PROBLEM POSING PERFORMANCE OF GRADE 9 STUDENTS IN SINGAPORE ON AN OPEN-ENDED STIMULUS

CHUA, Puay Huat <puayhuat.chua@nie.edu.sg>
National Institute of Education, Nanyang Technological University, Singapore

YEAP, Ban Har <banhar.yeap@nie.edu.sg>
National Institute of Education, Nanyang Technological University, Singapore

Abstract

This is an exploratory study into the individual problem posing characteristics of 152 Grade 9 students (aged 15) from four secondary schools in Singapore. The subjects were novice problem posers in that they were not given any training in problem posing skills. Each student was asked to write down a problem for their friends with the final answer as 60^0. Students also solved their own problems. The relationship between the structures of the posed problems, the topics involved in the problems and the solutions were discussed. Students' self-reported metacognitive regulatory strategies, the effects of achievement levels and of gender were also discussed. It was found that direct proposition type of problems occurred in about half of the posed problems. The presence of problem over-conditioning was not significant across achievement levels and gender. Students' confidence in their posed problems were found to be related to some of the metacognitive strategies at the property noticing phase, problem construction stage and during solution checking.

Keywords: Mathematical Problem Posing, Problem Solving, Angle Measure, Metacognition

The importance of problem posing in relation to mathematical explorations has been highlighted in various literatures. There are studies linking mathematical problem posing to creativity (Silver, 1994, Haylock, 1987) and to mathematical competence (Ellerton, 1986). Specifically the relationship between mathematical problem posing performance and problem solving abilities has also been studied in recent years. In fact, English (1997) noted that both problem posing and problem solving are closely related and that the process of problem posing in fact draws heavily on the processes of problem solving.

Silver and Cai (1996) made a distinction between two notions of problem posing. Firstly, problem posing can be construed to be a case of a generation of new problems from a mathematical situation. Secondly, it can also be interpreted as the reformulation of a given problem in which there is an intention to uncover the deeper underlying structures of a given question or problem. In this case, it is one strategy in problem solving where the solver tries to answer related questions which will give insights to the original problem. For the present work, the focus will be on the nature of problem posing itself and not as part of a problem solving heuristic. Problem posing is used here as the formulation of new problems from a mathematical stimulus.

The inclusion of activities in which students generate their own problems had also been strongly endorsed by the National Council of Teachers of Mathematics (1991). It is believed that such activities can provide a glimpse of students' understanding of mathematical concepts and processes and their attitudes towards problem solving. As part of the metacognitive aspect of the national mathematics curriculum framework, problem posing is also strongly encouraged in the classroom. In the Ministry of Education document, *Mathematics Syllabus (Lower Secondary)* (CPDD, 2001), students are encouraged to "create, formulate or extend problems." (p.16)

One strand of the studies in problem posing involves developing problem posing as an instructional intervention to improve problem solving skills and to improve disposition towards solving. Some of these include work done by Gonzales (1994, 1998) on using problem posing to improve on preservice teacher training and Manouchehri (2001), who worked on an instructional model for promoting problem posing in a sixth grade classroom. Another strand of work goes into analyzing the problems posed in terms of their surface structures. For example, Marshall's schema theory (Marshall, 1995) was used in the study by Charalambous, Kyriakides and Philippou (2003) on the problem posing skills of primary school students. For both strands, the contexts of these studies mainly involved mathematical word problems and largely on arithmetic. Subjects of these studies varied between students of various grade levels to undergraduates in pre-service teacher preparation courses.

Lesser work is being done in the area on the cognitive processes of mathematical problem posing itself and the regulation of these processes. Christou, Mousoulides, Pittalis, Pantazi and Sriraman (2005) had proposed in their study on 143 Grade 6 students in Cyprus, a few processes that can be used to describe problem posing. Selecting quantitative information is one of the processes involved in posing problems. It is mostly linked to tasks that require students to pose problems that are appropriate to specific given answers. Such a process involves the ability to focus on the context of the problem structure and the relationship between the given initial information and the subsequent information that the students created to make the posed problems coherent. This is an important skill in building connections across domains of knowledge and sense making in mathematical exploration. One purpose of this study is to look at the selecting process involved in problem posing in the area of school geometry in Singapore. Students' posed problems in this area will shed light into how they perceive linkages between the different topics within school geometry and into how they construct their problems. The control of this cognitive process is also an important aspect in the study of problem posing.

