PEDAGOGICAL CONTENT KNOWLEDGE OF A NOVEL TEACHER: A CASE FROM THE TEACHING OF STATISTICAL GRAPHICAL REPRESENTATION

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This study examined the pedagogical content knowledge (PCK) of a novel teacher, specifically on her knowledge of instructional representations of a specific topic: statistical graphical representation. For the purpose of this study, a content analysis was used to determine a System of Dimensions and Indicators of the PCK in order to organize and structure the variety of characteristics and concepts of its components. The results reveal that teacher PCK is influenced by teacher conception of mathematics and, results on the use of a limited repertoire of educational strategies for the teaching of graphical representation.

Keywords: pedagogical content knowledge, statistical graphics, data representation, teaching statistics

INTRODUCTION

From the pioneering work of Shulman (1986), pedagogical content knowledge (PCK) has provoked a great interest as a model to improve teachers' training (Scharton, 2007) and as an object of study (Lancaster, 2007). Although PCK has raised enough interest in Mathematics Education for some years, it has been a theoretical construct little explored in Statistics Education (Batanero, 2002; ICMI/IASE, 2006, Sorto, 2007).

Defined as "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstration – in a word, the ways of representing and formulating the subject to make it compressible to others… includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (Shulman, 1986, p.9), PCK conceives a new didactic perspective of teaching that consideres not only direct instruction and knowledge of general didactic but it searches teacher's understanding of what it has to be learn, how concepts have to be taught and the wisdom of practice, from the understanding of how students learn, understand, solve problems and develop their critical thinking about a particular content (Shulman, 1987).

The review and analysis of the theoretical framework of PCK, highlights a comprehensive overview of the elements that underlie the theory proposed by Shulman and colleagues (Shulman, 1986 and 1987; Shulman & Sykes, 1986) which is also assumed by other researchers (eg Chinnappan & Lawson, 2005; Chen, 2004; An, Kulm & Wu, 2004). It allows a specific view of the knowledge of the didactic of a specific content that according to Pinto & Gonzalez (2006) is not designed to remove, ignore or devalue other components of knowledge in the task of teaching, but for defending, proposing and justifying a body of knowledge on the specific content amalgamated with each other to fill the vacuum of the knowledge of a teacher about a specific subject. PCK literature has grown significantly in the last twenty years thanks to the development and research on teacher's knowledge, and it has led to a widespread interest in the understanding of its conceptual nature and the learning how to study teacher's PCK. This has represented a challenge for researchers, as teacher's cognition can not be observed directly; by definition, PCK is a construct internally complex, difficult to measure or evaluate (Baxter & Lederman, 1999) that consists of what teachers know, what they do and the reasons for which teachers do things.

PCK allows for a specific view about the knowledge of the didactics of a subject matter, which incorporates, according Shulman (1986), at least three basic components: the knowledge of the subject to teach, the knowledge of instructional representations and the knowledge of students. This paper presents a study about the PCK of a novel teacher on her knowledge of instructional representations.

McDiarmid, Ball &Anderson (1989) use the term *instructional representation* to name "a wide range of models that can communicate the learners something related to the subject, such as: activities, questions, models and analogies, for example". They state that with this knowledge, the teachers create units of instruction, make sheets of work, design activities, develop explanations, think of questions and respond to their students. Sometimes these are modified ideas of other representations, curricular texts, courses or workshops for teachers, shared experiences with colleagues, but other times are their original inventions. We will centre specifically on the knowledge of instructional representations of a specific topic: statistical graphical representation. We will try to describe, from what the teacher knows about graphical representation, how this knowledge is transformed in the classroom into knowledge that can be taught and how these representations are used.

Graphic representation is an object of study and interest at all levels of statistics education. A statistical graph is a "construct which was developed in specific cultural contexts to mediate interpretation of data… an activity which is related to a complex range of elements and processes" (Monteiro & Ainley, 2006, p.1) and it is considered as a reasoning tool to learn something new about the context it represents, gain new information or learn from the data (Pfannkuch, 2006). For this reason, teachers should develop PCK that will help them design activities to increase the conceptual understanding of statistical graphs in their students, based on the relationships between the main components of the graph and the necessary process of its interpretation (Friel, Curcio & Bright, 2001). However, several studies in statistics education (Burgues, 2002; Sorto, 2004, Monteiro & Ainley,2006) have concluded that the problems about graphic representation, the conceptual mistakes, the difficulties to explain an abstract idea in simple terms and not to be able to anticipate or correct students mistakes or difficulties of thought, as well as a limited repertoire of specific strategies to maximize learning, clearly exemplify the poor preparation of teachers of statistics. It is important, therefore, carry out studies that will help them understand more how to learn to teach in the school (Shaughnessy, Garfield & Creer, 1996; Lee & Nelson, 2004).

