"USING MODELS AND REPRESENTATIONS IN STATISTICAL CONTEXTS" AS A SUB-COMPETENCY OF STATISTICAL LITERACY – RESULTS FROM THREE EMPIRICAL STUDIES [1]

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Statistical literacy includes basic competencies of using representations and statistical models. Research on these aspects of statistical literacy on the theoretical and empirical levels can help to describe students' competencies and the complexity of tasks. For this reason, this paper presents a hierarchical competency model for "using models and representations in statistical contexts" as a sub-component of statistical literacy. This competency model was used to design a corresponding test, which was administered to more than 220 secondary and 40 university students. The results support the competency model and suggest growth in competency across the age groups.

INTRODUCTION

Even though statistical literacy is known to encompass a broad spectrum of content aspects, several empirical studies suggest asserting a one-dimensional structure of competencies in this domain. Corresponding competency models and didactical considerations can contribute to a better understanding of the complexity of tasks and the description of students' competencies. However, some of the approaches are divergent. For instance, the complexity of manipulation of data by reduction (Kröpfl, Peschek, & Schneider, 2000) on the one hand and the understanding of statistical variation (Watson, Kelly, Callingham, & Shaughnessy, 2003) on the other hand are two different aspects which can describe students' competencies in the domain of statistical literacy.

Integrating these two approaches, we developed a competency model for "using models and representations in statistical contexts", which we present in this paper. This competency model covers a sub-component of statistical literacy. We used the competency model for the development of tests assessing students' competency in that domain. The aim of three corresponding empirical studies was to evaluate the tests and to gather first insights into the competencies of German secondary and university students. In these studies, the competency model was verified empirically. Moreover, competency differences between different age groups suggest that there are progresses of the students in the competency "using models and representations in statistical contexts" across the years.

In the paper, we will (1) give an overview on the theoretical background of the studies. After having (2) deduced the research questions, we give (3) information on design and samples. Having presented some central results (4), we will discuss the findings (5).

1 THEORETICAL BACKGROUND

Statistical literacy focuses – in a somewhat analogous way like mathematical literacy (cf. Organisation for Economic Cooperation and Development (OECD), 2003) – on abilities of the learners necessary for participation as responsible citizens capable of critical evaluations of information presented by statistical representations or by using statistical models. As Wallman (1993, p. 1) describes, "'Statistical Literacy' is the ability to understand and critically evaluate statistical results that permeate our daily lives – coupled with the ability to appreciate the contribution that statistical thinking can make in public and private, professional and personal decisions". Moreover, specific metacognitive, volitional and motivational aspects should be included in competency models for statistical literacy. This is analogous to the notion of mathematical competency and can be seen in line with the approach by Gal (2004), who emphasises the importance of attitudes, beliefs and a critical stance.

Models describing requirements of statistical literacy can help to structure observations of deficits in statistical literacy (cf. e.g. Batanero, Godino, Vallecillos, Green & Holmes, 1995; Shaughnessy, 1983). Such deficits can consist in problems when interpreting representations of data, which are based on the use of elementary statistical models aiming at reducing the original information. Kröpfl, Peschek, and Schneider (2000) argue that the aspect “dealing with manipulations of data by reduction” can be used as a criterion for describing requirements in the domain of statistical literacy. This aspect can be regarded as a requirement of dealing with information given in the form of data, emphasising the idea that overviews often can only be generated by reducing the information contained in the original data set. For example, a mean value does not contain information about the broadness or variance of a distribution any more.

Another facet of dealing with statistical information can be seen in considerations by Watson and Callingham (2003; cf. also Watson et al., 2003): According to this approach, the understanding of statistical variation is an important component of statistical literacy. Deterministic viewpoints for instance, as observed also by Engel und Sedlmeier (2005), can be an obstacle for interpreting statistical data containing variation. Considering the understanding of statistical variation as central criterion, Watson and Callingham (2003, cf. also Watson, 1997) postulated a hierarchical concept of statistical literacy, which was verified empirically with a test instrument and codings verifying a one-dimensional Rasch model.