Livingston (2003) referred to metacognitive regulatory processes as those that one uses to control cognitive activities and more importantly to see to the meeting of the cognitive goal. These involve planning, monitoring of cognitive activities and checking of outcomes of those activities. The other purpose of this study is to illuminate some of these regulatory processes that are involved in problem posing.

Method

a) Subjects

This is an exploratory study about the individual problem posing characteristics of 152 Grade 9 (aged 15) students from four secondary schools in Singapore. The subjects were novice problem posers. Besides their classroom experience in asking questions, they were not given any specific training in problem posing prior to this study. The decision to locate the study with these students was that few such studies had been made in this area in Singapore.

b) Task

Each student was asked to freely write down a problem for his or her friends to solve with the final answer as 60^0. Students also solved the problems they had posed. By solving their own problems, the students can make explicit the selecting process as they construct the problem structures. Silver (1990) in her study on problem posing involving number sense, also noted that such open ended stimulus tends to provide good opportunities for students to be engaged in generative aspects of mathematical thinking.

Much of what constitutes a problem is dependent on the context in which the problem is posed. When students posed problems to friends, they do so with some perceived knowledge of their friends' familiarity of topics and methods, cognizance of common errors made by friends and the time taken for their friends to complete the tasks. Such perceived knowledge is captured in students' posed problems.

c) Questionnaire

Immediately after completing the task, students were asked to complete an 18-item questionnaire as shown in Table 1. The purpose of the questionnaire is to get a snapshot of their metacognitive regulatory strategies during their posing and solving. Each item has a 4 point Likert scale with 1 being strongly disagree and 4 being strongly agree. This instrument is an adaptation of Goos, Galbraith and Renshaw (2000) metacognitive survey for secondary students in the Australian state of Queensland in their study on the metacognitive aspects of students solving combinatorics problems. In order to make the questionnaire more appropriate for the students in this study, some of the questions were changed. Further modifications had also been made to take into account the different phases of metacognitive regulatory behaviour in problem posing. These phases were the results of an earlier work made by the authors as they worked on the think-aloud

Table 1
Metacognitive Regulatory Strategies

Code	Statements	Phase
4A	I read carefully when there is an important information	PN
6A	I am good at recalling what my teacher had taught	PN
8A	I ask myself questions about the information before I begin	PN
9	I use my own examples to make what is given more meaningful	PN
13A	I try to use my own words when I read the new information	PN
1A	I check periodically if I am getting the problem that I want	PC
2AA	I consider other possibilities to a problem before I ask it	PC
10AA	I find myself checking on my understanding as I posed the problem	PC
11A	I draw diagrams to help me understand while posing the problem	PC
17A	I think about the method of solution first before I pose the problem	PC
5A	I ask if I have considered all possibilities to my problem while solving it	CS
14A	I go through over new information that is not clear	CS
15AA	I constantly look back at the problem as I start the solution	CS
16A	I check my solution as I worked on it	CS
3A	I know how well I have done once I finish the problem	LB
7A	I ask if there was an easier way to pose after I finish posing the problem	LB
12A	I ask if I could have posed a different problem after I finished	LB
18A	I like the problem that I posed	LB
PN: Property Noticing, PC: Problem Construction, CS: Checking Solution, LB: Looking Back		

protocols of 10 students prior to this study. Students were then engaged in the same task of creating problems with the final answer as 60^0. Property noticing describes the initial phase before students start the active construction of their problems. In this phase, students make associations with the topics that first come to their mind when confronted with this stimulus. Within the problem construction phase, students draw upon their earlier experiences about topics to come up with problems. Simultaneously they are also checking the solutions to their posed problems and retrospectively going back to their earlier posed problems (and modifying when necessary) and see if their solutions make sense. In the last phase, students reflect back and evaluate their work.

One limitation to this study is that the sample of 152 students can not meant to be representative of the students in all the secondary schools in Singapore. The sample size also does not allow for factor analysis of the metacognitive regulatory strategies.

