may determine the pleasure of intellectual work and leave, both in spirit and in the character, a trace that will last for a lifetime .. "Polya (1945).

ABSTRACT

This paper is framed within the Project of Improvement of the Teaching of Mathematics in Chile. According to the agreement with the Japanese International Cooperation Agency (JICA), and within the program of International Cooperation between the Chilean and Japanese Governments. A research applied on students of Primary Education at the County of Pelarco is presented due to the pretty low results in the national and international evaluations Students are exposed to problem solving. In order to develop this project, a unit using the Japanese Lesson Planning, Polya Model was designed to be implemented inside the classroom. The methodology was of the quantitatively and qualitatively type, through an interpretative analysis of the content. Categories were established according to the Polya 4 stages categorization model, thus allowing to analyze the outcomes of the students. As conclusions, it is worth mentioning to say that students showed difficulties and obstacles at the beginning, due mainly to the verbal (spoken) or written comprehension of the problems, in the mathematical descriptions and in the mathematical communication. However, this fact is reverted with the methodology used, showing significant progresses at the end of the experience.

INTRODUCTION

As a result of the poor results obtained by students in the national test (SIMCE) and internationals (PISA y TIMSS), which depict that Chilean students have been much lower than the international media, and not showing significant differences in the last years. There is a permanent concern at all educational levels in order to improve the quality of the mathematical learning in Chile. To understand this problem thoroughly, it is necessary to analyze the mathematical formation in the last decades, since it has been aimed at exercising preferently and at the management of the mathematical operation and to the basic algorithmic outside contexts. This fact has not allowed the students to understand its usefulness in the present world. This problem has widely increased at schools that deal with medium to low socio economic strata, and even more, at the rural and marginal areas (Aravena & Caamaño, 2007).

Problem solving, with some exceptions, is far away from the classrooms of our country. It can be proved that mathematics is neither related to reality nor to the other areas of knowledge. Furthermore, there exists a division inside mathematics itself (Aravena & Caamaño, 2007).

There is a great amount of research that depicts the importance of problem solving in the development of the thinking and uppermost skills. Besides, different skills for the problem solving are shown as Heuristic methods that easy work for students.

[1] This present article is based on a wider research which has been directed by Dr. María Aravena Díaz in her Seminary to obtain a Postgraduate on Mathematical Education, which was developed by the following teachers of Basic Education: Carrión A. Zunilda; Garrido E. Fernando; Miño V. Elena; Muñoz B. Myriam; Fuentealba C. Eduardo; Morales G. Sandra.

Starting with Polya works (1957), which have been taken as a referent by most of the researches, we can find different proposals that evidence the importance that problem solving has in the formation of the present mathematics (Schoenfeld, 1982; Mayer, 1986; Polya, 1985; Schoenfeld, 1987; Schoenfeld, 1988; Santos, 1992; Shumizu, et al., 2007).

Problem under Research

Based on the above mentioned problematic, and on the researches made at an international level on problem solving.

It was put forward a work proposal in order to introduce students of Primary Education in the problem solving method. For that purpose**,** the Lesson Study Model was introduced in the design of the lesson plan that aims at essential elements in the development of the mathematical reasoning, with the caution of adapting it to the socio-cultural Chilean reality, and to the 4 Polya stages. Which were introduced in the development of the class. Based on the exposed above, we put forward the following problem:

¿Which is the potential of work of students in Primary Education on solving problems with verbal and numeric enunciates using the 4 Polya stages?

Objective of the Research.

To validate a methodological proposal based on the Japanese model of Lesson Study, with the back up of George Polya problem solving method, proposing an improvement in the comprehension and problem solving with verbal or written and mathematical enunciates in mathematical and daily situations in students of Primary Education.

General Hypothesis:

The present results show that students of Chilean Primary Schools show difficulties and obstacles, which may be regulated using a detailed planning according to the **Lesson Study** Model and to the Polya model, as a methodological classroom strategy.

Sub-hypothesis or particular hypothesis.

H_1: Students of the Chilean Primary Education evidence difficulties and obstacles when facing problems with verbal enunciate*s*. For example:

Reading Comprehension, organization and interpretation of data, mathematization (algorithms, properties, formulate regularities) and mathematical communication.

H_2: Applying the Polya Method, based on problem solving, students improve their learning levels significantly, being able to develop comprehension skills, mathematical reasoning and mathematical communication.