Regarding content areas related to statistical literacy, Holmes (1980) identified five main components: data collection, data tabulation and presentation, data reduction, probability and the field of interpretation and inference. Even though this indicates a broad range of possible aspects in the domain of statistical literacy, there are overarching ideas helping to create indicators for components of statistical literacy, such as the understanding of statistical variation introduced above (cf. the results of Watson & Callingham, 2003). Another such idea is “graphical representations of data and manipulation of data by reduction”, a criterion which can even represent empirically the competency spectrum of statistical literacy (Reading, 2002). An overarching idea can also be seen in the metaphor of data-related “reading”. This metaphor can be understood to include dealing with statistical variation and manipulations of data by reduction. As a starting point, the approach of Curcio (1987) can be cited to describe the metaphor of data-related “reading”. Curcio defines three levels of competency, 1-“reading the data”, 2-“reading within the data”, and 3-“reading beyond the data” in order to classify requirements when working with statistical representations like diagrams. Even though there are intersections with research approaches of “reading as a social practice” and linguistic reading competency (cf. Luke and Freebody, 1997; OECD, 2003), we understand the metaphor of data-related “reading” as broader, including considerations on dealing with underlying models (cf. Kuntze, Lindmeier & Reiss, in press). Indeed, own modelling abilities (Blomhøj & Jensen, 2003; Maaß, 2006) often play crucial roles when learners have to produce or to make sense of statistical representations. This seems to be the case in particular when statistical variation comes into play. Accordingly, in the domain of statistics, it seems almost impossible to completely separate modelling from dealing with representations. As argued above, both aspects are linked to the metaphor of data-related “reading”.

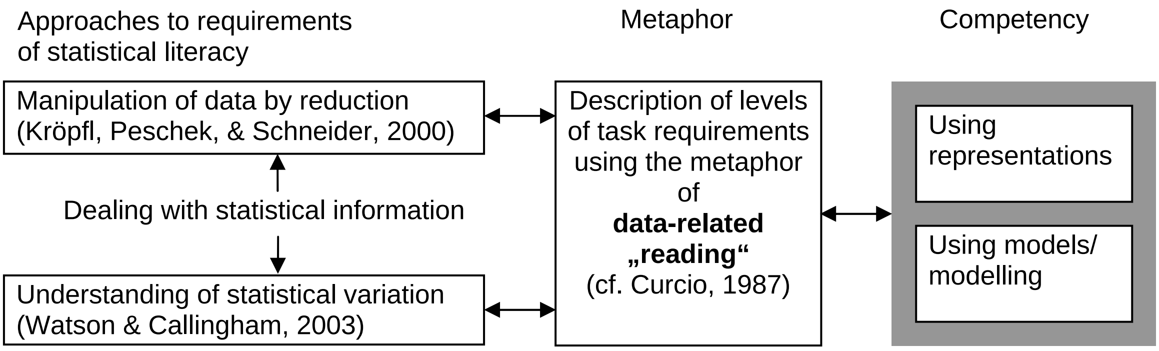


Figure 1: Aspects of requirements in the domain of statistical literacy linked to the competency “using models and representations in statistical contexts”

Accordingly, and in the sense of the overview in Figure 1 on the considerations made above, we deducted a competency model for “using models and representations in statistical contexts" consisting of different levels of competency. Table 1 gives an overview by providing short descriptions of the levels of competency. For this competency model, the recently introduced German standards (Kultusministerkonferenz (KMK), 2003) were used as an additional reference: against this background, the competency model described in Table 1 covers sub-areas of "modelling" and "using representations" in the domain of "data and chance". The competency model is an extension of the model presented in Lindmeier, Kuntze and Reiss (2007) and Fröhlich, Kuntze und Lindmeier (2007) by the forth level of competency. A sample item for the first and fourth level of competency is given in Figures 2 and 3.

Table 1: Levels of competency for using models and representations in statistical contexts

Level I	One-step use of a representation or work within a given model (e.g. reading a given value from a diagram, completing a given diagram for given data)
Level II	Two- or multi-step use of representations or changing between two given models (e.g. comparing data including a transformation step or referring to a mathematical concept)
Level III	Multi-step use of representations including the use of a non-given model (e.g. own modeling activities supporting a cumulative interpretation of data given in diagrams)
Level IV	Multi-step use of representations and/or use of a non-given model requiring an adequate dealing with statistical variation, respectively (e.g. own modeling activities on the basis of diagrams that require dealing with statistical variation)

2 RESEARCH QUESTIONS

Based on the theoretical background described above, a core aim of this research is to examine the competency model shown in Table 1 empirically. For this purpose, it is of particular interest whether a set of test items designed to fit to the competency levels supports the hierarchical concept asserted theoretically. In particular, we liked to find out whether the competency "using models and representations in statistical contexts" is empirically one-dimensional.

