CONCEPTIONS AND MISCONCEPTIONS OF AVERAGE: A COMPARATIVE STUDY BETWEEN TEACHERS AND STUDENTS

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The present study aims to analyze students' and teachers' conceptions and misconceptions of average. A paper and pencil test has been designed, comprised of three questions, which was undertaken by 287 Brazilian students and teachers. They were: 54 pupils from the 4th grade, 47 from 5th grade, 61 students beginning undergraduation on Pedagogy, 82 students concluding undergraduation on Pedagogy and 43 primary school teachers. The test was undertaken in a collective way. Except for the teachers of the group, all the groups tend to confuse average with addition of the values, or the maximum value. In spite of an evolution on the understanding of the concept of average according to the educational level of the groups, some misconceptions were found within the teachers' group.

Key-words: statistics literacy, average, misconception

STATISTICS LITERACY

The role of the school in citizens' formation has been the focus of research of Educators in Brazil and in the world. Results of large scale evaluations, such as the *Program for International Student Assessment* – PISA, have brought the Brazilian educational system to a concern. The PISA is a comparative evaluation program which its main objective is to produce registers about the effectiveness of the educational systems in students' formation to carry out an active role as citizens, in society. Instead of only knowing which content of curriculum students have learned, the focus of their evaluation is centered in establishing whether 15 years-old students enrolled in schools are able to use the school knowledge in usual situations of daily life. The PISA searches to evaluate the students' capacity to analyze, to think and to communicate effectively when they state, formulate and solve mathematical problems in such a variety of domains and situations related to daily life, defining this type of evaluation as "mathematical literacy".

The analysis of the nature of the problems which surround people's daily lives shows that, even more, involves statistical concepts and procedures, such as graphs, tables, averages, which subsidize people decisions. This knowledge about how to interpret and evaluate critically the statistical information has been denominated statistical literacy by various authors (Gal, 2002; Watson and Callingham, 2003).

In the concept of Gal (2002), an adult from an industrialized society is considered statistics literate, when he/she is able to interpret and evaluate critically statistical information discussing and communicating his/her understanding about the implications of that information and of the conclusions provided. On the base of this definition, basic statistics concepts and procedures are implicit, such as graphics and tables, measurement of central tendency and variability. Watson and Callingham (2003) proposed different hierarchical levels of the statistical literacy.

By recognizing the importance role played by school in the development of statistical thinking and competence in citizen formation, most countries include the study of basic statistical concepts and procedures in their Elementary Education curriculum (Batanero et al., 1994). Brazil has also approved this tendency and, throughout the *Parâmetros Curriculares Nacionais* for Elementary Education, officially introduced data handling from the first years of elementary school, as the fourth area of school mathematics (Brasil, 1997 e 1998) and more formal teaching of the statistics in Secondary Education (Brasil, 2002 e 2006).

However, the statistics teaching in Basic Education (elementary and secondary school) still faces many challenges. According to Batanero (2000), it is a paradox to demand of a mathematics teacher to teach Statistics, when in the undergraduation course; he/she had no specific teaching formation. Furthermore, the research on the Didactics of Statistics is starting and we have just been aware of the main students' difficulties. Besides, the research on Didactics of Statistics has just begun and we have just got across the main students' difficulties. These results will be available to teachers.

In this context, a comparative research has been developed to investigate competencies and conceptions of basic school students, undergraduate students and teachers, in relation to reading of tables, graphs and measures of central tendencies. In this article, we present only the results in relation to the comprehension of concept to media.

CONCEPT OF MEDIA

According to Pollatsek, Lima and Well (1981), the arithmetic average is not only a more basic concept of Statistics and of the experimental science, but also the most used in the every day life. In general, when inferring both in the academic field and in the every day life, we use average or comparison between averages.

Average provides a register that can be interpreted as a typical value and that can represent, in certain circumstances, a group of data. Besides that, it is a base for the calculus of other measures such as standard deviation, variation of factor, of correlation, amongst others.

For non gathered data, the simple average is calculated as a quotient between the sum of all the variable values and the number of observations involved in the sum. For pondered data or gathered, the values of the variable must be pondered by their respective weights or frequencies, in that case, the average is called pondered, such mathematical notations are shown as follow:

Simple Average $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ Pondered Average $\overline{X} = \frac{\sum_{i=1}^{n} f_i X_i}{n}$

where X_i are the assumed values by the variable, n is the number of the data and f_i is the weight or frequencies.