II. BACKGROUND.

In different epochs and conditions, it has been proposed that doing mathematics is problem solving mostly. For ages, scientists have tried to understand and teach the necessary skills for that purpose. However, such history can be divided into two big moments: before Polya´s works (1945) and after these ones. In the first stage, Socrates first progresses outstand, especially in the construction of a square of equal area to the double of a given square, showing all the strategies and techniques of its solving process. Another support is observed in the work of Descartes where you can find suggestions for those who might like to solve problems easily. Besides, we can find significant help in Euler´s works, which are related to the techniques that he used and to the heuristic education he employed with his disciples.

The present stage is started with the works by Polya (1945), who impulses greatly to this process with his work *"How to solve it*", and lately, with "Mathematical and Plausible Reasoning" (1954) y "Mathematical Discovery" (1965). Such works are a precious referent to the different groups that have worked and given support to this line.
Since the 70´s, as a result of the crisis produced by ¨modern mathematics¨, problem solving becomes the central axes of mathematics. (Schoenfeld,1985), with the outstanding creation of Curricular Standards by the National Council of Teachers of Mathematics in the United States, that was assumed by several countries, becoming the fundamental objective in the Teaching of Mathematics and the axes of the curriculum in the 80´s. According to Rico (1988) and Brown (1983), this became the most important innovation, and also became an autonomous field of systematic research. Another important stimulus was done by the Cockcroft Report (1985), a comprehensive analysis of 11 mathematical themes, in England and Wales treats the main areas of difficulty, this has been referent for the research in this field. The 90´s stated a breakthrough in a systematic work with numerous publications, becoming an autonomous area within the Mathematical Education.
Dealing with the teaching of mathematics by problem solving as a subject of study, is becoming an outstanding issue. Besides, the different strategies of solving known as heuristic method, easy the work of students, and have been an international referent. But, in spite of all the proposals that aim at problem solving, the problematic has not still been solved in many countries and conditions, where they do not work with problems, and the Mathematics that is taught is completely far from context. As a result, students do not develop major skills they need, according to our present society.

Problem Solving Model. Japanese Lesson Study.

The results obtained by Japan in the International Tests of Mathematics, where it has got the first places, have caught the attention in our country. In an analysis of the text "Japanese Lesson Study in mathematics at a Glance" (Shizumi, et.al, 2005), the mathematical work in the classroom is focused on:1) Teaching with the ¨Problem Solving Method´, (2) Teaching with the ¨Method of Discussion¨ and 3) Teaching with the ¨Method of Problem Discovery¨. The problem solving method that roots on the theories developed by Dewey, Polya and Wallas, is the most used in the Japanese schools (Figure 1). The stages it is based upon are: (1) ´Comprehension of the problem¨, (2) ¨Development of a solution by oneself´, (3)´Progress through the Discussion´, and (4) ´Conclusion¨. Each one of these steps is very well organized in the classes and is kept in the so called Annual Didactic Plan, according to national standards for the curriculum, and whose objective is to develop the comprehensive skill and creative reasoning. The following parts are taken into account: Connection of the previous content with the new one; development of the content and beginning of another one without any visible connection; subdivision of units to allow the spiral study.

The didactic plan of teaching, referred to as how to develop the classes aims at: (1) create situations or problems centred in the recognition of problems, in the recognition of regularities or properties that empower the inductive mathematical thinking and the search of a new information; (2) development of mathematical creative activities that stimulate the search of regularities; and (3) development of innovating teaching strategies in order to back up different ways of thinking and promoting the pleasure of learning. The purpose is to ensure that children learn by themselves through the stimulation of ideas about the necessary kind of knowledge for solving a problem. In the design of a Lesson Plan, the required didactic material is studied deeply, which acts as a bridge or link for the students to develop their own ideas. Problems are prepared beforehand, going ahead to the ideas of the students, understanding the quality and efficiency of the ideas and developing questions in order to stimulate the solution (Aravena, 2007). In table 1, the three problem solving are shown upon the Japanese Model. We can find the 4 Polya. stages

The 4 Polya stages	The 5 Dewey stages	The Wallas 4 stages
1. Understanding of the problem 2. Sketching of an action plan 3. Execution of the plan 4. Reconsideration and retrospection	1. Experimenting a difficulty 2. Defining the difficulty 3. Generating a possible solution 4. Proving the solution by reasoning 5. Verification of the solution	1. Preparation 2. Incubation 3. Illumination 4. Verification.

Table 1. Problem Solving Models based on the Japanese Method.