Results

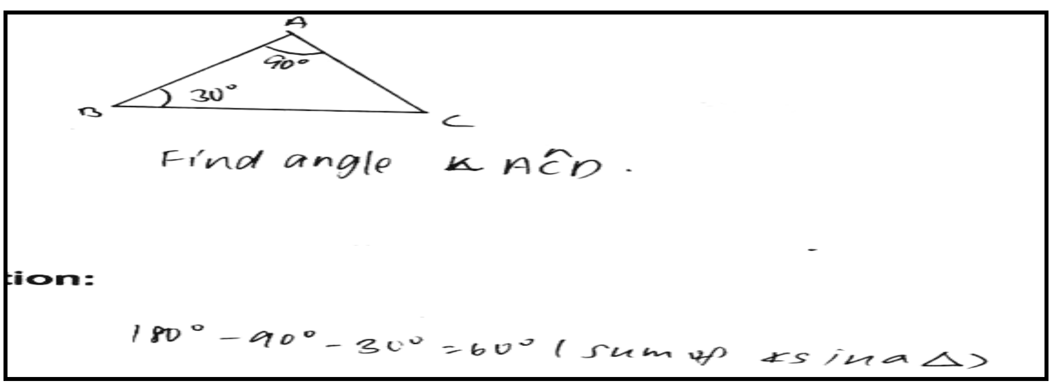
The results of analysis of students' posed problems, their solutions and the questionnaire responses are presented in two parts. In the first part, problems posed are described in terms of the types of problem structures, the domains of knowledge used and the solutions to the problems. Secondly, discussions are made on students' self-reported metacognitive regulatory strategies. Specifically the relationship between the different strategies used and the types of posed problems, students' gender and students' achievement levels are also discussed. All results are discussed at 5% level of significance.

a) Characteristics of Posed Problems

i) Problem structure

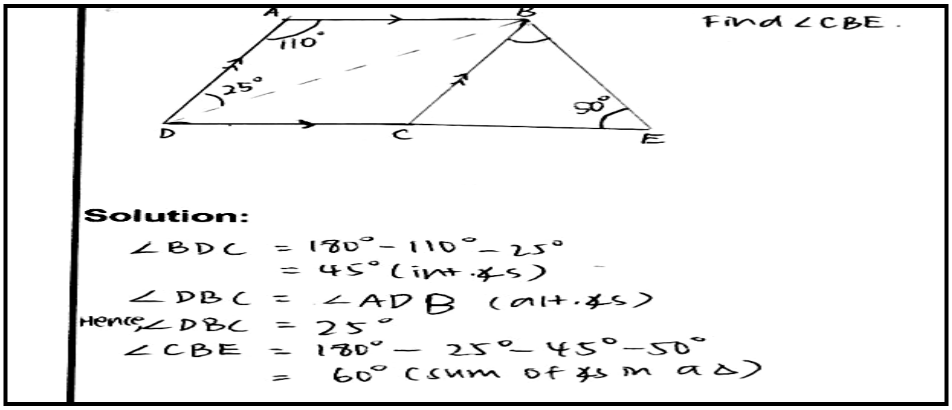
There are direct proposition problems where the solutions require single-step solutions as shown in Figure 1. Each of the direct proposition problems involves a single topic. These problems account for 50.7% of the total posed problems. Their solutions involved some forms imitative reasoning. For example, the solution may involve the recalling of a

Figure 1
Example of a direct proposition problem



simple algorithm like finding the angle sum in a triangle as in Figure 1 or a possible memorized answer like $\cos^{-1}(0.5) = 60^0$ to solve "what is x if $\cos x = 0.5$?" These may suggest that students just created them from what first came into their mind without trying to create linkages with other topics. The other possibility is that these are problems which students perceived their friends are able to solve. They also reflect what can be commonly found as exercises in school textbooks or perhaps problems which they commonly encountered in their classroom learning experiences.

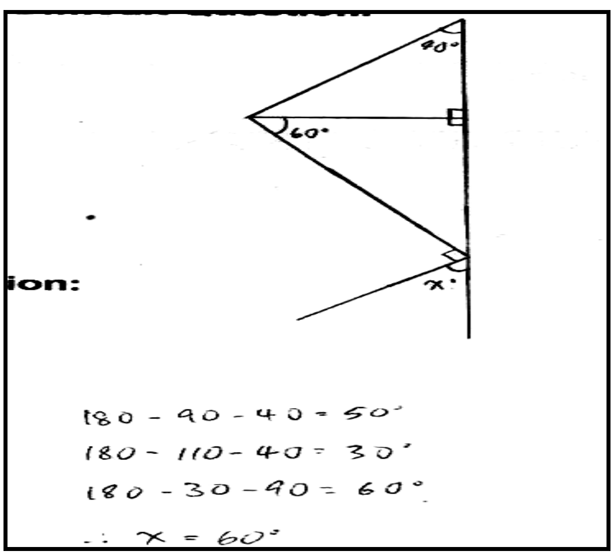
Figure 2
Example of a multiple topic problem