The concept of average is intimately related to the comprehension of the properties which according to Strauss and Bichler (1988) are:

a) the average is located between the extreme values
   (minimum value ≤ average ≤ maximum value);
b) the sum of the deviation from the average is zero (Σ(Xi – average)=0);
c) the average is influenced by each and by all the values (average = ΣXi/n);
d) the average does not necessarily coincide with one of the values which are composed by it.
e) the average may be a number which does not have a correspondent in the physical reality (for example, the average number of children per couple can be 2.3);
f) the calculation of average takes into consideration all the values including the negative and zero;
g) the average is a representative value of the data from which has been calculated. In spatial terms, the average is the value which is closer to all the values.

Knowing how to calculate the average does not imply its comprehension as shown in Batanero et al (1994). One of the difficulties is to interpret average when whole numbers are involved and the result is a decimal number, such as an average number of children by a couple equals to 2.3. The literature points out some reasons for this lack of comprehension. Watson (1996) and Selva and Borba (2005), researching with young children, points out to their lack on decimal number understanding. Cazorla (2003), investigating 840 students in undergraduation courses, showed that students' do not understand average as the ratio between two magnitudes: number of children and number of couples. In these studies, students, from elementary school to university level, do not consider two proprieties in average: (d) it does not coincide necessarily with one of the values of the variable; and (e) it can be a number without a physical correspondence in reality.

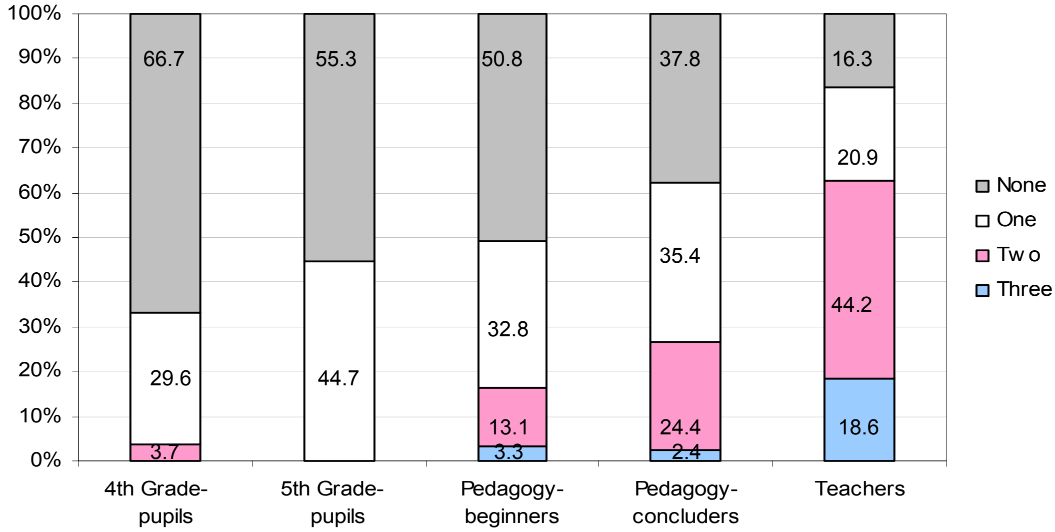
Another difficulty concerns the use of pondered average. Pollatsek and others (1981) have asked to 17 psychology students to calculate the average weight of 10 people in a lift, distributed as follow: four women with 120 pounds of average weight and six men with 180 pounds of average weight. The majority of the students have used the arithmetic average of 120 and 180. This result has also been found in Cazorla (2003) which, besides that, has found that 44.5% of her subjects have been limited to sum the weight of 10 people. The sum of the values of the variable in the place of the average is other conception (with no statistical validity) commonly observed in students' answers, what allows the inference that they do not know or take into account that the value of the average must be between the extreme values.

METHOD

In order to analyze the concepts of students and primary school teachers from the city of São Paulo – Brasil, an exploratory research has been drawn. A paper and pencil test was undertaken for 287 subjects in the city of São Paulo. The subjects were: 54 pupils of 4th grade (10 years-old), 47 of pupils of 5th grade (11 years-old), 61 students beginning undergraduation on Pedagogy, 82 students concluding undergraduation on Pedagogy and 43 primary school teachers. The test comprised 7 activities about reading, interpreting and constructing tables and graphs and 3 activities of the concept of average. It has been applied in a collective way and solved individually. The present work only refers to the analysis of three activities which involve the average (Annex 1).