METHOD

This research, was qualitative court using a case study approach centred on the knowledge of an university teacher (Mrs. Alice), with exclusive training in Mathematics, who teaches Statistics to students of Education. Mrs. Alice (in the successive CA making reference to the Case Alice) has three years of teaching experience in Statistics, has a Master's degree in Mathematics and her interest is in abstract mathematics. In addition, she is a teacher committed to teaching those who likes to learn.

Data collection included different techniques: a) contextual and biographical interview on the planning of the classes about statistical graphic representation (E1), b) didactic questionnaire on graphic representation (Cases 1 to 4), c) in depth interview about her responses to the questionnaire (E2), d) analysis of class materials for the teaching of graphic representation, including class notes (Not) and the field (Cam), exercises (Eje), exams (Exa) course program (Pro), textbooks (Bib), students notebooks (Lib), presentations (Pre), Web pages used (INT). Interviews were taped and transcribed entirely to be analyzed later. The uses of these different procedures allowed for triangulation.

The didactic questionnaire dealt with graphical representation. It consisted of four hypothetical situations of teaching and learning, in which graphs of Stem and Leaf, Histogram, Bars and Pictogram, were respectively used. These situations were designed from cases where students were expected to identify conceptual errors in the construction and interpretation of graphics, select, criticize, evaluate and solve problems about the writing, interpretation and communication of results from the graphic. For its design, the learning objectives of the graphical representation at the level of statistical thinking were defined according to Garfield (2002); a data base of 66 items that measured graphical representation at different cognitive levels was identified and elaborated; a questionnaire was firstly administered to students in order to explore mistakes, conceptions and conceptual difficulties; the type of questions addressed to the teacher to explore her PCK were identified according to a System of Dimensions and Indicators of the PCK.

How the system was built? Firstly, a conceptual and methodological review of research in mathematics education regarding the teacher's professional knowledge was conducted, followed by an analysis of the content using an empirical approximation, inductive – deductive (see Figure 1), to analyze the components of PCK. Thus, a System of Dimensions and Indicators (SDI) of the PCK was determined (Pinto & Gonzalez, 2006); this system includes three categories: the content knowledge of the subject to be taught, the knowledge of instructional representations and the knowledge of the students.

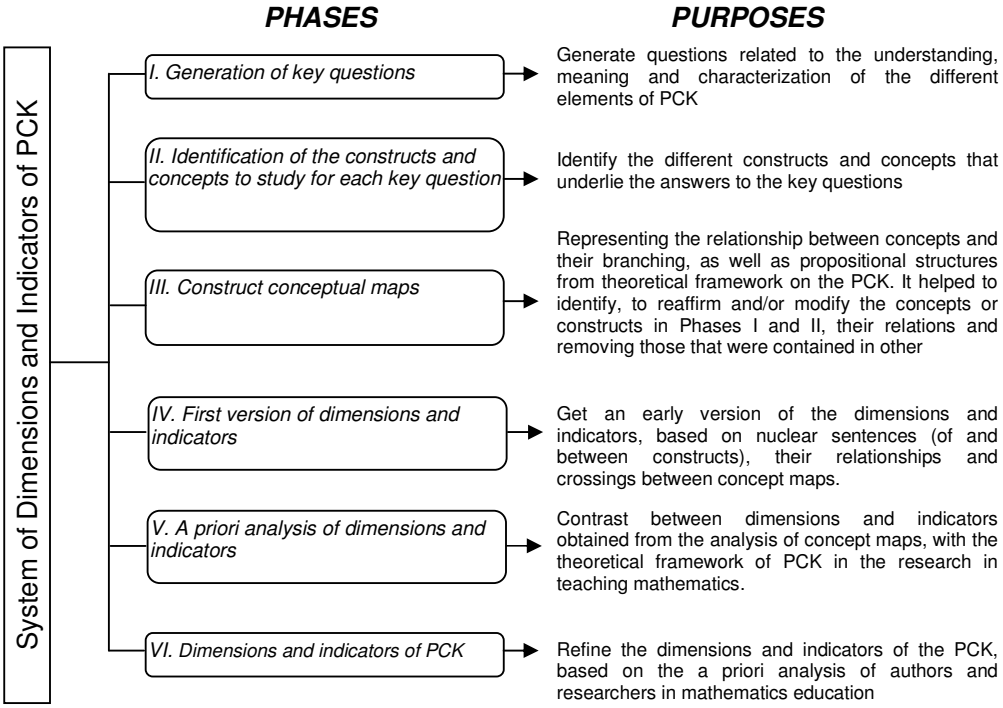

Figure 1. Phases and purposes for the construction of the SDI of the PCK

This SDI allowed for identifying and analyzing the meanings underlying the different components and features of PCK that have been studied in math education, to estimate conceptual links generated by research and to define our object of study.