The rest of the problems involve multiple steps and are situated in a combination of topics. One example is shown in Figure 2. These questions are good examples of how students were able to link topics together in their problem construction. In the problem in Figure 2, the student involved the uses of the geometric properties of a parallelogram, a trapezium and a triangle. Justifications of the steps were also made by using of the properties of alternate angles and the angle sum of a triangle.

Within these non-direct propositions, there are problems which are over-conditioned. An over-conditioned problem contains extraneous information which does not contribute to the solution. In Figure 3, angle 40^0 is not needed as part of the solution. This occurs in

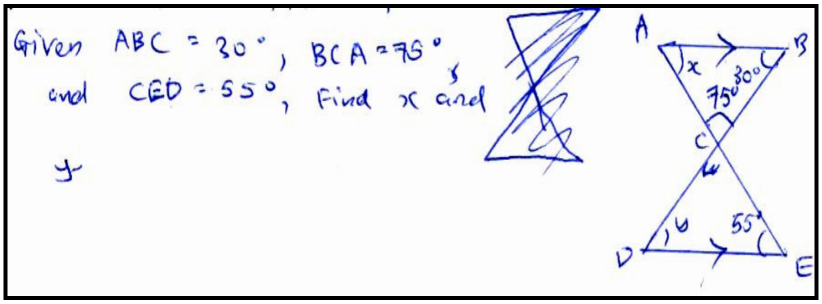
Figure 3
Example of an over-conditioned problem



20.4% of the total posed problems. Yet there are other problems that contain inconsistencies in their structures. The angle sum of a triangle is violated in Figure 4.

The problem was constructed without considering the linkage to the other parts of the diagram. The student perhaps was trying to impress his or her friends the sophistication the problem by including more information. Of the posed problems, 17.1% of them are inconsistent problems.

Figure 4
Example of an inconsistent problem



ii) Domains of Knowledge

The problem in Figure 1 involves a single topic about the sum of angles in a right angle triangle. For single topic problems, students also made use of the topics on alternate angles, complementary/supplementary angles, corresponding angles and trigonometric ratios. Multiple topics, namely, the use of the properties of angles in a triangle and alternate angles are found in the problem in Figure 2. Across the four schools, students used a variety of topics in school geometry in their posed problems. Of these topics, the use of angle sum in a triangle is most prevalent. It is observed in 43.4% of the work with others using the topics about complementary/supplementary angles (24.3%), circle properties (21.1%) and alternate angles (15.1%).

As shown in Table 2, the use of the topic on triangle is also prevalent among problems that involve multiple topics (64.2%) rather than in single topic problems (27.1%). Perhaps students are more familiar with this topic. Over-conditioning is also not significantly associated to problems with multiple topics, $\chi^2_{(1)} = 0.588$, p = .366.

Table 2
Analysis of posed problems involving multiple topics

		Multiple Topics (P10)			$\chi^2_{(1)}$	**Asymp Sig. (2 sided)**
		Absent	**Present**	**Total**		
Use of Angle Sum in Triangle (K3)	Absent	62 (72.9%)	24 (35.8%)	86	21.01	.000
	Present	23 (27.1%)	43 (64.2%)	66		
Use of Circle (K5)	Absent	78 (91.8%)	42 (62.7%)	120	19.06	.000
	Present	7 (8.2%)	25 (37.3%)	32		

iii) Solution

There are varying numbers of steps in students' solutions to their posed problems. They range from the single step direct proposition type to more involved types like the problem in Figure 2. In the course of working through their solutions, students demonstrated they were able to justify their steps appropriately. Most of the solutions involve the recall of solution algorithms in school geometry and trigonometry. Except for four solutions that have computational errors, the rest of the students' solutions are found to be correct.

b) Achievement and Gender

Across the four schools, students' scores in the standardized national Primary School Leaving Examination at Grade 6 are used to classify the achievement levels. Students are classified either as High Achievers (HA), Average Achievers (AV) or Low Achievers (LA). The distribution of gender and achievement levels is shown in Table 3.