RESULTS

Graph 1 shows the percentage of individuals who answered correctly between zero to three activities. We have observed that more than half of the students from the 4th and 5th grades and from beginners undergraduation students on Pedagogy have got no activity right, however, this rate increases in relation to concluders undergraduation students on Pedagogy and, even more, in the teachers group. Even though, it is possible to observe a percentage of 16.3% in this last category. On the other hand, we noticed that none of the groups of students reached a percentage of success in the three questions higher than 3.3%. Furthermore, the group of the teachers had stood out from the others, presenting a significantly higher performance, in spite of only 18.6 % have succeeded on the three questions.



Graph 1. Percentage of the individuals, according to the number of the correct answers per group.

By analyzing the subjects' performance, measured from the average of right answers, those differences have been statistically significant ($F(4,282) = 20.552$; $p = 0.000$). However, the only group that has presented a clearly higher performance to the others has been the teachers group, according to the Turkey test presented in Table 1.

Table 1. The subjects' development on activities of average by groups.

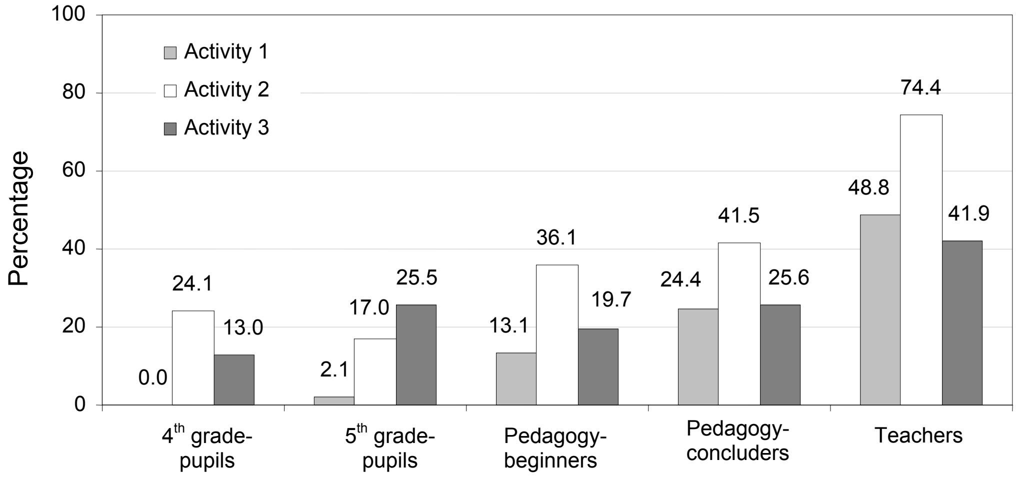
Groups	N	Average of score (*)	Standard deviation
4th grade pupils	54	0.37a	0.56
5th grade pupils	47	0.45ab	0.50
Beginners undergraduation students on Pedagogy	61	0.69 bc	0.83
Concluders undergraduation students on Pedagogy	82	0.91 c	0.85
Teachers	43	1.65 d	0.97
Total	287	0.80	0.87

(*) Averages with equal letters do not statistically differ according to Turkey test.

Graph 2 shows the performance of subject per activity. It is possible to observe that students from 4th and 5th grades have obtained a close performance and have not gone beyond 25.5% in none of the activities; students from the course of Pedagogy (beginners and concluders) have presented a slightly higher performance to the students from the primary grades of elementary school, although lower than 41.5% and the teachers have presented a much higher performance, mainly in Activity 2 (74.4%), even far from expected, as they have a degree level and frequently use the concept of average in their professional practices.

By analyzing the performance in the activities, we could notice that students encountered more difficulties in activity 1, specially 4th and 5th grades students. Activity 2 has presented a much less degree of difficulty amongst students, except for the ones from the 5th grade.

