Teachers' beliefs about mathematical problem solving, their problem solving competence and the impact on instruction: A case study of three Cypriot primary teachers

Constantinos Xenofontos and Paul Andrews
University of Cambridge

Abstract
In this paper we report a case study of three Cypriot primary teachers, with respect to their mathematical problem solving beliefs and competence, and the impact of these on instruction. Semi-structured interviews were carried out with the teachers. Each of them was invited to solve a purely mathematical non-routine problem and explain simultaneously the solution process. The teachers prepared a lesson based on the problem and taught it to their classrooms. Our findings suggest that teachers' mathematical problem solving beliefs, competence and instructional practices are in a complicated relation that cannot be explained in terms of cause-and-effect.

Introduction
Mathematical problems and problem solving
In starting this literature review it is important, particularly as the word *problem*, even within the domain of mathematics education, frequently means different things to different people (Borasi, 1986; Blum & Niss, 1991; Nesher, Hershkovitz & Novotne, 2003; Wilson, Fernandez & Hadaway, 1993; Goos, Galbraith & Renshaw, 2000), to consider how it is defined. A common definition is that a mathematical problem presents an objective or goal with no immediate or obvious solution or solution process (Blum and Niss, 1991; Schrock, 2000, Polya, 1981; Nunokawa, 2005). In summarising the work of Schrock (2000) and Wilson *et al* (1993) we suggest that a mathematical problem must meet at least three criteria; individuals must accept an engagement with the problem, they must encounter a block and see no immediate solution process, and they must actively explore a variety of approaches to the problem.

According to Chapman (1997) problem solving means different things to different people, having been viewed as a goal, process, basic skill, mode of inquiry, mathematical thinking and teaching approach. However, most research in the area seems to regard problem solving as the process of achieving a solution (Chapman, 1997; Blum & Niss, 1991; Boekaerts, Seegers & Vermeer, 1995; Franke & Carey, 1997; Hart, 1993). Famously, Polya (1981:ix) described it as a means of "finding a way out of difficulty, a way around an obstacle, attaining an aim which was not immediately attainable" and it is on this conception that we focus our work.

Various writers, including Polya (1945), have developed frameworks for analysing the problem solving process. Polya's model comprises the four phases of understanding the problem, devising a plan, carrying out the plan, looking back. Other models, frequently based on Polya's, include Kapa's (2001) six phase and Mason et al's (1985) three phase. The latter suggest that problem solving comprises entry, attack and review. However, space prevents a lengthy discussion on the details of these models and their similarities and differences, although it is our view that their resonance with Polya's is close and not difficult to discern. From the perspective of this study, we tend towards Mason et al's (1985) model as their *attack* phase appears not to necessitate a predetermined plan in the manner of Polya's *devising and carrying out a plan*.

In the context of Cyprus, the location of this study, much research on problem solving has been undertaken over the last few years (Christou & Philippou, 1998; Elia & Philippou, 2004; Gagatsis & Elia, 2004; Gagatsis & Shiakalli, 2004; Nicolaidou & Philippou, 2003). However, the focus of every study has been on students' understanding, beliefs, abilities and attainment. Such work, when seen against the international trend for teacher-focused research on problem solving to address instructional effectiveness rather than teacher competence (Chapman, 1997, Thompson, 1985) or the impact of beliefs (Thompson, 1984) highlights well a field ripe for development.

In this paper we report on the mathematical problem solving beliefs and competence, and the impact of these on their instructional choices, of three Cypriot primary teachers. Beliefs, and their impact on teachers' instructional choices, have been the subject of extensive investigation in mathematics education. However, despite this extensive research, ambiguity regarding terminology has caused confusion to the area (McLeod, 1988; Pehkonen & Pietila, 2003; Törner, 2002). In this paper we draw on Raymond's (1997: 552) definition in which teachers' mathematics beliefs, in relation to the nature of mathematics, its teaching and learning, refer to their "personal judgments about mathematics formulated from experiences in mathematics".

Methodology

A case study investigation was undertaken in March, 2007. The participants, who were all teaching 11 and 12 year-old children at the time, were given the pseudonyms Mrs Antigoni (22 years of teaching experience), Ms Electra (newly appointed teacher) and Mr Orestis (second year of teaching). Initial semi-structured interviews focused on four thematic areas relating to how colleagues viewed themselves as teachers of mathematics, their espoused beliefs about the nature of problem solving, their perceptions of themselves as problem solvers and, finally, their beliefs about the management of problem solving in classrooms. After the interviews, each teacher was invited to solve a mathematical problem and explain simultaneously the solution process. The problem presented was the following:

"On the grid paper you have been given, each little square is equal to one square unit. How many isosceles triangles can you make which will satisfy all of the following three criteria?