Table 3
Distribution of Gender and Achievement Levels

	Male	**Female**	Subtotal
LA	11	29	40
AV	31	41	72
HA	17	23	40
Subtotal	59	93	152

The students' achievement levels are found to be not significantly associated to the types of posed problems, domains of knowledge and their solutions. For example, there is no strong evidence to suggest that HA students produce more multiple topics type of questions compared to students in the other levels. In Table 4, achievement levels and the presence of over-conditioned problems are also found not to be significantly related in this study. This suggests that the problem posing performance of students to the given open-ended stimulus is not strongly influenced by how well they had performed in their standardized tests.

Across the achievement levels, there are also no significant associations with the students' metacognitive regulatory strategies except for the presence of checking during the solution phase. There is some association between HA students and their self-declared use of checking in their solutions. Lower number of HA students checked their solutions compared to the other two groups of students. This perhaps reflects the HA students' confidence in their solutions to their posed problems and hence the lesser need for checking.

Table 4
Achievement Profiles, Over-Conditioning and Use of Checking

		Achievement Profiles				$\chi^2(2)$	**Asymp Sig. (2 sided)**
		LA	**AV**	**HA**	**Total**		
Over Conditioning (P8)	Absent	31 (77.5%)	54 (75.0%)	36 (90.0%)	121	3.71	.156
	Present	9 (22.5%)	18 (25.0%)	4 (10.0%)	31		
Use of Checking in Solution (CS) (Q16A)	No	17 (42.5%)	43 (59.7%)	29 (72.5%)	89	7.49	.024
	Yes	23 (57.5%)	29 (40.3%)	11 (27.5%)	63		

Problem structures are also found not to be significant in discussing gender. Like in achievement levels, the over-conditioning feature in problems is also found not to be significant across gender. Table 5 shows some significant results from the questionnaire survey and gender. At property noticing phase, more females than males agree that they were not good at recalling what the teachers had taught. For both gender, close to half of the students (46.7%) felt that they were good at recalling what had been taught.

More males reported that they had considered other possibilities to the posed problems as they were solving the problems. But most of them (69.5%) did not check their solutions. More males than females are also found to like the problems they had posed compared to females. For all the students, 88 (57.9%) reported they like their posed problems.

Table 5
Questionnaire Responses and Gender

		Gender			$\chi^2(1)$	Asymp Sig. (2 sided)
		Female	**Male**	**Total**		
Good at recall (PN) (Q6A)	No	56 (60.2%)	26 (44.1%)	82	3.788	.038
	Yes	37 (39.8%)	33 (55.9%)	70		
Think of method of solution first (PC) (Q17A)	No	46 (49.5%)	18 (30.5%)	64	5.320	.016
	Yes	47 (50.5%)	41 (69.5%)	88		
Consider all possibilities to problem (CS) (Q5A)	No	45 (48.4%)	18 (30.5%)	63	4.755	.022
	Yes	48 (51.6%)	41 (69.5%)	89		
Use of Checking in Solution (CS) (Q16A)	No	48 (51.6%)	41 (69.5%)	89	4.76	.029
	Yes	45 (48.4%)	18 (30.5%)	63		
Like posed questions (LB) (Q18A)	No	46 (49.5%)	18 (30.5%)	64	5.32	.021
	Yes	47 (50.5%)	41 (69.5%)	88		

From Table 6, among students who reflected that they knew how well they had done once they had finished posing their problems, 87.8% of them checked their solutions and 86.7% drew diagrams to help them understand as they constructed their problems. Similarly, of those who felt that they had done well, a high number also reported that they asked questions about the information during the property noticing phase of their problem posing. The strategy of asking questions at the property noticing phase, drawing diagrams during their problem construction phase and checking of their solutions appears to account for the higher confidence in their knowing of how well they have done in their problem posing.