A second research interest was to gather first evidence about the competency of learners of different age groups in the domain of using models and representations in statistical contexts. Even though there are results that competency growth might be limited (cf. Serrano, 1996, cited by Batanero, Godino & Roa, 2004, p.7; Engel & Sedlmeier, 2005), we asserted that the learners' competency increases during secondary schooling as found by Reading (2002).

In short terms, the study aimed at providing evidence for the following research questions:

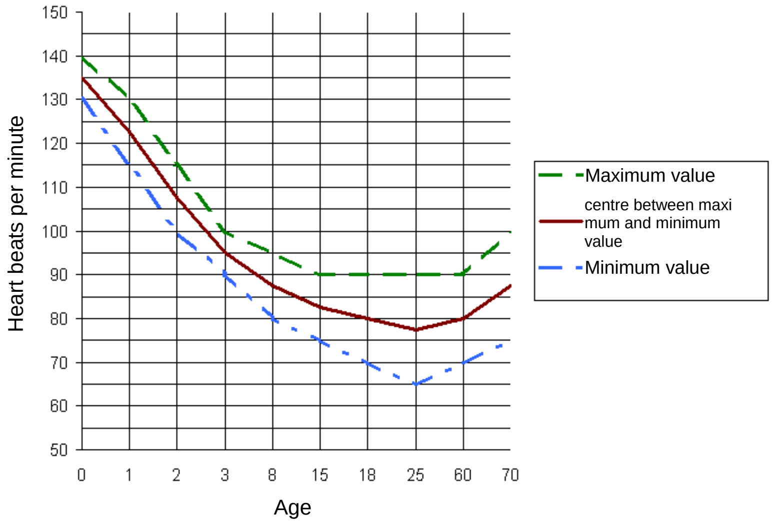
1. Is it possible to describe the test by a one-dimensional Rasch model and are the predicted competency levels verified empirically?
2. What competencies can be observed for fifth- and eight-grade students, as well as for university teacher students?

3 DESIGN AND SAMPLES OF THE THREE STUDIES

FIRST STUDY

For the first study, a test including items for the competency levels I, II and III was developed. This test was enlarged by three reference items from the TIMS Study corresponding to the three levels of competency. A sample item of this test for competency level III is presented in Lindmeier, Kuntze and Reiss (2007), a sample item for competency level I is given in Figure 2. The test was administered to grade 5 and grade 8 students in two German academic track secondary schools. There were N=187 students (99 female, 88 male) participating in the study. 112 of these students were 5^{th}-graders (52 female, 60 male) and 75 were 8^{th}-graders (47 female, 28 male).

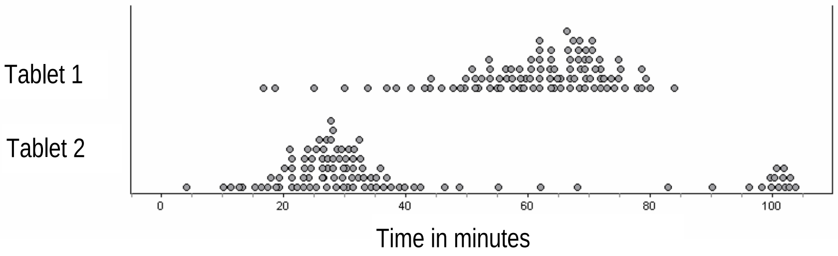
Kevin has found the following diagram about the pulse rate of humans right after waking up, the so-called resting pulse rate.



Note down the maximum and minimum value for a 15-year-old!

Figure 2: Sample item for level of competency I (first study)

A company produces two sorts of headache tablets. Both sorts have been tested in a laboratory with respectively 100 persons suffering from headache. The diagram below shows, how long it took until the headache was over. Each point represents one test person.



Dr. Green: Tablet 1 is the better one!

Find counter-arguments!

No, because ________________

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____

Dr. Jenkins: Tablet 2 is the better one!

Find counter-arguments!

No, because ________________

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Figure 3: Sample item for level of competency IV (second and third study)

The answers of the students to the test items were coded in a dichotomised way according to their correctness (one code per item; 1: correct solution; 0: incorrect solution). The coding was done separately by two raters. A total score was calculated for each student summing up the scores of the items. Moreover, partial scores on the different levels of competency were calculated.

SECOND STUDY

The sample of the second study consisted of N=227 students in grade 8 (112 female, 108 male, 7 unspecified due to missing data). These students came from 9 classes of two German academic track secondary schools. For both schools, these were all of the classes of this grade.

In the second study, two parallel tests were developed, containing items of all four competency levels (cf. Tab. 1) in an analogous way, respectively. A sample item for the level of competency IV is presented in Figure 3. For an appropriate answer, an understanding of statistical variation is required.