RESULTS

During the process of data collection and the interaction with CA, were identified important aspects of PCK on graphical representation in statistics. Specifically, the study focused on some dimensions and indicators about the knowledge of instructional representations. First we will centred in the conceptions of CA on the meaning of learning and teaching graphic representation, teachers´ conception about students difficulties and the usual manner in which she teaches. Secondly, we analyse the CA's knowledge of the statistics curriculum and particularly on statistical graphing, we focused on the exploration of the purposes, goals, the teaching planning and the emphasis that she gave to the activities used in classroom. Finally, we explore the knowledge of instructional representations: activities, questions, examples and tools used for teaching graphic representation.

Conceptions about learning and teaching graphical representation

To learn Statistics, CA considers that is important for students to "discover, and dealt with some doubts, some questions" (E1.CA.27). Learning statistics mean:

"Let them know the meaning of the word statistics, what is it, for what does it serve, and more than anything else, they must understand the concepts and apply them. That is what I am worried about the most, that they can apply those concepts in their daily lives and in their professional practice "(E1.CA.101).

According to CA, a student that study Education must learn Statistics because "he has to work with information… use it well and not just manage it well, but also they must know how to get that information" (E1.CA.51). She said that statistics is important for students of Education because they need to look for, collect data and draw conclusions from the information, but when referring to "information" she did not specify what this means.

To achieve this purpose, CA performs the following sequence: asks the students to do exercises, checks whether or not they are correct in order to see what difficulties the student has (either individually or in group), and finally with those students who have difficulties she makes them questions to try for them to identify their own errors (E1.CA.102, Case 1 and 2). The same is done when she teaches graphic representation (E1.CA.103).

CA says that "interpretation in general" is what makes it harder for students. That students do well in the operational aspect (i.e. make calculations and construct graphs), but "if you change the problem or if you ask the questions differently or expect for them to interpret the graph or ask them to tell you what they have just made… they can't explain. Some good students can, but not all; I feel that this is one of the [difficulties]… interpretations" (E1.CA.95). However, when asked about the specific difficulties that her students present on the topic of graphical representation, Alice couldn't identify them. The same thing happened when asked about the reasons that students had to give certain answer to specific questions (Case 1, 3 and 4 and E2.CA.11 and 50):

"Mmm [she was thinking] I wouldn't know to tell you in representing data [which makes it difficult]… Mmm ... No, there is not [I have not identified what is most difficult for them] "(E1.CA.97-98)

"[She was thinking] I do not know (mmm). I'm still surprised at the response they gave ... It is what I said to you, I am thinking about it, if I saw this, I think that would be very hard for me thinking that answer "they fail to distinguish between one graph and another" (E2.CA.34)

CA's conception about statistics teaching is linked to her answers and the way she thinks about certain concepts. Repeatedly, CA said that she likes her students work during the lessons, because she thinks, in that way, she can monitor what they are doing and they have an opportunity to ask her questions and clarify their doubts, so she can identify their difficulties they face when doing exercises because she can ask them exploratory questions. Similarly, Alice uses frequently as a resource to make intercalated questions (Case 1, 2 and 4), in order for students to understand the usefulness of the concept, its value and meaning. According to CA, she looks first to relate the concept to something they already know, i.e.:

"In this sense the average ratings are familiar to them. Then, we may ask them 'Well, when you get your average grade, which is this meaning? Why do you use only one grade? During the period the schools demanded from you to choose a subject, how did you pick that grade?'… They began to say ' because there were so many exams, they were to be graded in this way and then we got this final grade, " and why they considered only this score? ". (E1.CA.38).

Upon completion of the sentence, "my method or strategy to teach statistics is…" Alice replied "I explain and give examples" (E1.CA.50). Figure 2 represents what she usually does in class (E1.CA.41-45), which keeps correspondence with the manner in which Alice plans all her classes (E1.CA.71): 1. She consults the lessons in the program; 2. She reviews bibliography; 3. She analyzes how appear the topics in different texts; 4 °. She tries to find examples applied to the education field and 5 °. She selects examples to be used with students.

