MATHEMATICAL MODELING AND ENVIRONMENTAL EDUCATION[1]

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Abstract:

The objective of this research was to identify possibilities for using mathematical modeling as a resource in the education of future elementary and middle school teachers. Specifically, the modeling activities were developed as part of projects related to environmental issues. The study was carried out in four schools: two rural schools, one urban school, and a fourth located on an island of fishermen, all in the state of Paraná, located on the southern coast of Brazil. It was carried out in two stages: the first consisted of a Participatory Environmental Diagnosis (PED) of local environmental problems, and the second involved qualitative and quantitative discussions of a specific problem identified in the PED, selected by the teachers in each locale, and modeled mathematically. The results indicate that it is possible, in the process of mathematical modeling of environmental issues, to provoke critical reflections among teachers regarding the role of mathematics and its uses in environmental debates.

Keywords: mathematics education, mathematical modeling, environmental education, teacher education.

1. Introduction:

The objective of this research was to identify possibilities, through the elaboration of projects and use of mathematical modeling, for discussing the uses of mathematics to understand local environmental problems.

It was developed in two stages: the first was composed of a Participatory Environmental Diagnosis (PED); and the second involved the elaboration of mathematical models and quantitative/qualitative discussions focused on a specific problem identified in the diagnosis and selected by the teachers in each locale.

Four groups were formed, with a total of nineteen teachers: the first group was composed of seven teachers from an urban school in Guaraqueçaba; the second consisted of four teachers from a rural school in Tagaçaba; the third included three teachers from a rural school in Serra Negra. All of these are located in the municipality of Guaraqueçaba. The fourth group was composed of five teachers from a school located on an island of fisherman known as Ilha das Peças.

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2. The first stage: Participatory Environmental Diagnosis (PED)

The objective of the PED was to encourage the participation of teachers in local environmental issues. Ten one-day meetings were held in each locale at intervals of at least 15 days. The meetings included procedures of information and reflection outlined in IBAMA/SMA/UNICAMP (1998) and included the following types of activities:

a) *Conceptual presentations:* aimed to discuss basic concepts from the different fields of knowledge associated with environmental issues, such as: social sciences, biodiversity, and sustainable development, among others. This activity was carried out by the researcher.

b) *Group activities:* with the objective of integrating the theoretical base with information collected about the local reality, in order to share, deepen, and reflect on the information, creating a process of construction of a local vision. In this process, the teachers recognized the need to understand their own reality in their own depositions.

c) *Empirical research:* allowed the teachers to experience every stage of the participatory diagnosis, including collection of relevant information to characterize local, and occasionally regional, problems.

The work began in the four sites with a survey of the environmental problems carried out by the teachers in their locales, in a process of unveiling that included identification of the actors involved, reflection on the characteristics of a participatory environmental analysis, and the necessary steps to carry it out.

These steps were used with the aim of reflecting on, organizing, ordering, and systematizing the construction of a basic instrument to guide the empirical research in each locale. The first step was to identify difficulties encountered by the teachers in the elaboration of the DEP, and afterwards, offer guidance by responding to the questions: How is a participatory environmental diagnosis carried out? What does it contain?

Through the activities and discussions carried out in groups, the teachers began to reflect on what such a diagnosis consists of and identify steps, which included:

1st step: identify an environmental problem or problems in your locale. The main questions were:

a) What can be considered a problem?
b) How to proceed to seek consensus regarding the definition (or not) of a problem?

2nd step: consult the local population about the main problems; why they are considered problems, and what can be done to resolve them. The main questions were:

a) Who should be consulted?
b) What are the best methodological procedures for this consultation?

3rd step: relate the available information about the various aspects and points of view regarding the problems raised.

4th step: faced with the information obtained, identify the blanks remaining to be filled to completely unveil the problem.

The methodology for consultation adopted by the teachers for the survey was *informal conversations*, i.e., without the use of an audio-tape recorder, but based on a previously established interview guide. Basically, they sought answers to the following

questions: What is a problem? Why is it a problem? What solutions are proposed? The number of people interviewed depended on the time the teachers had available to carry out the survey.

3. Results of the first stage:

As a result of the first stage of this study, in which the teachers carried out the DEP, the following environmental problems were identified in each locale:

Table 1. Survey of environmental problems in the study sites.