It can be inferred that the strategy for the developing of the class looks for empowering the methods, concepts, and forms of mathematical reasoning, which, are always aiming at all dimentions and levels and at the formulation and the solving of problems. It deals, then, with an integrating strategy in order to provide mathematics to the development of the mathematical reasoning, and also, to the communication of methods and processes.
The strongest point of the Japanese teaching is the way how the teacher or group of teachers organizes the analysis of the planning, and the way they feedback the process, they call ***"Lesson Study"***.
The three elements which are discussed in the study of a class are: (1) to design the lesson plan according to the national curriculum and to the principles that guide the method. This is, the formulation and discussion of the problems that will be worked by the students, focusing the classes towards the solving of such problems and the discussion of the didactic materials;(2) To develop the class work with the presence of the group of teachers; (3) To analyze the class with the team in order to detect the good points and the difficulties through the discussion and group reflection; y (4)to establish the adjustments for the feedback and the revision of the lesson plans. This modality, where teachers from different schools can be included has allowed them to reinforce and feedback the process and to establish improvements, as well as help other teachers to develop their own pedagogical practices.

III. METHODOLOGY

The methodology was qualitative and quantitative, a pretest-postest were designed, and a didactic unit that took into account the elements described in the model Lesson Study: (1) Mathematical problems that aim at: (a) development of the thinking (b) search for regularities and (c) solving of situations within the context of the students. (2) analysis of the required didactic material, studying such material deeply, so that it can act as a bridge or link between the students to develop their own ideas: (a) study of the problems and possible solutions on account of the teachers and (b) group revision of the possible difficulties and goals, discussing the quality and efficiency of the ideas, grounding and deepening the questions and possible answers.

Design of the Unit Planning. Problem Solving

Objective of the Unit: Solving problems from different areas of knowledge and from daily life that may involve the basic mathematical operations.
Didactic Material: Problems are prepared where regularities can be found and the mathematical background can be analyzed in each one of them, being solved by each one of the 6 teachers who took part in the experience, using methods and personal strategies. Afterwards, each one of the solving strategies is discussed by the team.
The use of the blackboard is considered, as a learning centre, so that the children show their different solutions and the class, as a group discusses the efficiency of each one of them and the obstacles and mistakes that have appeared.
Students Behaviour: An analysis of the possible difficulties is made. The following aspects are considered: reading of the problem, conditions, restrictions, processes, types of solutions, previous knowledge, use of the didactic material.

Design of the lesson plan of each class. In the following Table an example of a class is presented.

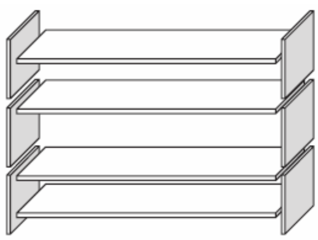
Class	Objetive	Situation or problem	Planning of the students work
Clase: (N° 4 out of 6 classes.) of 90 minutes.	Understand The meaning of the cocient and of the rest in order to apply it to daily situations.	***Shelves*** For building shelves a carpenter needs the following: -4 long pieces of wood, -6 short pieces of wood, -12 small hooks, -2 big hooks, - 14 screws The carpenter has 26 long pieces of wood, 33 short pieces of wood, 200 small hooks, 20 big hooks, and 510 screws in the store. ¿How many complete shelves can this carpenter build? **SKETCH FOR THE SHELF** 	***a) Understanding of the problem::*** *(Time 15 minutes approximately*) -Reading of the instructions and understanding of the situation. -Clearing of the problem situation through the discussion among students. ***b) Development of a solution by themselves:*** *(Time 25 min)* -The students think, discuss, and work on the problem and find solutions.. -The teacher goes round the classroom commenting, orientating, making suggestions to the students who show difficulties to face the problem ***c) Progress through discussion:*** *(Time approx. 30 minutes)* -A member of the group gives their solution to the class - The students share ideas, exchange opinions about the qualities, advantages, and disadvantages of each solution, identifying likenesses and differences. ***d) Conclusion:*** *(Time 15 minutes approx.)* -Summary of the key points appeared in the class. -Consolidation of the ideas. -Record in the notebooks. ***e) Self evaluation or Auto evaluation*** *(Time 5 minutes)*

Table 2. Lesson plan 4 out of 6, taking the Japanese method Lesson Study

Methods and Instruments of Analysis

For recognizing the real progress of the work by the students**,** an interpretative analysis of the content was made. For that purpose, the following categories were designed beforehand: (1) ***Understanding of the problem***: identifying data and conditions. (2) ***Sketching of an action plan***: description of mathematical relationships that interpret the process. (3) ***Execution of the plan***: discovering the regularities using the algorithms and properties. (4) ***Verification of results*****:** Checking processes and checking results.

Interpreting the data and communicating the solutions (Polya, 1985; Aravena & Caamaño, 2007).