1. *The area must be nine square units.*
2. *One of the vertices is at the given point.*
3. *The other two vertices are on grid points too."*

Problems of this nature, which are non-routine and purely mathematical (Blum & Niss, 1991) are never found in the National Textbooks of Cyprus.

The final phase involved each teacher in preparing and delivering a lesson based on the same problem. The lessons were observed with the ways in which the teachers presented the problem, managed the classroom during problem solving and their approach to responding to students' questions being the primary foci.

The aim was to 'sketch portraits' of the three participants, regarding their beliefs on problem solving, their problem solving competence and the impact of the former on their instructional practice. The work of Alba Gonzalez Thompson (Thompson, 1984) was particularly influential. Data analysis was framed by the three-phase problem solving model of Mason *et al*. (1985). The framework used for analysing classroom observation data (teachers' instructional practices during the teaching of the problem solving activity) was chosen *a posteriori* since we did not want to apply predetermined categories. For this purpose, six out of the ten mathematical didactics (table below), proposed by Andrews (2007), were used. The mathematical didactics are teaching strategies used by teachers in order to facilitate students' understanding.

Activating prior knowledge	The teacher explicitly focuses learners' attention on mathematical content covered earlier in their careers in the form of a period of revision as preparation for activities to follow.
Explaining	The teacher explicitly explains an idea or solution. This may include demonstration, explicit telling or the pedagogical modelling of higher level thinking. In such instances the teacher is the informer with little or no student input.
Sharing	The teacher explicitly engages learners in the process of public sharing of ideas, solutions or answers. This may include whole-class discussion in which the teacher's role is one of manager rather than explicit informer.
Coaching	The teacher explicitly offers hints, prompts or feedback to facilitate their understanding of or abilities to undertake tasks or to correct errors or misunderstandings.
Assessing or evaluating	The teacher explicitly assesses or evaluates learners' responses to determine the overall attainment of the class.
Questioning	The teacher explicitly uses a sequence of questions, perhaps Socratic, which lead pupils to built up new mathematical ideas or clarify or refine existing ones.

Table 1: Mathematical Didactics

Sketching the three portraits

A. Mrs Antigoni

Beliefs about teaching/learning mathematics and about herself as a mathematics teacher

For Mrs Antigoni, mathematics is the most important lesson in the primary curriculum. A 'good' mathematics teacher should give students many examples so that "*they can adopt some procedures easily in order to solve similar problems*". Also, he/she should accept critique, either good or bad, in order to improve. She believes that all of her mathematics teaching aspects require improvement. However, she does not make any allusions to how she could improve her mathematics teaching. She only refers to external factors that could change and therefore improve the conditions under which she teaches (i.e wishes that children would concentrate more during the lessons and that, pupils would have greater self-confidence). She claims that the quality of her teaching would be improved if the number of pupils per classroom decreases.

Beliefs about the nature of problem solving

A mathematical problem is "*a procedure, which requires you to discover which information is given, to rank the given points, to find what the problem asks you to do and then solve it*". The given information should be clear and accurate so that *all* the children understand what has to be done. Problem solving is the achievement of a goal, either set by the problem solver or by others. Teaching *via* problem solving can be feasible only if children know the mathematical procedures. Otherwise, if students do not know how to solve procedural exercises, they will be very disappointed and they will not have the motivation to try and solve more difficult problems. Both finding a problem's solution and the journey towards the solution are important. The correct answer matters a lot since "*in the future, students will be asked to take mathematics exams, for which they have to achieve a high mark. If the steps they follow are correct but still the answer is wrong, then they will not attain a good grade*".

Beliefs about her competence as a problem solver

Mrs Antigoni views herself as an average problem solver and is bored of problems which require a more advanced level of thinking because she has been tackling simple problems with simple solutions throughout the years of her teaching experience. Sometimes she views problems in a superficial way and often has to revisit a problem many times in order to understand what is required for its solution. Practice is for her the only way of improving her problem solving abilities. Difficult mathematical problems are a cause of anxiety to her. When she has to solve an advanced problem she tries to relax and concentrate on the given information. When she faces difficulties during the solution process, she either recalls previous knowledge regarding the solution of similar problems or tries to solve the problem algebraically. If the problem is to be taught to primary school children, she tries to simplify the solution, because "*children in primary schools do not know algebraic equations*".