Problems/locales	Ilha das Peças	Guaraqueçaba	Serra Negra	Tagaçaba
Garbage	X	X		X
Sewage treatment	X	X	X	X
Poaching of wild animals		X	X	
Palm-heart cutting		X	X	
Stationary telephone poles	X	X		X
Transportation/roads		X	X	X
Flooding	X			
Burning		X	X	
Crabs	X	X		
Hygiene		X	X	
Leisure	X			X
Water	X	X		
Predatory fishing	X	X		
Education	X			X
Health	X	X		X
Loose animals	X	X		
River pollution		X		X
Snails	X	X	X	X
Deforestation		X		
Sanitary inspection		X		X
Security	X	X		
Urban inspection		X		

Following the completion of the DEP, with the local environmental data in hand, the second stage of the research began. Mathematical modeling processes were carried out aiming to construct the instruments of qualitative/quantitative understanding of the environmental realities, experienced through mathematical concepts, resulting from the search for solutions to the questions reflected in the themes. The following themes were selected:

Table 2. Themes selected by the teachers.

Locales/ Projects	Water	Garbage	Hygiene
Ilha Da Peças	X		
Guaraqueçaba		X	
Serra Negra			X
Tagaçaba		X	

4. Second stage of the research: Mathematical Modeling

The work was carried out by the same groups in the second stage of the research, using pedagogical assumptions based in mathematical modeling.

According to MEYER & CALDEIRA (2001), although many approaches are referred to as mathematical modeling, most of them include:

1st stage: the *formulation* of the question, in which the critical attitude is revealed at the moment the essential aspects of each problem are selected for inclusion in the mathematical model. This formulation includes the establishment of the question itself, but also its expression in a language from the mathematical universe, i.e, a mathematical problem.

The following mathematical questions were constructed by the teachers in each locale:

- Ilha das Peças: What would be the capacity of a water deposit capable of meeting the needs of the island for one day?
- Guaraqueçaba**:** What would be the quantity and weight of the paper, plastic, and pencil shavings discarded during one week (Monday to Friday) in the following classrooms: pre-school – 1st grade; 2nd grade; 3rd grade; 4th grade in the municipal school?
- Tagaçaba: How much garbage is produced in Tagaçaba?
- Serra Negra: What is the hygiene situation of the 1st, 2nd, and 3rd grade students in the Serra Negra Municipal School?

Collection of data to try to answer these questions was carried out entirely by the teachers, in some cases with the participation of their students and, in the case of Tagaçaba, with the participation of the community in the collection and selection of garbage.

2nd stage: *resolution*. The resolution of the problem that was expressed mathematically was, obviously, only approximate. Here a critical view of the adequacy of the mathematical instrument is also necessary, given the use of a mathematical instrument for non-mathematical ends: mathematics as a means of understanding reality. The same critical attitude is also necessary in the evaluation of the precision of the response acquired, as well as the evaluation of the results.

To find solutions to the questions, the mathematical concepts used were basically operations with natural numbers (addition, subtraction, multiplication, division), measures (perimeter, area, and volume), geometry (plane and spatial figures), and information handling (graphs and tables).

3rd stage: *evaluation.* In addition to evaluating the mathematical results for the problem studied, it is also necessary to critically analyze the adequacy of this solution as a response to the concerns of the community: its problems, life, quality of life, and environment. While objective characteristics were highlighted in the process of evaluation, subjective aspects were also raised, as the evaluation process is composed not only of the mathematical evaluation, but can also include evaluation processes that are important for the community problem – and may even involve the solution of the problem by the community itself. This assumes taking a position, making a commitment, and critical engagement, which led us back to the beginning of the process, given the context of the starting point, which is necessarily inserted in a dynamic environment and can, therefore, lead to problems that are studied and abandoned, or problems that continue to be restudied. In the case of Ilha das Peças, for example, the work continued after the project ended, and the water deposit was built by the inhabitants of the island.

In summary, based on the construction of the diagnosis, we began with an environmental problem selected by the teachers, which was then modeled mathematically and, as such, understood in a new way. In the attempt to resolve the problem proposed by the model, the mathematical contents became mobilized and were used as mathematical tools, as means for achieving a larger goal: quality of life.

Each teacher's experience with learning mathematics was a determining factor in the establishment of the mathematical contents and methods needed to determine the solutions to the mathematical questions. On the other hand, these solutions were obviously not the only ones, given the fact that they were determined by the themes and contents chosen by each group.

4.1 Development of the group work.

4.1.1 Ilha das Peças

The theme chosen by the teachers on Ilha das Peças for the mathematical modeling project, as shown in Table 2, was "Water". This choice was motivated by the experience of the island's inhabitants, where potable water is often in short supply or unavailable. Due to the many septic tanks installed on the island, the quality of the potable water has become totally compromised. The alternative sought by inhabitants of the island, despite their location in the middle of the Atlantic Forest, is to pipe drinking water from the continent.