THIRD STUDY

The third study focused on the competencies of 40 university students. The sample consisted of prospective teachers (31 female, 9 male), who had not had any previous statistics or probability course on the university level. There was a reference group consisting of N=115 secondary school students (8^{th} grade, 60 female, 52 male, 3 unspecified) having worked with the same set of items like the university students.

4 RESULTS

FIRST STUDY

The results of the first study support the hierarchical order of the competency levels and conform to a one-dimensional Rasch model (for details cf. Lindmeier, Kuntze & Reiss, 2007). A comparison with the anchoring items from the TIMS Study gives a possibility of making an empirical link to TIMSS results. With the exception of one of the TIMSS items which resulted to be not as complex for the students as expected, all the TIMSS items fitted to their theoretically predicted level of competency.

Figure 4 shows the results for the three levels of competency of this test.

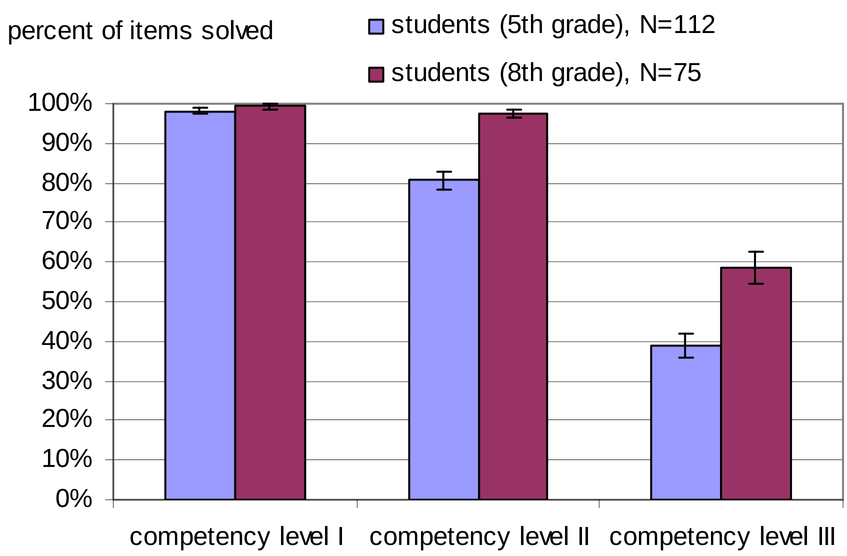

Figure 4: Percentages of correctly solved items on different levels of competency (first study)

As far as the student's competencies especially in grade 8 are concerned, a ceiling effect was observed. As shown in Figure 3, these students solved more than 90% of the items on the levels of competency I and II correctly. However, the competency results for students in grade 5 are significantly lower. As far as the total score is concerned, the grade 5 students solved on average 73.7% of the items (SD=14.2%) for the competency "using models and representations in statistical contexts". The grade 8 students attained a mean score of 86.9% (SD=11.0%). These mean values differ significantly from each other (T=7.13; df=181,10; p<0.001; d=1.04), showing a strong effect. Also for the levels of competency shown in Figure 4, there are highly significant differences for the levels of competency II (T=6.89; df=155.91; p<0.001; d=0.95) and III (T=4.09; df=185; p<0.001; d=0.60).

SECOND STUDY

Consequently, in the second study (cf. Kuntze, Lindmeier & Reiss, in press), a test including the full spectrum of items from all four levels of competency was administered to 227 high-attaining students in grade 8 (German "academic track", 112 female, 108 male, 7 unspecified due to missing data). Again, the results support the one-dimensional model, which could be established using Rasch-analysis.

Another result of the second study was that the items of the two parallel tests showed a consistent hierarchical pattern also with respect to the anchoring items contained in both of the tests.

As far as the competency values are concerned, there was no ceiling effect like it had been observed in the first study. The results indicate that basic competencies especially according to the first and second level of competency were available for a big majority of the students. However, tasks requiring an understanding of statistical variation showed much lower rates of correct answers. The data of a sub-sample of the 8th- graders participating in the second study is presented in Figure 5, as this sub-sample was also considered as a reference group for the third study. More detailed information is given in Kuntze, Lindmeier & Reiss (in press).