The management of problem solving in her classroom

The school programme, according to her, does not allow any time for problem solving activities in classroom. Students should not spend more than three to five minutes on a problem. Her mathematics teaching is based on the national textbooks of the Ministry of Education and Culture. She brings mathematical problems in her classroom she creates herself. It is easier and more practical for her to write problems similar to those in the national textbooks rather than to search for them on the internet or in books. Most of her students feel nervous when they have to solve mathematical problems. Only few of them will try to reach a solution while the rest quickly abandon every effort. When students face difficulties during problem solving, she tries to give them some hints that will help them to proceed. Students should be encouraged by the teachers with expressions like "*well done, you are on the right track. Now try to think what else should be done*".

Solving the problem

Mrs Antigoni's *Entry* to the problem began by acknowledging the central goal of the activity (finding triangles that satisfy all three criteria) and by introducing the formula for the area of triangles. After finding four of the 12 triangles of the group of "base 6, height 3", Mrs Antigoni began examining another case instead of finding the rest of the triangles in the same group. Soon after that, she demonstrated her boredom and was ready to abandon the problem. She continued her effort, mostly because of my interventions. Eventually she found all 36 solutions, though with great difficulty. A wide range of emotions were observed; from her initial enthusiasm to the desire to abandon the problem.

The lesson based on the problem

Mrs Antigoni's lesson was a procedural explanation of the problem by the teacher. Children were given very little opportunities to demonstrate their thinking since the teacher was explicitly explaining every step during the teaching period. Two mathematical didactics were observed, *Activating Prior Knowledge* and *Explaining*.

Activating Prior Knowledge

At some point in the middle of the lesson, she asked students to bring to mind a previous lesson they had on symmetry. She suggested that students could use the rules of symmetry to find the triangles.

Explaining

Generally speaking, the lesson was based on the teacher's explanations and step-by-step guidance. Selectively, I present the two following selected incidences which confirm and justify my claim:

(a) When children were given the problem, the teacher told them to start by using the formula for the area.

(b) A student suggested that there could be a triangle with base 4 units and height 4.5 units. The teacher told him that they were to work only with integers and that decimal numbers could not be used. She did not offer any further explanations on this.

B. Ms Electra

Beliefs about teaching/learning mathematics and about herself as a mathematics teacher

Mathematics is for Ms Electra a very important lesson, since it is in the primary and secondary curricula of every educational system around the world. Its importance can be seen in daily life due to its many applications, like money, measurements and so on. She feels able to teach mathematics only if she is well prepared. She claims that she does not actually know mathematics, since she lacks advanced knowledge in this field. A 'bad' mathematics teacher is one who teaches procedurally and does not allow children to build their own conceptual understanding. A 'good' mathematics teacher approaches mathematics teaching in an interdisciplinary way, and tries to facilitate students' learning and to make children appreciate mathematics. She wishes she would feel more secure in teaching lessons based on non-routine problems.

Beliefs about the nature of problem solving

A mathematical problem is like every other problem; it is a situation that requires a solution. This solution can either be approached through a simple procedure or certain strategies have to be employed. "*There is a wide range of mathematical problems; at one end of the scale, there are very simple ones and at the other end, there are some which are much more complicated*". A mathematical problem has a verbal and a numerical part. The ways the given information and the question of the problem are expressed comprise its verbal part. The numerical part is included in the verbal, in the form of numbers and symbols. Problem solving is the procedure during which the problem solver understands the problem, separates the given information from what he/she is asked to achieve and uses all the elements of the problem in order to reach a solution. Teaching *via* problem solving is a method of direct approach of a mathematical concept and is a way of connecting a mathematical concept with daily life situations. Teaching *via* problem solving can be used as a way for introducing new concepts.The journey towards the solution of a problem is much more important than the solution itself. The solver has to find her/his own way which will eventually lead her/him to the solution. Only then has the problem solver really understood the problem and its solution and can explain her/his journey to others.

Beliefs about her competence as a problem solver

Ms Electra labels herself as an insecure problem solver. As she claims, "*solving problems is not something I enjoy doing and furthermore I do not have the skills for it*". If she does not *have to* solve a difficult mathematical problem then she is not interested in solving it. She would only try to solve a difficult problem if she had to present it to students and that would cause her anxiety. Mathematical problems in particular make her nervous, but "*if I really had to solve it, I would try and use every possible means, because I am a really stubborn and proud person*". When she faces difficulties during the process of solving a problem, she either asks for advice from someone more experienced or she temporarily abandons the problem until she relaxes and then revisits it and keeps trying until she reaches a solution.