This situation led us to reflect on the possibility of having a water cistern on the island. Thus, the mathematical problem was to find an answer to the following question: *What capacity would the cistern need to have to meet water supply needs on the island for one day?*

The first answer was to determine the number of people that use water in the island. This led to a discussion regarding the flow of people present on the island – local residents and tourists. Thus, we initially raised the following questions:

1. *What is the number of houses of residents and of tourists on the island?;*
2. *What is the flow if people during high and low tourist seasons?;*
3. *What is the total number of people who use potable water on the island in one year?;*
4. *How much water does each person use per day?;*
5. *How much water is consumed per day in the maintenance of a home?*

The first answers began to emerge according to the teachers' knowledge regarding the locale, as a result of intuition and experience acquired in some activities. The opinions of children and students were also taken into account in some cases. Based on the responses, some tables were created to provide us with an overview of the data.

Faced with these data, we carried out various simulations, for example:

1. Quantity of water used per day by local residents for personal hygiene and home maintenance;
2. Quantity of water consumed on weekends;
3. Quantity of water used on a festival on the island, when approximately five thousand people gather.

Thus, we constructed various simulations and always interpreted them in terms of real situations.

The first idea was to work with the arithmetic (number concept and basic operations) needed to arrive at a better understanding of the phenomenon, as well as to show that the mathematics taught in the early grades is necessary to achieve this understanding.

The second step was to seek an answer to the initial question: *What size of cistern is needed to meet the water supply needs on the island for one day?*

It was decided to take into consideration only the local residents. This led us to build a model (using data collected by the teachers) of a cistern with an approximately 90,000 liter capacity.

At that moment, we began to notice that arithmetic alone would not suffice. It was necessary to introduce concepts of geometry (perimeter, area, volume). In the end, we carried out some simulations with cisterns or various shapes and sizes, and concluded that it should measure 3m x 5m x 6m.

4.1.2 Guaraqueçaba

As shown in Table 2, the theme chosen by the teachers was garbage in the classroom. The procedure followed for the research was as follows: at the end of every class, the teachers responsible for the class swept the floor, collected the garbage, counted it, weighed it using a letter scale, and wrote the information down in a table, thus creating tables and graphs.

This was done for all the classes involved in the research. All together, the following tables and graphs were made:

1. quantity and weight of paper collected per day in each classroom;
2. quantity and weight of plastic collected per day in each class;
3. weight of pencil shavings of each grade per day;

All graphs were created without using any technology. We had no computers in the school, so they were created using graph paper, and the results were discussed with the teachers.

The results showed a significant amount of garbage produced in the classrooms, and led to a discussion about the importance of minimizing garbage production in and out of the classroom.

4.1.3 Tagaçaba

As presented in Table 2, the problem chosen by this group was garbage in the locale. The teachers of this group were interested in addressing the following question: *How much recyclable garbage is produced in Tagaçaba?*

Other questions arose from this initial question, such as:

How much money would be raised by selling the recyclable garbage produced in one year?

How much would it cost to buy the school?

The development of the modeling process involved a sample of ten houses in Tabaçaba, and for 30 days, the teachers gathered garbage, separated the paper, plastic, glass, and aluminum, and weighed the material, creating tables with the data.

After the table was created, we discovered the price per kilo of each type of material: plastics, paper, glass, and aluminum, and then proceeded to answer the following question: How much would the teachers earn if they sold the garbage collected in one year?

Based in the results, it was concluded that, despite the very low value, it was nonetheless worthwhile to recycle garbage as a way to preserve the environment and reuse materials that become garbage in the community.

4.1.4. Serra Negra

Among the environmental problems detected in Table 2, the teachers of Serra Negra chose the theme of personal hygiene among students.

This choice of theme was due to the teachers' perception that many students were coming to class without meeting conventional standards for hygiene: soiled clothing, unbathed, hair uncombed, smelling of urine, among other things. There was a certain resistance in the group to working with this theme as we agreed that it was not very related to mathematical issues. We nonetheless insisted on working with the theme, although we did encounter difficulties.

To obtain the data, the following steps were followed:

1. Elaboration of a close-ended questionnaire about various forms of hygiene for the students to fill out;
2. Tabulation of the data and transformation into graphs.

The questionnaire addressed the following themes about the students: sanitary conditions in their homes; bathing conditions; clothing; towel use; use of toothbrush; use of toilet paper; among others. The results pointed to possibilities for discussing the importance of personal hygiene with the students.