The results of activity 1, presented on Table 2, allow us to infer the lack of subjects' understanding of some of properties of average presented by Strauss and Bichler (1988). The first misconception to be identified was to consider average as the sum of the values of the variable. That conception, with no statistics validity, has been more frequent amongst 4h grade students and has decreased to 1.2% in the undergraduating students and has disappeared in the group of teachers.



Graph 2: Percentage of success in the activities per group.

Another conception of that type, which also has appeared in the group of students, with a decreasing trajectory, has been the understanding that the average can only result from constant values and equal to it, for example, the individual marks only the option (3,3,3) as correct, ignoring the other two with repeated values.

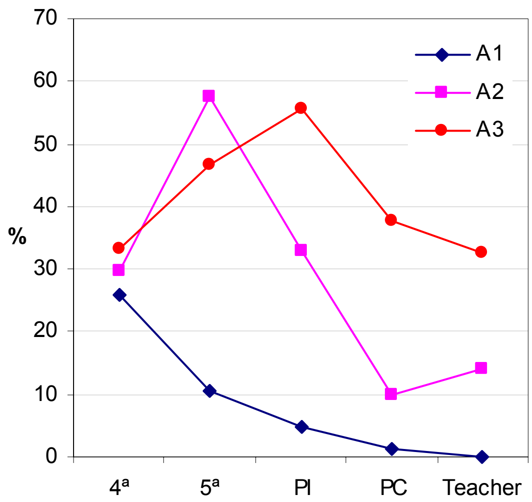
Table 2. Misconceptions in relation to the average according to the individuals.

Misconceptions	4th grade students	5th grade students	Pedagogy beginners	Pedagogy concluders	Teachers	Total
The average as a sum (1,1,1)	25.9	10.6	4.9	1.2	0.0	8.0
Mark only (3,3,3)	18.5	10.6	11.5	7.3	0.0	9.8
The average as a constant (1,1,1and 3,3,3 and 9,9,9)	5.6	0.0	13.1	1.2	4.7	4.9
The average cannot be constant (1,3,6 and 1,3,5 and 1,2,6)	0.0	14.9	6.6	3.7	0.0	4.9
The values do not overcome the average (1,1,1 and 3,3,3)	0.0	23.4	4.9	1.2	0.0	5.2
Multiples of 3 (3,3,3 and 9,9,9)	3.7	2.1	1.6	7.3	2.3	3.8
One of the numbers has to be the value of the average (3,3,3 and 1,3,6 e 1,3,5)	1.9	4.3	9.8	4.9	4.7	5.2
Blank	7.4	2.1	9.8	19.5	4.7	10.1

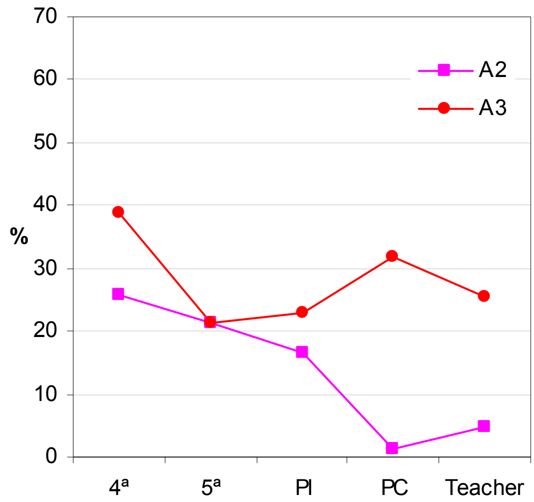
The lack of variability can also be another misconception, mainly to beginner students of Pedagogy. (13,1%), who marked the three alternatives in which the values were constant (1,1,1; 3,3,3 e 9,9,9). We have also identified another misconception, in the opposite way, which consider that the values cannot be constant (1,3,6 e 1,3,5 and 1,2,6), found in 14.9% of 5th grade students. Those students (5th grade) seem to believe that the values of the variable cannot overcome the value of the average, once 23.4% have marked the alternatives (1,1,1) and (3,3,3). Detecting conceptions has been more difficult in the group of the undergraduating students of

Pedagogy, as 19.0% have not answered the test. Two other alternative conceptions have appeared – *consider the average as a multiple of the average* and *consider that one of the values of the variable has to coincide with the average* – but they do not overcome 10.0%.

