THE DECISION-MAKING AS A SCHOOL ACTIVITY

María Candelaria Espinel Febles
University of La Laguna (mespinel@ull.es)
Ana Teresa Antequera Guerra
IES Luis Cobiella Cuevas (aantegue@yahoo.es)

ABSTRACT
A problem is chosen from the PISA 2003 Report, and then extended to include questions involving a choice of preferences and decisions. These questions are posed to secondary school students aged 15 to 16, whose problem-solving and heuristic decision-making skills are then analyzed. We discovered that students have difficulty using mathematics when assigning a weight to achieve an objective, though they are able to recognize the functional use of the rules of choice used in society.

INTRODUCTION

Some of the real world choices an individual may have to make require the use of mathematical calculation tools in order to establish a preference or analyze a situation. Multi-criteria analysis constitutes a way to model decision processes and involves the decision to be made, any unknown events which may affect the results, the possible courses of action and the result(s) of said actions. Using multi-criteria models, the decision maker assesses the possible implications each course of action may entail so as to obtain a better understanding of the relationships between the actions and the objectives. There are various mathematical procedures for summarizing the values yielded by each alternative with respect to all the criteria considered in the analysis. The best known mechanisms are those which use linear weighting (scoring), i.e. the simple sum of each attribute's contributions. This is a very fruitful field of Operational Research in which improvements and variants are constantly being developed, along with growing applications in various contexts (Berumen and Llamazares, 2007).
For the specific case of a discrete decision problem, the information available is realized through an evaluation of n attributes for a set of m alternatives, and presented in a double entry matrix in which the attributes are shown as columns, while the associated alternatives appear in the rows. The decision is made by fixing the criterion to be used in deciding the best solution (Romero, 1993).

In "The Best car" activity in PISA 2003 (MEC, 2005), a decision problem is posed using a data table with four attributes and five alternatives. It then asks for the value of a linear function to be calculated which ranks the alternatives, so that a linear function can then be weighted to yield a specific objective. We broadened this activity, asking that the best car be chosen by applying two rules, the first taking into account the number of first places, and the second by deleting the high and low values and adding the remaining scores.
In this paper we analyzed the students' results for the activity proposed. The study was done with secondary education students (abbreviated ESO in Spanish, Obligatory Secondary Education), most of them around 15 years of age.
The objectives guiding our research were:
. To see whether the students understood the decision rules and applied them correctly. The problem was easy to understand since the context, in which the problem was presented, to choose the best car, is familiar to most youngsters.
. To see if the students could identify other situations or settings where the different rules of choice presented in the activity, or other applicable rules, can be used.

. To observe whether the students used a mathematical process in other contexts requiring that a choice or decision be made, since the goal was to promote the development of the student's ability to take a critical stance given the rules of choice that are applied in society.

. To verify the validity of the activity designed via a questionnaire on fair decisions and well-founded judgments. This problem is part of wider research into the design and implementation of activities involving game theory and negotiation models using mathematics.

Theoretical framework

The salient points and the framework guiding our research were oriented around the studies conducted by the Organization for Economic Cooperation and Development (OECD) on the use of mathematics by students, which in the case of Spanish students shows their unwillingness to think beyond applicable procedures or routine problems in mathematics.

In Spain, one of the curriculum changes made to secondary education in 2007 involved the use of competencies in the curriculum. Mathematical competencies are understood as an individual's ability to use mathematics to meet his needs as a constructive, committed and thinking citizen. This is expedited through the promotion of activities that involve the problem solving and discrete mathematics.

At the present time, problem solving is provided as part of the primary and secondary education curricula. A problem is defined as a conflictive situation that requires a solution for which an explicit procedure is not known beforehand. Many of the text books more commonly used by school children include guidelines for the solving of problems, almost always following the phases described by Polya (1945). These phases encourage the use of heuristics, considered as exploratory methods or algorithms used in the problem-solving process in which the solution is found by evaluating the progress made in the search for the final result. This is why heuristics are "golden rules," conjectures, intuitive solutions or simply common sense. A problem is no more than a tool for thinking mathematically (Schoenfeld, 1992) and requires training individuals who can think for themselves and who can critique and reflect on the solutions.

The curricular content dedicated to problem solving, mandatory until the age of 16, offers the chance to work with discrete mathematics. This subject is, however, not well-known by practicing teachers, and the lack of available material and activities further serves to prevent this subject from being taught to the same level as in other countries (DeBellis and Rosenstein, 2004).