The management of problem solving in her the classroom

Time parameters set by the school programme do not allow much time to spend on problem solving activities, states Ms Electra. Most of the problem solving activities she uses concern the introduction of new mathematical concepts. She does not spend time on problems if they are not related to a particular mathematical concept she wants to teach. The time a student should spend on a problem depends on her/his abilities and on the degree of difficulty of the problem. The majority of the students in her classroom do not like solving problems due to the insecurity children feel. Her class do not respond with enthusiasm when she presents them with mathematical problems. When her students face difficulties during the process of solving a mathematical problem, she helps each student independently. She asks her students questions that will help them to think rationally. She does not directly explain to children what to do; she prefers that they discover the way by themselves. Suggesting her students work in pairs is another method she uses. Students can learn a lot through discussion and exchange of ideas while working with mathematical problems.

Solving the problem

Ms Electra seemed a little nervous when she was told she would be given a mathematical problem. Nevertheless, during the process of solving the problem, she managed to control her initial anxiety and successfully navigated through the problem's difficulties. As soon as she realised that the initial way she was following was not appropriate, because of the three criteria, she revised her thinking and applied a new way. After discovering a pattern, she easily generalised her ideas and found all 36 solutions.

The lesson based on the problem

Ms Electra's lesson was mostly based on students' sharing and explaining their ideas to their classmates. Also, Ms Electra was particularly interested in offering individual and group feedback and assessing her students' understanding. This was achieved either through a group discussion, where the teacher was managing rather than explaining, or through questioning, where the teacher used to challenge her students with questions. Analytically, selected incidents confirming the mathematical didactics employed by Ms Electra are presented below.

<u>Sharing</u>

(a) A student argued that the height that begins from the vertex between the two equal sides of an isosceles triangle divides the triangle into 2 equal parts. The teacher asked the student to demonstrate his ideas on the board and explain it to his classmates.

(b) At a point where most of the students had found all of the triangles with base 2 and height 9 and base 6 and height 3, the teacher asked if there was anyone who would like to explain analytically her or his way of thinking to the others. One student went to the board (where a grid paper was presented though an overhead projector) and explained to her classmates how she found all the triangles with base 6 and height 3. In the meantime, the other students asked that student questions on some of her steps.

Questioning

The teacher asked the students to number the characteristics of isosceles triangles.

Student: *The sum of their angles is 180°.*

Ms Electra: *Is this a characteristic that only isosceles triangles have?*

Student: *No, it applies to all triangles.*

Ms Electra: *How important do you think this information is for our problem?*

Student: *We don't need it.*

Assessing and coaching

When students were working either individually or in groups, Ms Electra went round the class and asked them to explain what they were doing. Her help and feedback was very encouraging. When the students faced difficulties, she challenged them with hints that would lead them to discover their own way.

C. Mr Orestis

Beliefs about teaching/learning mathematics and about himself as a mathematics teacher

Mr Orestis views mathematics as a way of thinking. Mathematics learning and the acquisition of particular techniques/information are for him totally different. The learning of mathematics can help students in their daily lives. More importantly, though, mathematics helps the learner in developing a more organised way of thinking. He feels confident in teaching primary mathematics but he would like to improve his mathematics teaching abilities as regards the introduction of new concepts. He wishes he knew better ways of developing the conceptual understanding of his students, because "*a good mathematics teacher is one who helps students in building a conceptual understanding of mathematics rather than finding answers mechanistically*".

Beliefs about the nature of problem solving

A mathematical problem is a situation which leads the solver "*on a mental adventure*". The solver has to use the given information methodically in order to arrive at a certain solution.
There must not be a standard way to the solution and problem should be clearly posed so that it does not lead the solver to misunderstandings. Problem solving is the successful manipulation of the given information in order to find a solution. Teaching *via* problem solving is not feasible for all mathematical topics, despite the fact that it should be at the centre of teaching and learning mathematics. The journey towards the solution of a problem is much more important than its solution. It does not actually matter if the solver did not reach a correct answer because of some procedural mistakes when using the numbers and so on. "*The emphasis is on the mental processes required in order to arrive at the solution*".