Instruments of Analysis. The didactic unit was validated through a triangulation of evaluators, selecting 10 problems to be incorporated in the classroom.. The Pretest and postest were validated through the Cronbach alfa, with the following results: pretest $\alpha = 0{,}9718$ y postest $\alpha = 0{,}9702$. Relative frequencies were used for the analysis of the data in each one of the categories. And, to give more validation to the study, the test t-student was used in a signification level of 0.05.The items selected belonged to the indicators that are related to the 4 Polya stages, and that were analyzed through triangulation of the teachers and the researcher, and being sent to expert juries afterwards. Besides, a qualitative and quantitative analysis was made considering the categories under study and each one of the elements described in the model **Lesson Study.** For that purpose, an observation list was used, that was triangulated by the team.

Sample. The Maule Region is historically recognized as one of the most deficient in national tests measuring, especially in state schools of medium to low socio economic strata, and even more, at the rural and marginal areas as well, where you can find this diversity most clearly. (Fuentes, et. al, 1996; Aravena & Caamaño, 2007). For that reason, the Santa Rita School was chosen at Pelarco County, which is a rural area of the Maule Region. The work was done in students of 5th grade, ages ranging between 9 to 11, that was composed of students from the area from different family environments, from a medium socio economic stratum, coming from season workers families. The experience was implemented during a month during the second term of 2007, a group of 13 students

IV. DISCUSSION ON THE RESULS

The present text reports the results according to categories of analysis regarding to understanding of the problem of the *verbal enunciated,* the students show difficulties in identifying the data and the restrictions, the deficient results in this item reach to 62%, in contrast with the postest where it is observed that they make a correct identification 53,9%. Regarding this aspect, we coincide with the researches from Aravena & Caamaño (2007) that show that this deficiency continues to be present at secondary levels. Moreover, this situation continues to be present at Superior Education in Chile. (Aravena, 2002). Another difficulty is to identify the hidden or unknown mathematics, so that it can lead them to describe the mathematical relationships, the percentages of correct ones reaches only up to 40%. On the contrary, in the postest 53% identifies them as correct.

Regarding to a *planning configuration* in order to solve the problem, where data must be represented graphically, the percentages are inverted, since in the pretest the percentage was higher in this item, reaching to a 70% of the correct answers against a 46.2% in the postest. Regarding to discovering regularities, the results show that 100% of the students is not able to explain none of the regularities at the beginning, nor explaining the operating involved, as the multiplying principle. On the other hand, in the postest, the results are much encouraging, although they are not high, they are significant all the same, reaching to 46,2%.

Regarding to the indicator execution of a plan, the results show that, at the beginning, the students show serious problems about developing the algorithms, reaching only to 24% of the right ones. On the other hand, in the postest, the results are much more favourable, since 53% of the students execute them correctly. With regards to the application of the multiplying principle, 0% of the students does not reach it correctly, against 38,5% in the postest. Although the result is not the most favourable one, there is a meaningful change regarding to the pretest.

As to the *verification of results*, where we have wanted to include the written version of the answer to the real problem, the interpretation of the data communicating the solutions, the verification of processes and the proving of the results, so as to see if these match into the problem. The analysis of the results confirms that before the experience, all the items do not go beyond 40%. On the contrary, in the postest the results surpass 53%. We coincide with Alsina (1998) y Aravena & Caamaño (2007) in this aspect. They show that the communication of results is one of the least worked out issues in teaching at all levels. So the students are not accustomed to giving an answer to the real problem, they just keep the mathematical problem.

QUALITATIVE DESCRIPTION

According to the categories of analysis, an interpretative analysis was done that was recorded by each one of the teachers. Through the afterthoughts and discussions inside the classroom, we detected that a planned work in the described terms was defined as:

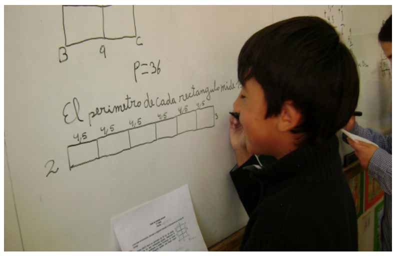
(1) The role of the teacher will be to pay attention to the processes and help students to develop deductive reasoning skills, giving chance to students to express themselves, explain the way they reasoned, and that they can also be able to detect their weaknesses and mistakes.

Picture 1. Data analysis of the problem

(2) The students have an active role in the construction and development of the mathematical problems, where group work is fundamental for communicating ideas and the reinforcement of their self confidence in learning how to think by themselves, to defend and express their ideas to the rest.

Picture 2. Reflection on the problem

(3) The classes came to be an accumulative product of dialogs, shown on the blackboard, where the students place the mathematical production to be analyzed by the class. It could be observed how they reason and think on the problem, the way the structure it, and how they enter into the mathematical problem without losing sight of the real problem.