Discrete mathematics is one of the branches of mathematics which has seen the most progress in the 20th century as a result of computers. Different learning centers and associations, such as the Freudenthal Institute (Doorman et al., 2007), the National Council of Teachers of Mathematics (NCTM), and the American Mathematical Society (AMS) have promoted its incorporation into curricula to include experiments and research involving mainly the use of graphs, matrices and combinatory, all of which are very useful in today's mathematics (Kenney and Hirsch, 1991; Rosenstein et al., 1997). Material and ideas for its development have also been provided by the arrival of texts and projects such as COMAP (1988), based on a set of examples of the applications of mathematics most relevant to everyday life. The past decade has seen a proliferation of textbooks (Parks et al., 2000) and web pages along these same lines, such as http://www.dimacs.rutgers.edu.

Some Mathematics Education researchers regard discrete mathematics as a chance to revitalize mathematics in school (DeBellis and Rosenstein, 2004; Rosenstein et al.,

1997). They see it as a chance for innovation and an opportunity to discover problems that are out of the ordinary (Goldin, 2004).
The problems associated with this field can also be used to build mathematical knowledge and to model situations, which help the student understand and control the world around him. Mathematical modeling can be viewed as a means for linking problem solving to the real world. While the term mathematical modeling itself may be in dispute, within mathematics it can be used to solve real problems and to include students in some of the phases involved with the modeling tasks (Stillman et al., 2007). The trend or perspective of models for teaching mathematics and solving problems is to teach real life situations in the classroom (Lesh and English, 2005).
It is within this framework and considering the points mentioned (development of mathematical and problem-solving skills and modeling of everyday problems) that we developed the decision-making activity involving several alternatives in a set context.

DESCRIPTION OF THE STUDY AND RESULTS

Design of the study

This paper details the results of a study conducted with 72 secondary school (ESO) students divided into four groups, with 35 third-year students in two groups, and 37 fourth-year students in the other two. Most of the students were between 15 and 16 years old. Also included are the results of an interview with a group of fourth-year students asked to justify their answers.
The activity used, "The best car," was taken in part from PISA 2003 (MEC, 2005). A discrete decision problem was formulated by means of the data table with four attributes and five alternatives shown below. The students were asked to calculate the value of a linear function which ranks the alternatives so as to then weight a linear function and obtain a specified objective. The PISA activity was expanded by asking that the best car be chosen considering two rules (rule 1: number of top rankings and, rule 2: eliminate highs and lows and add).

Car	Safety (S)	Fuel consumption (C)	Exterior design (D)	Interior space (H)	Score
Ca	3	1	2	3	
M2	2	2	2	2	
Sp	3	1	3	2	
N1	1	3	3	3	
Xk	3	2	3	2	

The scores are interpreted as follows:
3 points = Excellent 2 points = Good 1 point = Acceptable

For the student, the activity had three parts:

. Question (a) consisted of evaluating a given simple linear function and choosing the best car:

$$\text{Total score} = (3x\ S) + C + D + H$$

. Question (b) required creating a weighted linear function to achieve a specific objective:

Total score = …. S +…. C +…. D +…. H,

which yielded alternative Ca as the best car.

. Question (c) required the students to apply two rules to select the best car according to each rule:

Rule 1: number of first place rankings,
Rule 2: sum of scores after eliminating highs and lows.

Analysis of the information

For the analysis of the results, the three questions in the activity were broken down into seven coded items as follows:

T = Complete table, A = Value of a linear function,
B = Assign weights to a linear function to achieve the objective,
CR1 = "First place" rule, CR2 = "Eliminate highs and lows" rule,
CE1 = "First place" choice, CE2 = "Eliminate highs/lows and add" choice.

The results of the data analysis are shown in four sections. The first includes the results on the success of each of the items. The second and third sections are dedicated to question (b) on the design of a function, and detail the results and strategies used by the students. The third section offers a comparison of the 72-student sample with the results of PISA for Spain and the OECD for questions (a) and (b). Along with the results there is a qualitative analysis of the interviews with one of the groups that took part in the study and whose students were asked to explain their solutions to the different questions in the activity, to express their opinions about the rules, to cite other situations where these rules are applicable and to name other rules of choice.

1. *Success of each of the seven items*

Table 1 shows the percentage of students who correctly solved each of the seven items comprising the three questions.