Beliefs about his competence as a problem solver

Mr Orestis feels confident at solving mathematical problems, and particularly those which refer to primary mathematics. Experience is for him a very important factor for someone to improve her/his problem solving abilities. Difficult mathematical problems cause him some anxiety, but if he had to solve a difficult problem he would overcome his anxiety and would do his best to solve it. When he faces difficulties during the solving of a problem, he revisits the problem and the given information and tries to connect them in a way that will lead him to the solution.

The management of problem solving in his classroom

The school teaching program, as he claims, does not offer many opportunities for problem solving activities. The time that should be spent on a problem depends on two factors, the degree of the problem's difficulty and the students' abilities. A teacher should be flexible as regards time because he has to avoid creating the impression that "*the fastest problem solver wins*". A whole teaching period (40 minutes) should not be spent on a problem, unless there is one and only objective for that lesson. "*A problem that requires an entire teaching period in order to be solved must be very demanding and students will have to use advanced mental processes*". His students do not like mathematical problems and they prefer simple procedural exercises with only one particular answer and one particular way of arriving at it. When his students face difficulties during the process of solving a problem, he gives some hints so that the student can find their own way towards the solution. If he observes that some difficulties are common among the students, then he gives them group feedback.

Solving the problem

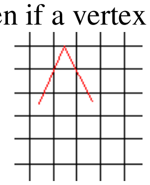
Mr Orestis was very calm when he was given the problem. Nonetheless, during solving the problem, he was methodical from the beginning. He identified all the pairs of numbers which could be the dimensions of triangles satisfying the three criteria. After drawing all the 12 triangles of the group "base 2, height 6", he generalised his idea and claimed that there were 36 triangles for the problem.

The lesson based on the problem

During the lesson based on the problem, Mr Orestis offered hints to his students on how they could proceed with the solution. However, a big part of his lesson, especially at the end of it, was mostly based on the teacher's demonstration of the solution. Selected incidents that justify the use of particular mathematical didactics are presented below.

<u>Coaching</u>

(a) The classroom was discussing the criterion of the problem which stated that all vertices should be on grid points. In Greek, there is no single word for "grid point", so the Greek translation of the problem referred to this as the "points where the horizontal and vertical lines of the paper intercept". The teacher asked the students if they understood what was meant. Some children argued that they were not sure. Then Mr Orestis drew the following shape on the board and asked the children if a vertex could be like that.



A student explained to her classmates that this could not happen. All children agreed.

(b) Mr Orestis asked the children: "How are we going to use the first piece of information that the problem gives us? 9 square units. Definitely, we are not going to draw triangles by chance".

<u>Explaining</u>

(a) A student asked if a triangle with base 4 and height 4.5 could be a solution. The teacher told him that only integers should be used, without offering any further explanations.

(b) At the end of the lesson, the teacher told the students that they should look at all the cases of triangles with certain dimensions. He chose to demonstrate the triangles with base 6 and height 3, since he had already presented some of those cases earlier. When the bell rang, Mr Orestis told the children that if anyone was interested in finding all the solutions, he/she should try at home.

Conclusions

Many researchers have examined the relationship between beliefs and mathematics achievement, and specifically the attainment regarding problem solving. However, as already noted, the foci of their studies were on students and not teachers. For instance, the study of Nicolaidou and Philippou (2003) has shown that there is a significant correlation between Cypriot primary students' attitudes towards mathematics, self-efficacy beliefs and performance on problem solving activities. Similarly, Mason's (2003) study, concerning Italian students, revealed that the strongest predictors for mathematics achievement were two problem solving beliefs (belief regarding perceived ability to solve time-consuming problems and belief that not all problems can be solved by applying step-by-step procedures). We assert, however, that the connection between teachers' beliefs about their problem solving competence and the observed competence is complicated and cannot be presented as a linear function.

The findings of the research reveal the complexity of the relation between teachers' problem solving beliefs and competence. Moreover, they show that the interrelationships of beliefs and competence on teachers' instructional practice are also complex with no simple cause and effect in much the same way as Thompson (1984) found with her three teachers. In closing we refer to Thompson's appeal for teachers (1) to experience mathematical problem solving from the perspective of the problem solver before they can adequately deal with its teaching, (2) to reflect upon the thought processes that they use in solving problems to gain insights into the nature of the activity and (3) to become acquainted with the literature on research on problem solving and instruction in problem solving. According to Cooney (1985), studies suggested that teachers may not possess rich enough constructs to envision anything other than limited curricular objectives or teaching styles and hence may be handicapped in realising a problem solving orientation. The use of a problem-solving approach demands not only extensive preparation but also the development of ways to maintain at least a modicum of classroom control and, perhaps most importantly, the ability to envision goals of mathematics teaching in light of such an orientation.