Table 6
Questionnaire Responses with Knowing How Well When Done

		Know how well when done (Q3A)			$\chi^2(1)$	Asymp Sig. (2 sided)
		No	**Yes**	**Total**		
Ask questions about information (PN) (Q8A)	No	30 (55.6%)	34 (34.7%)	64	6.216	.010
	Yes	24 (44.4%)	64 (65.3%)	88		
Draw diagrams to help understand (PC) (Q11A)	No	20 (37.0%)	13 (13.3%)	33	11.575	.001
	Yes	34 (63.0%)	85 (86.7%)	119		
Use of Checking in Solution (CS) (Q16A)	No	31 (57.4%)	12 (12.2%)	43	35.005	.000
	Yes	23 (42.6%)	86 (87.8%)	109		

Conclusion and Implications

To encourage a variety of problem structures, the classroom teacher needs to broaden the types of problem experiences being presented to students. The teacher can capitalize on the informal activities situated in students' daily activities and get students to the habit of recognizing mathematical situations wherever they might be and making connections to various aspects of school geometry. Otherwise, students would only be comfortable with constructing direct proposition type of problems which does not allow them to explore the inter-connectedness of topics. This ability to make connections is an important skill in mathematical exploration.

Perhaps, to get students to have more confidence in their problem posing, the teacher can encourage students to ask more questions about the given stimulus during the property noticing phase, to draw diagrams during the problem construction phase and to check their solutions. These metacognitive strategies appear to help novice problem posers in this study to have more confidence in their work.

Since achievement levels and gender are not significant across problem posing performance, classroom problem posing activities should be encouraged for all students. The teacher can also make use of students' problem posing work as teaching points. For example, the teacher in discussing students' posed problems, can sensitize the class to issues about the inconsistencies in problem structures or to the notion of over-conditioning in problem construction. Perhaps such discussions may help to produce better problem posers and may contribute to students' engagement in more quality mathematical inquiry in the classroom.

The teacher in teaching is also involved in posing problems. The school curriculum planner can look into ways of promoting the teacher's competency in problem posing just like the way problem solving heuristics are made known to teachers. The very way in which the teacher asks questions can affect that shared spirit of investigation between the teacher and the students. Appropriate use of varied problem types which may depart from the textbook exercises may bring about a better quality of classroom interaction. But such teacher's behaviour is also dependent on the teacher's beliefs and perceptions about problem posing itself and about the teachers' views on the nature of mathematics. This is an issue that warrants further investigation.

References

Christou, C., Mousoulides, N., Pittalis, M., Pantazi, D.P., & Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *Zentralblatt f□r Didaktik der Mathematik*, *37*(3), Retrieved September 21, 2007, from http://www.umt.edu/math/reports/sriraman/Int_Reviews_Preprint_Cyprus_Sriraman.pdf

Charalambos, C., Kyriakides, L., & Philippou, G. (2003). Testing a comprehensive model for measuring problem solving and problem posing skills of primary pupils. In N.A. Pateman, B.J. Dougherty & J. Zilliox (Eds.), *Proceedings of the 27th PME, vol 2* (pp. 205-212). Honolulu : PME.

Ellerton, N. F. (1986). Children's made-up problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17,* 261-271.

English, L. D. (1997). Promoting a problem-posing classroom. *Teaching Children Mathematics, 4,* 172-181.

Gonzales, N.A. (1994). Problem Posing: A neglected component in mathematics courses for prospective elementary and middle school teachers. *School Science and Mathematics, 94*(2), 78-84.

Gonzales, N. A. (1998). A blueprint for problem posing. *School Science and Mathematics, 98*(8)*,* 448-456.

Goos, M., Galbraith, P., & Renshaw, P. (2000). A money problem: a source of insight into problem solving action. Retrieved September 5, 2007, from www.cimt.plymouth.ac.uk/journal/pgmoney.pdf

Haylock, D.W. (1987). A framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics*, *18*, 59-74.

Livingston, J.A. (2003). *Metacognition: an overview*. Maryland: NY: ERIC Clearinghouse No. TM034808. (ERIC Document Reproduction Service ED 474 273)

Manouchehri, A. (2001) A four-point instructional model. *Teaching Children Mathematics, 8,* 180-186.

Marshall, S.P. (1995). *Schemas in problem solving*, (pp.62-112). New York: Cambridge University Press.

Ministry of Education of Singapore. (2001). *Mathematics syllabus (Lower Secondary),* (pp.16-92). Singapore: Curriculum Planning and Development Division.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA : NCTM.

Silver, E. A. (1990). Contributions of research to practice: Applying findings, methods and perspectives. In Cooney, T.J., Hirsch, C.R. (Eds.), *Teaching and learning mathematics in the 1990s,* (pp. 1-11). Reston, VA: NCTM.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal of Research for Mathematics Education, 27*(5), 521-539.