THIRD STUDY

The *third study* aimed at gathering first evidence about how the competency of "using models and representations in statistical contexts" might evolve through secondary schooling till adulthood. 40 university students (prospective teachers, 31 female, 9 male) were asked to answer the same test like a sub-sample of the grade 8 students in the second study. The university students had not had any previous university course in the domain of statistics and probability. The results indicate higher competency values of the university students. Looking at the total scores, the university students reached a medium score of 72.9% of all items (SD=11.3%), whereas the grade 8 reference group students solved 61.7% of the items correctly (SD=13.5%). The difference corresponds to a strong effect (T=4.67; df=153; p<0.001; d=0.89). However, the rate of correct answers of the university students to tasks requiring an understanding of statistical variation on competency level IV was still less than 50%.

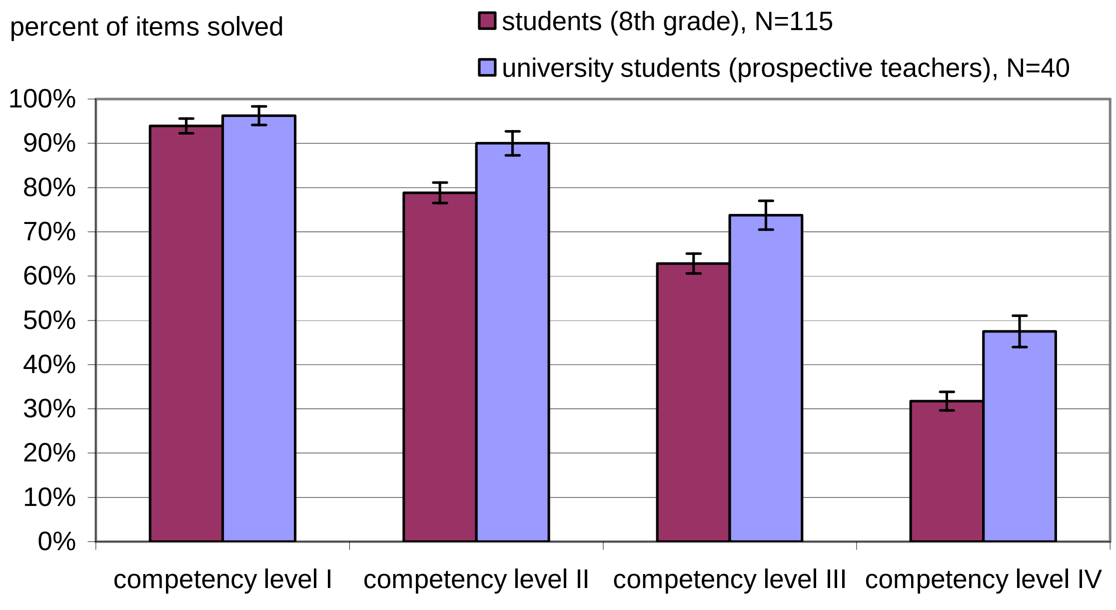

Figure 5: Percentages of correctly solved items on different competency levels (third study and sub-sample of the second study)

For the results on the levels of competency shown in Figure 5, the differences on the levels II (T=3.13; df=97.78; p<0.01; d=0.52), III (T=2.78; df=80.04; p<0.01; d=0.49) and IV (T=3.78; df=153; p<0.001; d=0.70) are highly significant, showing medium effect sizes.

5 DISCUSSION

The results of the three studies indicate that the competency model was verified empirically.

Moreover, first empirical evidence for the competency "using models and representations in statistical contexts" of German students was gathered. The results suggest that some basic abilities linked to statistical literacy are available for a big majority of the learners of the sample. However, tasks requiring an understanding of statistical variation seem to remain challenging to many students. This seems to be the case also for the university students participating in the third study.

As an additional result of the studies, the test instrument for the competency "using models and representations in statistical contexts" can be used now for further studies. A corresponding research interest consists in evaluating learning environments using the test instrument. For example, in a current research project we evaluate a computer-based learning environment allowing self-regulated learning processes (cf. Kuntze, Lindmeier & Reiss, in press). The test development and the evaluation of the test is a prerequisite providing us with an instrument for examining the competency development of the students working with the learning environment.

For future studies, it would be particularly interesting to compare the competencies of students in different German secondary school types, as only high-attaining students took part in the first and second study.

Furthermore, an adaptation of the test for comparisons on the international level also in relation to existing test instruments could contribute to a more complete empirical picture of statistical literacy. In particular, it would be interesting to strengthen empirical links to research on graphical thinking competencies (e.g. Wu, 2004). As already described in the theoretical background section, these competencies of data-related reading and the evaluation of graphs on the one hand and "reading" with an understanding of statistical variation on the other hand are two central aspects of statistical literacy.

NOTES

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