Picture 3. Work on the blackboard

(4) The reconstruction of the class that they developed in their notebooks becomes a potential source, since it reflects the evolving mathematical knowledge. The reproduction of the work carried out, leads them to rethink on how to reconstruct the class, allowing them to display significant notions, keeping an order in the content, deepening the understanding and thinking the learning process over. This allows to test the meta cognitive skill through the self regulation of the knowledge

(5) The communication of concepts and mathematical processes was presented in a natural way, since the not penalizing of mistakes makes students express themselves freely.

(6) The evaluation was focused on recognizing what students are able to do instead of on what they are not. The auto evaluation or self evaluation that students did in each class allowed them to deepen in their own processes and regulate their learning process by themselves.

ANALYSIS OF THE RESULTS THROUGH THE TEST *T- STUDENT*

In order to provide more validation to the study, an analysis was done through the test *t-student* for individual samples. Equivalent items were taken in both tests and that were essential in problem solving according to the Polya four stages Model

Name Items	Pretest		Postest		Change		t-student
	Mean	SD	Mean	SD	Mean	SD	
Understanding the Problem							
Identifies data	1,92	1,320	2,85	1,068	0,92	0,862	**
Identifies unknowns quantity	2,00	1,211	2,92	1,038	0,92	0,954	**
Organizes details in writing	2,23	1,301	2,62	1,044	0,38	0,961	NS
Identifies operations	2,08	1,382	2,69	1,251	0,62	1,044	**
Setting up a Plan							
Represents solution graphically	2,85	1,281	2,69	1,182	0,15	1,068	NS
Discovers regularities	0,92	0,277	2,23	1,363	1,31	1,581	***
Applies multiplicative principle	0,92	0,277	2,38	1,261	1,46	1,266	***
Plan execution							
Solves algorithm division	1,38	1,261	2,77	1,481	1,38	1,557	NS
Solves algorithm multiplication	1,54	1,450	2,54	1,506	1,00	1,633	***
Expresses written regularity found	0,92	0,277	1,69	0,947	0,77	0,927	***
Verification of results							
Expresses written response to the problem	1,85	1,405	2,62	1,121	0,77	1,166	**
Interprets and communicates data solutions	1,92	1,382	2,77	1,013	0,85	0,689	***
Checking of results	2,23	1,301	2,69	1,032	0,46	0,660	**

** P<0,05; * **P<0,01

Table 3. Results of the Test t student considering the 4 Polya stages

In table 3 we can observe that in10 out of 13 items that were related to both tests, there are significant differences (p<0,05) and highly significant (p<0,01), favouring the postest. Thus, regarding to: (1) ***Understanding of the problem*****;** it is observed that students show difficulties at the beginning in the identification of the unknown mathematics and in the restrictions of the problem. (2**)** ***Sketching of an action plan;*** to the solving regarding the solving, the description of the mathematical problem according to data problems. (3) ***Regarding the plan execution;*** It is established that students do not get to use the algorithms and their properties correctly. Also, it is very difficult for them to find the implicit regularities in the problems, especially when establishing a relationship between the mathematical data and the real problem.4) V***erification of results***; where it is asked to communicate the solutions according to the problem. We coincide with previous research that shows that this aspect is very little treated and worked out at all levels (Alsina, 1998). However, there are not significant differences regarding to the developing of schemes in order to organize data, especially in problems that are inside of a certain context. Based on the analysis shown, we can conclude that an organized planning in the above mentioned terms, allows students to use the problem solving skill for understanding the mathematical concepts and processes in a wider range. Thus, developing significant mathematical skills, what allows us to validate the sub hypothesis H_1 y H_2, and the hypothesis put forward in this research.

V. CONCLUSIONS AND DIDACTIC IMPLICATIONS

At a methodological level, it can be concluded that: (1) the planning designed let the students to have an active role in the solving of mathematical problems. (2) the group work let them develop self confidence in capacities and skills for learning and thinking by themselves.(3) an evaluation focused on recognizing what students are able to do instead of stressing the skills they are not able to get, can become a fundamental issue for students to lose their fear to make mistakes. (4) The use of the blackboard for students to expose their ideas allows the group to reconstruct their knowledge. (5) The importance of the role of the teacher for letting students express freely, considering mistakes as a natural thing. Finally, we can restate that an organization in the above mentioned terms is a promising one if we want to develop major skills. Lastly, it would be quite convenient to apply this experience in similar situations so as to validate this study, and focus it on new studies from the qualitative point of view, using case study to know how students reconstruct their mathematical knowledge deeply..

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