The data were tabulated and then transformed into tables and graphs. The next step was to carry out some mathematical simulations with the period of dental treatment of children. In this way, the teachers identified and resolved the following problems:

1. Knowing that the dentist is available three times per week and treats three children per day, how long would the last child have to wait to be treated by the dentist?
2. How many children would the dentist have to see per day in order to see all the children in one month?

After resolving all the problems, we discussed the viability of more timely dental treatment for the children.

5. Discussion of the Results:

The themes raised in the PED increased the teachers awareness regarding the indispensable partnership of the school in the search for solutions to questions faced by the community. The choice of the theme by the teachers was unanimous, as they recognized the urgent need to discuss and resolve the problem identified.

This approximates us to the ideas of SKOVSMOSE (2001) when he calls attention to the role of mathematics in society, and shows us the need to bring mathematics education and critical education closer together. Such an education is characterized by the critical engagement of teachers (and students) in the process, with dialogue and the teacher-student relationship occurring in a democratic process.

The study also showed that it is possible, in the process of learning about mathematics and environmental issues, for teachers to develop a critical competence, without it being imposed, but rather constructed through the experiences of the actors themselves; and that it is also possible to discuss a curriculum that questions the functions of school contents, the interests and assumptions reflected in these contents, and finally, the possible social interventions they can catalyze (or slow down), and under what circumstances.

The approach adopted for the study, based on the theoretical-methodological assumptions of mathematical modeling, showed us that it is possible for teachers to become involved in situations in their social context, and that the problems encountered there, when perceived as relevant, and because they are objectively existent social problems, lead to the critical engagement of the teachers in their communities.

From this perspective, teaching would no longer be developed through lectures and repetitive classroom activities, in the insulation of school buildings, but would situate the teacher in a process of reflection-formulation-action that leaves behind individuality in favor of the action of studies (CALDEIRA, 1998; BARBOSA, 2001; MONTEIRO, 1991; BORSSOI, & ALMEIDA, 2002).

Modifying the teachers' customary daily work habits of class preparation based on textbooks, this process required them to identify specific environmental phenomena that could inform their practice, and work not only with environmental concepts, but also mathematics, to interpret the situations presented to them.

The result was that teachers no longer tried only to respond to questions in the textbooks for their students, but constructed their own questions, as well as attempting to respond to them. Herein lies the critical-creative seed defended by D' AMBRÓSIO (1996).

For this to occur, a proper environment, where the focus was on the research, was necessary. The teachers were researchers of the environmental problems as well as the mathematics needed to understand them in a more meaningful way. This process of curiosity and challenge is what motivated them to do the work.

In the sphere of relations between the School and Society, we perceived, through the study, that the school is neither the neutral institution depicted by the ideas of functionalism, defended by Talcott Parsons, in which the school is capable of bringing an end to social inequality through the transmission of norms, values, and knowledge that assure social integration; nor does it play the role characterized by reproductionism, based on the theoretical studies of Bourdieu and Passeron, Althusser, Baudelot, and Establet, and of Bowles and Gintes (MOYSES, 1997), who see the school as reproducing social

inequalities and serving as ideological apparatus for the State, destined to perpetuate the system. In our case, the function of the school more closely approximated the ideas put forth by the so-called *critical theories,* which emerged in the 1980s, and sought to recover the positivity of the aforementioned trends, seeking to overcome the innocent fragility of functionalism as well as the immobility present in the reproductionist theories (GIROUX, 1986; 1997).

Within this conception, it is possible to verify the value that the school should have without falling into the notion of neutrality or uselessness for social transformation. As stated by CORTELLA (2001; p.136), "yes, the School can serve to reproduce injustices, but concomitantly, it is also capable of functioning as an instrument for change; the elite use it to guarantee their power, but because they are not aseptic, it can also serve to confront them".

In the context of the present study, school education and teachers have relative autonomy; it was possible to see the relation between School and Society as two-way; not completely independent, as seen by the functionalists, nor entirely dominant, as seen by the reproductionists. For this reason, the study made it possible to journey through this contradiction, always creating opportunities for building effective spaces for innovation in the educational practice developed by each teacher in his/her classroom.

Regarding the pedagogical aspects, in addition to communing with the ideas of mathematical modeling, we also identified very much with what is being denominated "Ecopedagogy" (GUTIÉRREZ & PRADO, 1999; GADOTTI, 2000; PADILHA, 2004), whose main theoretical basis lies in the struggle for a culture of sustainability, promotion of meaningful learning, and attributing meaning to everyday actions. The study closely approximated the work of those authors who defend a pedagogy of questioning, democratic and committed to solidarity, that invites teachers and students to guarantee the sustainability of each of our everyday acts as human beings who share life on this planet with other living beings.

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