Table 1: Percentage of students answering each item correctly

Ítem	T (a)	A (a)	B (b)	CR1 (c)	CR2 (c)	CE1 (c)	CE2 (c)
%	93	100	57	96	93	83	82

The results of the study with 72 students yielded a high percentage of correct answers. There were no noticeable differences between the groups of third- and fourth-year students. Ninety-three percent of the students completed the table they were given with the score for each car (item T). All the students managed to correctly solve question (a) and apply the rule, i.e. to find the value of a linear function (item A). Most, over 90%, applied rule 1 (CR1) and rule 2 (CR2). Over 80% also chose the correct car by applying the rules. The biggest difficulty was posed by question (c) on the weighting required to have a given car (Ca) be the best. Slightly over half of the students answered this question correctly.

The activity shown here and consisting of 7 items was part of a larger questionnaire with 23 items on game theory and decision making, and which was intended to analyze the development of critical thinking and reasoning skills in secondary school students. The validity of the activity was determined using the Rasch methodology (Linacer, 2007), which provides a correlation between students and items and which confirms that

assigning or selecting weights so as to achieve a desired result is difficult when compared to the other items formulated in the questionnaire (www.ince.mec.es/pub/pisamanualdatos.pdf pp. 64-81).
We now present a detailed analysis of this question.

2. Results of the question to weight a linear function

To analyze item B (assign weights to a linear function), we established a coding system for Blank, Wrong, Acceptable and Right answers, where "Acceptable" was used when the assigned weights yielded an equal score between the target car and another car. Table 2 shows the number of answers for each category.

Table 2: Results for questions (b)

Categories	Blank	Wrong	Acceptable	Right
N = 72	2	18	11	41

In the results in Table 2, note how the majority of students, 41, answered correctly. But one fourth of them, 18, gave a wrong answer and were unable to assign weights to a linear function so as to achieve the desired objective. If to these we add those students with blank or acceptable responses, a total of 44% of the students failed this question.
As concerns question (b), the reasons the students interviewed gave for choosing the weights are:

. They assigned a higher weight to the higher scores
. They assigned weights at random
. They used trial and error.

We believe the failure of these students, who are nearing the end of their obligatory schooling, to come up with a weighted linear function to yield a given objective could have negative repercussions in their future lives as members of society.
Next we analyze the correct and incorrect strategies used by students when assigning weights in this question.

3. Listing and analyses of strategies for linear weighting

Table 3 summarizes the strings of four weights provided by the students, and assigns them to the Right, Acceptable and Wrong categories, along with the value of the object function for each attribute or car and the number of students offering said choice. When a cell shows several weights, the number of students choosing those weights is shown instead of the values.

Table 3: Answer patterns according to strategies for achieving an objective

Grade	*Weight*	*Total Scores*	*Frequency*
Right	3 1 1 3	21, 16, 19, 18, 20	9
	3 1 1 4	24, 18, 21, 21, 22	7
	3 1 2 4	26, 20, 24, 24, 25	4
	3 1 1 6	30, 22, 25, 27, 26	4
	4 1 1 4	27, 20, 24, 22, 25	3
	2 1 1 3	18, 14, 16, 17, 17	2
	4 2 1 4	28, 22, 25, 25, 27	2
	5 1 2 5 ; 5 2 1 5 3 0 0 3 ; 2 1 0 2		4
	4 1 1 3 ; 5 1 1 3		2
	3 1 2 6 ; 4 2 1 6		4

	1 0 0 2 ; 4 2 3 6		
Acceptable	3 1 2 7	23, 18, 22, 21, 23	5
	2 1 2 3	20, 16, 19, 20, 20	2
	4 1 2 3 ; 6 1 2 3 6 2 4 6 ;15 1 2 3		4
Wrong	3 2 3 3 ; 3 3 2 3 2 3 4 2 ; 3 3 8 3 2 1 5 2 ; 2 3 2 2 3 2 2 3		9
	9 2 2 3 ; 9 3 2 3 9 3 3 2 ; 4 2 3 2 4 2 2 1 ; 3 1 3 2 1 1 1 3		9

Of the 72 students, over half, 41, chose the proper weights which, as shown in Table 3, are needed to have the first value be the highest of the five. A total of 17 different weights were given as answers. The most frequent was (3 1 1 3), given 9 times, which results in a score of 21 for car Ca, and (3 1 1 4), appearing 7 times and resulting in a score of 24.

The most frequent strategy was to assign high values to S and H and low values to C and D. Note how some students changed the last 3 (weight of H) to a 4, possibly to obtain a larger difference between the two highest scoring cars. This same strategy seems to have led to other weightings such as (3 1 1 6) and (3 1 2 6).