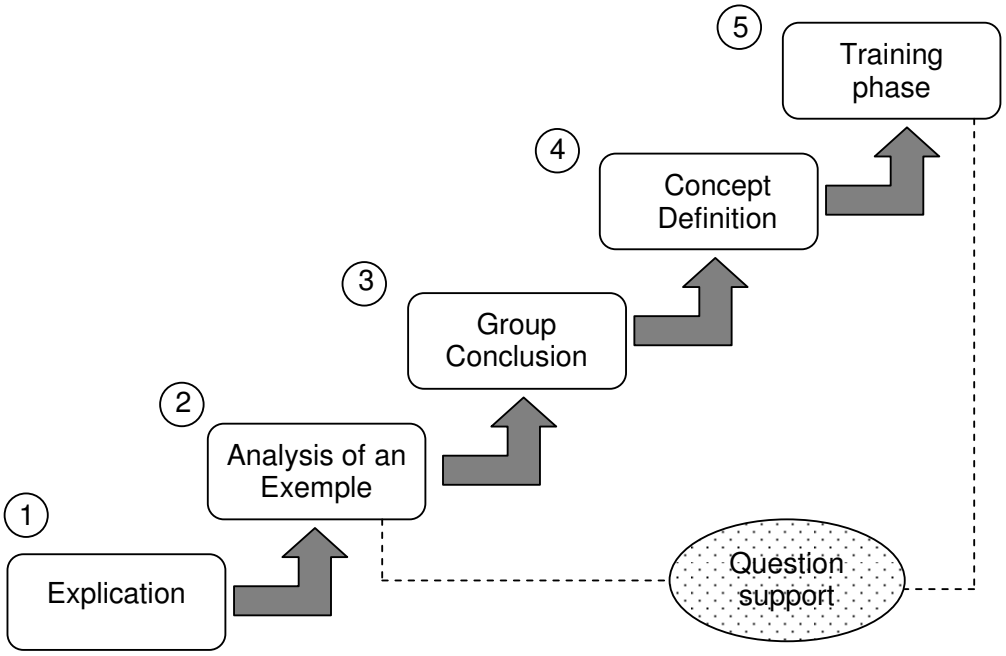

Figure 2. Habitual manner of CA's lessons

How she chooses the exercises and examples that she uses with her students? Alice has the following sequence (E1.CA.72 in 77 Cases 1 and 4): *First*, she looks for exercises that address the meaning (concept definition); *second*, she shows their use (application) in a way that its easy for students to understand; *third*, she gives them exercises to apply what they have just seen (exercise) and *fourth*, she finds and gives them exercises to teach different ways to resolve them.

Mrs Alice´s Knowledge of the curriculum of statistics and of graphical representation

According to CA, the main goal of statistical learning is that students of Education become aware of the meaning of the subject, of the concepts and in some cases they may apply. In contrast, the goal of the subject statistics is the interpretation of information obtained from an educational research (Pro.CA.01).

In dealing how CA conceptualizes the teaching of graphic representation, she said firstly what she hoped to accomplish with her students' achievement of the subject:

> "... Well if they have the data at hand they may know what kind of representation is the best to the specific problem on one hand; in addition, if you just have a representation, they must deduct and know what is it about this representation "(E1.CA.66)

When she was requested to provide an example of what she does in the classroom to achieve this purpose, she said "it is difficult to find examples related to the area of Education" (E1.CA.68). She recognised the need to find suitable examples for students of Education and she doesn't spend much time in class for the graphical representation (E2.CA.16, 60, 95, 116).

Specifically about the analysis of the curriculum of graphic representation, Alice doesn't know why some graphics are not included in the program, as the Stem and Leaf graphic: "... I had never asked. No ... I do not know exactly "(E2.CA.16). On the other hand, she recognises in several moments that certain contents and the development of certain cognitive levels of graphic representation are not achieved and that she does not provide a variety of exercises on different cognitive levels in the planning of her classes.