Graphs 3 and 4 show the results of the quantitative analysis of the activity. Once observing that the most frequent conceptions were the sum of the values or as a maximum point of those values, we decided to analyze the conceptions across the activities. Only 4th grade students have shown to take account the average as a sum of the three activities, for the other groups there is a big variation when compared to the three activities, the biggest different being amongst students from the 5th grade. It is important to emphasize, nevertheless, that the group of the teachers that have not presented that type of conception in activity 1, have presented it in activities 2 and 3, as shown in Graph 3.



Graph 3. The average as a sum.



Graph 4. The average as the maximum point.

Graph 4 also shows how different representation implies in different comprehension of the same concept, as it is possible to find different percentile of individuals who show comprehension that the average is the maximum point to all groups investigated by us. Considering the three activities, the individuals have succeeded better on the second, which presented data in a graph of bars and requested the calculus of the average, offering three possibilities of answer. It is possible to have the visual representation of a geometrical interpretation of the arithmetic average, so the graph seems to have helped on the comprehension of the average.

FINAL CONSIDERATIONS

The data analysis reveals that, in general, the average has presented as a difficult concept to be understood. We have observed a much bigger appropriation of that concept due to the education, but we have found that active teachers still present conceptions with no statistic validity.

Amongst the conceptions of that type, we have found a bigger number of individuals getting confused with the average and the sum of the values, what reinforce other studies (Pollatsek et al., 1988; Batanero, et al, 1994, Cazorla, 2003). However, what has called our attention is that incomprehension that occurred both amongst children and also amongst students of the undergraduation course (future teachers). Another common conception was noticed when the average got confused with the maximum value of the data, what seems to be related to the lack of comprehension of property of average, pointed out by Strauss and Bichler (1988) that the average can only take values from the extremes, what has also been detected, mainly amongst younger students. Another finding was that various individuals have shown to belief that the average has to coincide at least with one of the values. Such results were also shown with all other studies about average.

Those results have shown how much it still is necessary to invest on teacher trainings, mainly to undergraduation. It is also necessary to turn accessible the results of the researches to the teachers, so that they can find subsidies for their practice. This is necessary to unable them to develop a teaching methodology to lead students to understand the concept of average, and then, interpret and evaluate critically statistical information.

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ANNEX 1

Activity 1 (A1): Letícia is doing a research prices of snacks to buy the cheapest and save her allowance. She has written down prices from three different places and has found out that the average price of the snacks is $ 3.00.

Mark T (True) or F (False) for the possible values that Letícia has found:

a) ( ) 1,1,1 b) ( ) 1,3,5 c) ( ) 3,3,3
d) ( ) 1,3,6 e) ( ) 9,9,9 f) ( ) 1,2,6

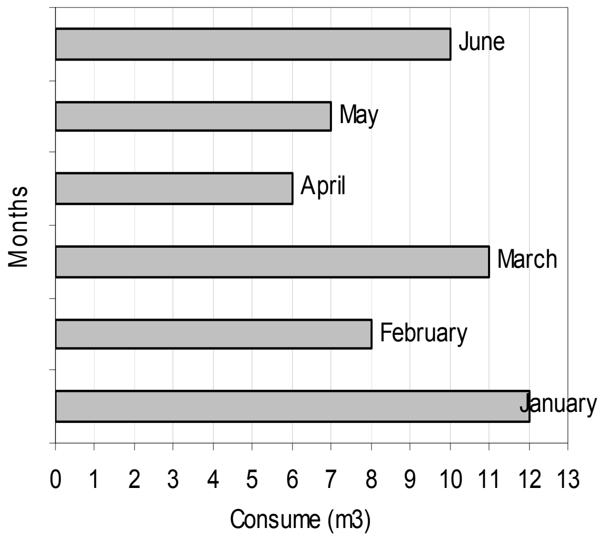
Describe the way you thought.

Activity 2 (A2): The graph shows the monthly use of water in a family during six months. Some students have calculated the average use of water of that family during six months. Read the conclusions of each student:

- João said that the average use has been $54m^3$.
- Carolina said that the average use has been $12m^3$.
- Marcelo said that the average use has been $9\ m^3$.

Who is right? ____________________

Why? ____________________________



Activity 3 (A3): Observe carefully the graph and answer:

What region in which the average of thefts is bigger?

( ) Northeast Region
( ) Southeast Region
( ) South Region
( ) All Regions
( ) None