The 11 cases graded as acceptable correspond to ties, and resulted from weights such as (3 1 2 3) or (2 1 2 3). Some students assigned weights to the attributes only to check the results with the alternatives they believed to be the most problematic, which may be why they failed to notice the tie.

The 18 students who answered incorrectly assigned weights like (3 2 3 3), (9 2 2 3) and other similarly nonsensical or absurd answers.

As already noted, it is troubling how so many students at or near the end of their mandatory schooling were unable to assign weights to achieve an objective. Differently stated, they were unable to analyze and work with data given to them so as to obtain the desired result.

4. *Comparison with the PISA results*

Table 4 shows the percentage of right answers for questions (a) and (b) given by the 72 students in the study, along with the results from Spain and the OECD. For question (a) we considered the result for item T, since in PISA the results of items T and A are combined.

Table 4: Comparison with the PISA results

	ESO N=72	Spain PISA	OECD PISA
Question (a)	93%	71.4%	72.9%
Question (b)	57%	22.2%	25.4%

For the 72 ESO students, the results are noticeably better than those for the rest of Spain and also than those for other OECD countries. For question (b), however, on the assignment of weights or the prioritization of alternatives to achieve a specified outcome, all three gave unsatisfactory results. The task involves an important mathematical process and evidences a lack of conceptual and algorithmic learning in those countries where the test was administered.

During the interview, the students were asked to give their opinions about the rules and to name other situations where these selection rules are applicable. In their answers, several commented that the rules applied to question (c) did not seem fair since both resulted in the least safe car being given the highest score, although they do accept that, in the case of cars, it makes sense to pick the car with the highest score. Moreover, they were able to identify these two rules with real situations and to come up with contexts, such as sports competitions, specifically synchronized swimming, ice skating, diving, dance contests, surfing and some television game shows, where said rules apply. They had problems inventing rules however, and most simply proposed summing the scores.

It was also apparent from the interviews that this was a new problem for them. They did not, for example, associate the scenario with functions, i.e. they did not see the activity as involving mathematics, such as finding the value of a function, and so they were not aware of any transfer of knowledge (Santos, 1997). They approached the problem as a novelty that they had never encountered in mathematics class.

CONCLUSIONS AND REFLECTIONS

The results from the study with third- and fourth-year secondary school students showed how they all completed the table with the scores for each car. They carried out a routine procedure given a direct instruction. We found that they understood the decision rules and, as expressed by those interviewed, were willing to apply them to their lives, though they did not see a connection to mathematics. Almost half of the students in our sample experienced difficulties in finding the right weights, and also failed to use the proper notation. In brief, it may be said that the students were able to read a table and to find the value of a function, but faced serious difficulties when assigning weights to reach a stated objective and said they knew of no method that would guarantee success.

The conclusions drawn from the data for the countries participating in PISA are a cause for reflection, in that while the students know how to apply rules to a data table, they do not know how to manipulate the data to reach a specified goal. It may be deduced from the OECD data, which agree with our own, that certain aspects of the educational system as it now stands are failing. Students do not have a procedure or a linear strategy they can apply, as they do when calculating an average or an expected value. Other studies (Cobo and Batanero, 2004) have shown that a large number of students experience difficulties when calculating and interpreting weighted averages in statistics. The correct weight when calculating an average involves the ability to apply the distributive property when adding a set of numeric values. Another valid analogy to demonstrate this problem is the “knapsack problem” or the “subset sum problem,” where given a weight or sum, the appropriate items have to be selected to fill the knapsack. This problem is NP-complete, but there are good heuristics that could be taught (Espinel, 1995). It seems students are not taught methods for selecting weights or values from among a given group to reach a total sum or weight.

We are currently designing and improving this and other activities so as to achieve the stated objectives of our research.

Of the three aspects considered in PISA (content, situation and competence), the “Best car” task is considered a public situation. It requires that the students resort to their mathematical understanding, knowledge and skills to evaluate the aspects of an external situation with repercussions in public life. Those competences or processes involved in the problem development are thinking, reasoning and argumentation (Niss, 2002). This is, in our opinion, a rich activity to take into the classroom (DeBellis and Rosenstein,

2004) since it covers a wide variety of mathematical fields, such as social choice, voting methods, social justice and the search for fair rules, and decision-making theory and the search for an optimum decision from a set of alternatives.

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