> "I think we did not do any exercise of that style that they may choose what it is that ... Only the advantages or disadvantages of each of them depending on what it was directed

towards. Basically we study definitions, we practice statistics examples, but we didn't emphasize which would be the best alternative in each case. At least I don't work on this point "(E2.CA.27)

In summarizing, CA teaches graphic representation in the same way she teaches on other statistical topics. Generally she emphasizes the construction of the graph and asks questions related to what students have "understood" of it, without specifying the meaning or the depth of this kind of questions. She focuses on the student to "take into account" the information of the graph, to identify descriptors of the graph and to analyze its elements for, so they can reconstruct it in order to make it clear for readers to know what they want to translate from the graph. She does not take into account the cognitive levels of graphic representation beyond statistics literacy, primarily because she isn't looking for the student to propose and solve problems involving the collection and analysis of data in a proper real context, or to evaluate arguments that were based on analysis of graphical representations or to identify conceptual and procedural errors in the selection, construction, interpretation and writing of graphic representations.

Knowledge of the strategies and of instructional representations

Based on the problem-situations and the in depth interview, we identified that Alice knows and uses three strategies to teach statistics: 1) setting definitions, 2) providing examples (sometimes by as way of demonstration), and 3) making explanations to revise characteristics of the concept.

E: What takes from you more attention [of the case]? (CASE 2).

A: I usually work with the definitions and say "good with these variables we use that, and with such variables such graphic" I… am imperative at that point so I say "this is for that and this is for that" and they did not question that because I am the teacher.(E2.CA.61)

This was reflected equally when we talk about graphical representation:

"No! In fact I don't work very much with graphics. It is basically the definition, some examples and some features such as the type of variable, the information, but I don' make a profound study" [of them] (E2.CA.96)

This situation is similar to Alice knowledge about specific strategies to teach graphing, she rarely:

- Tries to work with some specific examples, because it is difficult to find those that are appropriate to an educational context (E1.CA.67-68)
- Provides information to students in order for them to determine which is the best representation in each case (E1.CA.69, Case 2 and E2.CA.41 and 97)
- Uses diagram examples (i.e. histogram) to see if the students handle well the scales of measurement and are able to distinguish the differences between groups (E1.CA.70 and E2.CA.71)
- Seeks for students to emphasize what kind of information is given, why something must be done in order to avoid that making the graphing something mechanical (E1.CA.73-78 and E2.CA.27).

About her knowledge of the resources to teach graphing, Alice expressed that she mainly rely on the textbook and Web pages. She added that she was used to handle maths books, but that the teaching practice has led her to include texts that are related to Education (E1.CA.72). Internet is used primarily to search for Web pages that provide information about the contents she teaches in statistics, such as "… schools or institutions where they are driving or working with education, and if they have statistic subjects, she looks for the way they manage these themes to get a bit more concerned" (E1.CA.74). She updates reading and consulting with other people she believes

they are specialized in statistics, but only "in terms of content, because how to teach I think they are like me" (E1.CA.115).

DISCUSSION AND CONCLUSIONS

The results of this study indicated that: 1) the teacher has a conception about the teaching centred in the transmission of mathematic knowledge that affects her way of teaching Statistics, which is based on a transmissive, rigid approach using a hierarchical structure of the contents; 2) her PCK corresponds to her initial formation, the textbooks she uses, the Internet pages she consults, and her shared experiences with other colleagues, as well as a lack of a specific training to teach Statistics; 3) her vision of the curriculum on graphical representation is focused on constructing graphs, performing statistical calculations, but does not deal with data interpretation and analysis; 4) the specific strategies she uses to teach graphical representation are: explanation and questioning problem solving in small groups, with few applications to the context of Education.

It was found demonstrated that her knowledge of instructional representations is associated in a significant way with her lack of reflection regarding the conceptions, mistakes and difficulties students have (Chinnappan & Lawson, 2005; An, Kulm & Wu, 2004). The results reveal that her PCK is influenced by her conception of mathematics and, consequently, she uses a limited repertoire of educational strategies for the teaching of graphical representation (Sorto, 2004, Monteiro & Ainley, 2006). Likewise, it is confirmed, from this study, that graphical representation is not used at the level of statistical thinking (Garfield, 2002). This is due to both, the PCK of the teacher and the school curriculum, which only asks for the most common graphs used for the construction and presentation of the results of research.

Finding of this research support the need for further analyse of teacher's PCK with mathematical instruction, teaching in different contexts than mathematics. This would help us to better understand teacher cognition as well as the origin and nature of PCK on the graphic representation in Statistics. The results suggests the need for creation of communities of practice, the accompaniment of teaching practice with the help of a mentor, changing curriculum for mathematics teaching in institutions that prepare teachers, the use of action-research to improve teacher PCK, and the development of virtual learning environments with a variety of alternatives to improve teachers training taking in account different dimensions and indicators of PCK.

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