References

Andrews, P. (2007) Negotiating meaning in cross-national studies of mathematics teaching: kissing frogs to find princes. *Comparative Education, 43*(4), 489–509

Blum, W. & Niss, M. (1991) Applied Mathematical Problem Solving, Modelling, Applications, and Links to Other Subjects: State, Trends and Issues in Mathematics Instruction. *Educational Studies in Mathematics, 22* (1), 37-68.

Boekaerts, M., Seegers, G, & Vermeer, H. (1995) Solving Math Problems: Where and Why Does the Solution Process Go Astray? *Educational Studies in Mathematics, 28* (3), 241-262.

Borasi, R. (1986) On the Nature of Problems. *Educational Studies in Mathematics, 17* (2), 125-141.

Christou, C. & Philippou, G. (1998) The Developmental Nature of Ability to Solve One-Step Word Problems. *Journal for Research in Mathematics Education*, 29, No. (4), 436-442.

Cooney, T. J. (1985) A Beginning Teacher's View of Problem Solving. *Journal for Research in Mathematics Education*, *16* (5), 324-336

Elia, I. & Philippou, G. (2004) The functions of pictures in problem solving. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2*, 327–334.

Franke M. L. & Carey, D. A. (1997) Young Children's Perceptions of Mathematics in Problem-Solving Environments. *Journal for Research in Mathematics Education*, *28* (1), 8-25

Gagatsis, A. & Elia. I. (2004) The effects of different modes of representation on mathematical problem solving. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2,* 447–454.

Gagatsis, A. & Shiakalli, M. (2004) Ability to Translate from One Representation of the Concept of Function to Another and Mathematical Problem Solving. *Educational Psychology, 24* (5), 645-657

Goos, M., Galbraith, P. & Rensaw, P. (2000) A Money Problem: A Source of Insight into Problem Solving Action. *International Journal for Mathematics Teaching*. (April, 13th). Electronic Journal.

Hart, L. C. (1993) Some Factors That Impede or Enhance Performance in Mathematical Problem Solving. *Journal for Research in Mathematics Education*, *24*, (2), 167-171

Kapa, E. (2001) A Metacognitive Support during the Process of Problem Solving in a Computerized Environment. *Educational Studies in Mathematics*, *47* (3), 317-336.

Kelly, C. A. (2006) Using Manipulatives in Mathematical Problem Solving: A Performance-Based Analysis. *The Montana Mathematics Enthusiast*, ISSN 1551-3440, vol. 3 (2), 184-193

Mason, L. (2003) High School Students' Beliefs About Maths, Mathematical Problem Solving, and Their Achievement in Maths: A cross-sectional study. *Educational Psychology, 23* (1), 73-84

Mason, J., Burton, L., & Stacey, K. (1982) *Thinking mathematically*. London: Addison-Wesley.

Nesher, P., Hershkovitz, S. & Novotne, J. (2003) Situation Model, Text Base and What Else? Factors Affecting Problem Solving. *Educational Studies in Mathematics, 52* (2), 151-176.

Nicolaidou, M. & Philippou, G. (2003) Attitudes towards mathematics, self-efficacy and achievement in problem solving. *In*: M. A. Mariotti (Ed), *European Research in Mathematics Education III*. Pisa: University of Pisa.

Nunokawa, K. (2005) Mathematical problem solving and learning mathematics: What we expect students to obtain. *Journal of Mathematical Behavior, 24*, 325–340

Polya, G. (1945) *How to Solve It: A New Aspect of Mathematical Method*. London: Penguin Books Ltd.

Polya, G. (1981) Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving. New York: Wiley.

Raymond, A. M. (1997) Inconsistency between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice. *Journal for Research in Mathematics Education, 28,* (5), 550-576.

Schrock, C. (2000) Problem Solving-What Is It? *Journal of School Improvement, 1* (2).

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A. G. (1985). Teachers' conceptions of mathematics and the teaching of problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 281-294). Hillsdale, NJ: Lawrence Erlbaum.

Wilson, J., Fernandez, M., & Hadaway, N. (1993) *Mathematical problem solving*. Retrieved from University of Georgia, Department of Mathematics Education EMAT 4600/6600 Website: http://jwilson.coe.uga.edu/emt725/PSsyn/PSsyn.html